

Mixed Integer Programming for Time-Optimal Multi-Robot Coverage Path Planning with Efficient Heuristics*

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Introduction

We study the Multi-Robot Coverage Path Planning (MCPP), which aims to coordinate the paths of multiple robots to completely cover the given terrain. A fundamental challenge of MCPP lies in generating cost-balancing coverage paths to optimize task efficiency, commonly quantified by the *makespan* (i.e., the maximum path cost of all robots). This challenge is further compounded when dealing with large-scale applications where the number of robots and the size of the terrain increase. We follow existing graph-based MCPP algorithms (Zheng et al. 2010) that represent the terrain to be covered as a 4-connected 2D grid graph G and then leverage STC (Gabrieli and Rimon 2001) to generate coverage path on a decomposed graph D of G for each robot by circumnavigating a subtree of G in Fig. 1. The STC paradigm effectively reduces MCPP into the Min-Max Rooted Tree Cover (MMRTC) problem, which results in an MCPP solution with an asymptotic optimality ratio of 4. We propose a Mixed Integer Programming (MIP) model to optimally solve MMRTC and prove its correctness. We design two efficient suboptimal heuristics to reduce the model size with a configurable loss of optimality. We prove that the two reduced-size models are complete (i.e., guarantee to find a solution if one exists) for all MMRTC instances. Our extensive experiments show that our MIP-based MCPP planner yields higher-quality solutions at the cost of longer runtime.

Our Approaches

In an MCPP instance (G, D, R) , we are given a terrain graph $G = (V, E)$ and its corresponding decomposed graph $D = (V_d, E_d)$, where each $v \in V$ is decomposed into four small adjacent vertices in V_d . Given a set $I = \{1, 2, \dots, k\}$ robots with a set $R = \{r_i\}_{i \in I} \subseteq V$ of initial root vertices, the graph-based MCPP problem is to find a set $\Pi = \{\pi_i\}_{i \in I}$ of k paths such that each $v \in V_d$ is visited by at least one coverage path for complete coverage and each π_i starts and ends at a decomposed vertex (e.g., the left-top one) of $r_i \in R$. The coverage time is thereby measured by the makespan, defined to be the maximum path cost among all paths in Π . For an MCPP instance (G, D, R) , its corresponding MMRTC in-

stance aims to find a set of k subtrees $\{T_i\}_{i \in I}$ such that each T_i must be rooted at $r_i \in R$ and each vertex $v \in V$ is included in at least one subtree. We define the weight $w(T_i)$ of any T_i as the sum of the weight w_e of every edge e in T_i . The optimal set of subtrees is the one that minimizes the maximum weight among all subtrees (i.e., *makespan*):

$$\{T_i^*\}_{i \in I} = \arg \min_{T_1, T_2, \dots, T_k} \max\{w(T_1), w(T_2), \dots, w(T_k)\} \quad (1)$$

MIP Formulation for MMRTC: We introduce two sets of binary variables $\mathbf{x} = \{x_e^i\}_{e \in E, i \in I}$ and $\mathbf{y} = \{y_v^i\}_{v \in V, i \in I}$, where x_e^i and y_v^i take value 1 if edge e or vertex v is included in the i -th subtree T_i , respectively, and 0 otherwise. Assuming that each edge has one unit of flow, we further introduce a set of non-negative continuous flow variables $\mathbf{f} = \{f_{e,u}^i, f_{e,v}^i\}_{e \in E, i \in I}$ to represent the amount of flow assigned to vertices u and v for each edge $e = (u, v) \in E$. Let τ denote the makespan and $e \sim v$ denote that v is one of the endpoints of edge e . Our MIP model for MMRTC is formulated as follows:

$$\text{(MIP)} \quad \underset{\mathbf{x}, \mathbf{y}, \mathbf{f}, \tau}{\text{minimize}} \quad \tau \quad (2)$$

$$\text{s.t.} \quad \sum_{e \in E} w_e x_e^i \leq \tau, \quad \forall i \in I \quad (3)$$

$$\sum_{i \in I} y_v^i \geq 1, \quad \forall v \in V \quad (4)$$

$$y_{r_i}^i = 1, \quad \forall i \in I \quad (5)$$

$$\sum_{v \in V} y_v^i = 1 + \sum_{e \in E} x_e^i, \quad \forall i \in I \quad (6)$$

$$f_{e,u}^i + f_{e,v}^i = x_e^i, \quad \forall e = (u, v) \in E, \forall i \in I \quad (7)$$

$$\sum_{\substack{e \in E \\ e \sim v}} f_{e,v}^i \leq 1 - \frac{1}{|V|}, \quad \forall v \in V, \forall i \in I \quad (8)$$

$$x_e^i \leq y_v^i, \quad \forall v \in V, \forall e \in E, e \sim v, \forall i \in I \quad (9)$$

$$x_e^i, y_v^i \in \{0, 1\}, \quad \forall v \in V, \forall e \in E, \forall i \in I \quad (10)$$

$$f_{e,u}^i, f_{e,v}^i, \tau \in \mathbb{R}^+, \quad \forall e = (u, v) \in E, \forall i \in I \quad (11)$$

where the constraints in the model can be grouped as follows: a) *Makespan*: Eqn. (3) ensures that τ equals the maximum weight among all subtrees, which is minimized in the objective function defined in Eqn. (2); b) *Cover*: Eqn. (4) enforces that each $v \in V$ is included in at least one subtree;

