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CoRe Challenge 2022/2023: Empirical Evaluations for Independent Set Reconfiguration Problems (Extended Abstract)

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Introduction

Combinatorial Reconfiguration (Ito et al. 2011; Nishimura 2018; van den Heuvel 2013) is an algorithmic concept that provides mathematical models and analysis for "transformations over state spaces." It has been extensively studied over the decade, mainly from the viewpoint of algorithmic theory. However, despite its significance, its practical research has not been extensively conducted so far. Therefore, there are still intriguing questions remaining. For example, (i) how can large instances be practically solved? (ii) What kind of instances are difficult? (iii) Which techniques are effective?

CoRe Challenge 2022/2023 [1,2] is an international competition series organized by the authors of this extended abstract to make initial answers to questions (i)–(iii) above. It aims to construct the technical foundation of practical research for Combinatorial Reconfiguration. Before this competition series, there did not exist benchmark instances, their formats, state-of-the-art solvers, etc. for Combinatorial Reconfiguration, although they are indispensable to conduct practical research. As the target problem of the competition series, we selected the independent set reconfiguration problem under the token jumping model (Kamiński, Medvedev, and Milanič 2012), which is one of the most well-studied reconfiguration problems (Bousquet et al. 2022; Nishimura 2018). Henceforth, we will refer to it as TOKEN JUMPING since this problem is often referred to as so. TOKEN JUMPING is known to be PSPACE-complete (Kamiński, Medvedev, and Milanič 2012), which implies that there exist instances such that even a shortest transformation requires a super-polynomial steps with respect to the input size under the assumption of NP$\neq$ PSPACE. The problem has been utilized to prove the PSPACE-completeness of several other reconfiguration problems. In this sense, TOKEN JUMPING is a theoretically central problem and is thus important, similar to the SAT problem for NP-complete problems.

We emphasize that CoRe Challenge 2022/2023 has accelerated practical research for Combinatorial Reconfiguration. For example, several papers have recently been published by participating teams (Bousquet et al. 2023; Christen et al. 2023; Hirate et al. 2023; Ito et al. 2023; Toda et al. 2023; Yamada et al. 2024).

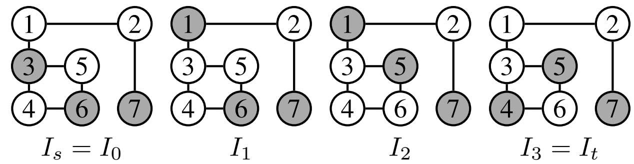


Figure 1: Instance/solution examples of TOKEN JUMPING.

TOKEN JUMPING

An *independent set* of an undirected graph G is a vertex subset of G such that no two vertices are adjacent. The term "token" means the same as a vertex in an independent set, establishing a one-to-one correspondence. A *reconfiguration sequence* under the *token jumping rule* shifts one independent set to another of the same size, changing one token at a time. The *length* of the sequence is the count of token moves.

Given an undirected graph G and two equal-sized independent sets I_s and I_t of G, the goal of TOKEN JUMPING (a.k.a., the independent set reconfiguration problem under the token jumping rule) is to decide the existence (yes or no) of a reconfiguration sequence between I_s and I_t under the token jumping rule. Figure 1 illustrates an example of TOKEN JUMPING. The graph consists of 7 vertices and 7 edges, and two independent sets are given as $I_s = \{3, 6, 7\}$ and $I_t = \{4, 5, 7\}$. For this instance, a reconfiguration sequence under the token jump rule exists. The sequence $\langle I_s, I_1, I_2, I_t \rangle$ illustrates one of the shortest sequences.