*Code: <https://github.com/reso1/MIP-MCPP>

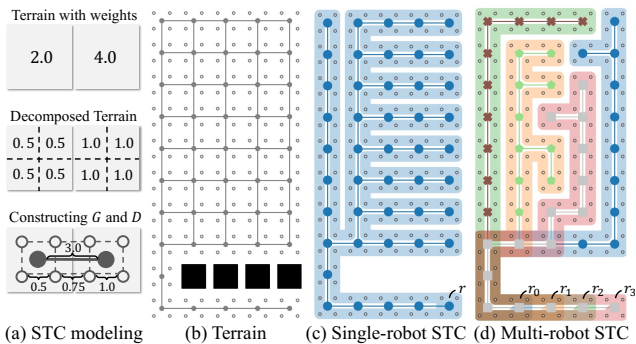


Figure 1: Graph-based STC in the instance: (a) local view of STC modeling of a weighted terrain to be covered; (b) black squares, color-filled markers, and hollowed circles represent obstacles, vertices of G , and vertices of D , respectively; (c)(d) the rooted subtrees (thin lines) in G and the resulting coverage paths (thick lines) in D .

c) *Rooted*: Eqn. (5) enforces each T_i is rooted at $r_i \in R$; d) *Tree*: Eqn. (6) ensures that each T_i is either a single tree or a forest with cycles in some of its trees, while Eqn. (7) and (8) eliminate any cycles in T_i . Together, these constraints ensure that any subtree is a single tree. With Theorem 1 proven in our full paper, we ensure that any solution of our model is feasible for its corresponding MMRTC instance.

Efficient Suboptimal Heuristics: We propose two heuristics, namely Parabolic Removal Heuristic (PRH) and Subgraph Removal Heuristic (SRH) in Fig. 2, to reduce the complexity of the above MIP model while sacrificing the optimality. Both heuristics work by generating an inferior graph H_i for each subtree T_i and preventing T_i from covering the vertices of H_i , and then replace the original terrain graph G for each T_i in the MIP model with a residual graph obtained by removing all the vertices and edges of H_i from G . For each T_i , H_i is generated by identifying its *sub-component* H_{ij} concerning each subtree T_j with $j \in I/\{i\}$ such that the vertices of H_{ij} are not to be included in T_i , since they are closer to the root r_j of T_j and thus inefficient to be covered by T_i . Specifically speaking, for PRH, it builds a parabola Ω_{ij} with r_i as its base and a hyperparameter α to control the parabola size, and then regards the graph induced by the inner area of Ω_{ij} be the sub-component H_{ij} of H_i for arbitrary $i, j \in I, i \neq j$. For SRH, it uses a Farthest-First-Search (FFS) to generate each sub-component H_{ij} to further construct H_i . The FFS is just a Breadth-First-Search starting from r_i with the queue prioritized by the distance from each vertex to r_i and a hyperparameter β to control the FFS tree size. Using Lemma 2 in the full paper, we proved that the reduced-size MIP models with both heuristics still ensure complete coverage.

Empirical Evaluation

We compare the coverage time and runtime of our MIP-based MCPP planner with state-of-the-art MCPP planners, MFC (Zheng et al. 2010), and MSTC* (Tang, Sun, and Zhang 2021). All MIP models are solved with warm-startup

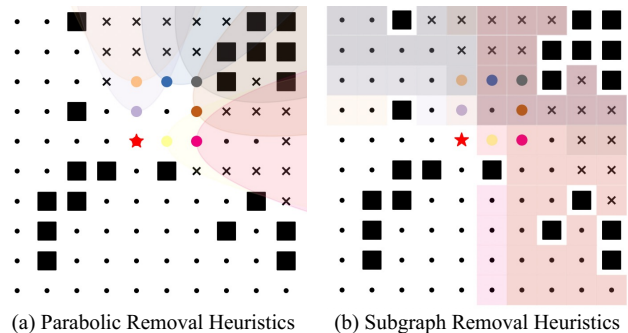


Figure 2: Proposed suboptimal heuristics on the root vertex (red star) to all other root vertices (colored circles). Black squares, black crosses, and black dots are obstacles, inferior graph vertices, and residual graph vertices, respectively.

which uses a valid initial solution to accelerate the model solving, and their reported runtimes include the computation time of their respective warm-startup solutions. Overall, our MIP-based MCPP planner consistently delivers superior solution quality at the cost of longer runtime, resulting in an average reduction in coverage time of 27.65% and 23.24% compared to MFC and MSTC*, respectively. The performance improvements are significant across different instance types. Specifically, the average reduction ratios compared to MFC and MSTC* are 22.61% and 41.03% for *maze* instances, 35.65% and 21.62% for *floor* instances, and 25.43% and 11.11% for *terrain* instances, respectively. Notably, our MIP-based planner performs exceptionally well for *maze* instances with relatively shorter runtimes. However, when the root vertices are clustered, particularly in a straight line, the proposed MIP models may result in a smaller reduction in coverage time. This is due to the increased likelihood of the inferior graphs of roots coinciding, requiring more time to converge.

Acknowledgements

This extended abstract is a short version of (Tang and Ma 2023) and was supported by the NSERC under grant number RGPIN2020-06540 and a CFI JELF award.

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