Highlights of the Results

We created and used 11 benchmark sets with 693 instances for evaluation. From the results of CoRe Challenge 2023, Table 1 summarizes the results of three representative solvers: AI-planning-based solver PARIS, BMC-and-ASP-based solver NuASP, ZDD-based solver ddreconf. The results in Table 1 are on the track for the shortest reconfiguration and single engine solvers with 30 minutes CPU time-limit. In turn, Figure 2 summarizes Graph track results, which take vertex numbers as inputs and aim to find instances of the specified numbers of vertices that need longer

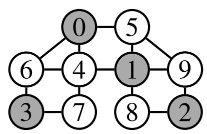
[1]https://core-challenge.github.io/2022/
[2]https://core-challenge.github.io/2023/

Benchmark sets	PARIS (AIP)	NuASP (BMC)	ddreconf (ZDD)
handcrafted	**5**	**5**	**5**
color04	198	**200**	76
queen	**46**	25	8
grid	**2**	**2**	**2**
ph-isr	**1**	0	**1**
square	6	0	**17**
power	4	0	**11**
sp	9	2	**15**
exp	9	5	**17**
wide	0	0	**53**
random	140	**194**	36
Total	420	**433**	241

Table 1: Results on the track for the shortest reconfiguration and single engine solvers with 30 min. CPU time-limit. Each figure shows the number of shortest reconfigurations found.

#Vs.	Length
10	*10
17	30
31	231
50	3069
59	11499
100	3145725

(a) Best results



(b) an optimal TOKEN JUMPING instance for 10 vertices: $\{0, 1, 2, 3\}$ to $\{2, 3, 4, 5\}$

Figure 2: Results on Graph track: (a) shows the list of the best results, where #Vs. means the number of vertices. Note that, for 10 vertices, three instances are proven to be optimal by (Bousquet, Durain, and Pierron 2022), i.e., the length 10 is the longest length of shortest reconfiguration sequences among all instances of 10 vertices.

shortest reconfiguration sequences.

Conclusion

This extended abstract introduces CoRe Challenge 2022/2023, an international competition series aiming to construct the foundation of practical research for Combinatorial Reconfiguration. To initiate that research, we offered 11 benchmark sets with 693 instances, their format, and the list of state-of-the-art solvers, and made them available through the competition series. With a 30-minute CPU time-limit, we could answer the initially discussed questions in Introduction: (i) The ZDD-based solver, ddreconf, found the longest shortest reconfiguration sequence of length 442175. The instance with the biggest graph, having 40000 vertices, was solved by PARIS. (ii) Instances requiring long reconfiguration sequences are usually hard in the sense that those are rarely solved by the participated solvers. (iii) Solvers vary in effectiveness, with no single best approach. These answers may be updated in the future, but together with the benchmark sets, formats, and lists of state-of-the-art solvers provided in the competition series, they can be expected to form the foundation of practical research. All materials are accessible on the CoRe Challenge 2022/2023 websites [1,2].

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References

Bousquet, N.; Durain, B.; and Pierron, T. 2022. Graph track description. In *Core Challenge 2022: Solver and Graph Descriptions, CoRR, abs/2208.02495*, 35–37.

Bousquet, N.; Durain, B.; Pierron, T.; and Thomassé, S. 2023. Extremal Independent Set Reconfiguration. *Electron. J. Comb.*, 30(3).

Bousquet, N.; Mouawad, A. E.; Nishimura, N.; and Siebertz, S. 2022. A survey on the parameterized complexity of the independent set and (connected) dominating set reconfiguration problems. *CoRR*, abs/2204.10526.

Christen, R.; Eriksson, S.; Katz, M.; Muise, C.; Petrov, A.; Pommerening, F.; Seipp, J.; Sievers, S.; and Speck, D. 2023. PARIS: Planning Algorithms for Reconfiguring Independent Sets. In *Proc. ECAI 2023*, volume 372 of *Frontiers in Artificial Intelligence and Applications*, 453–460. IOS Press.

van den Heuvel, J. 2013. The complexity of change. In *Surveys in Combinatorics 2013*, volume 409 of *London Mathematical Society Lecture Note Series*, 127–160. Cambridge University Press.

Hirate, T.; Banbara, M.; Inoue, K.; Lu, X.; Nabeshima, H.; Schaub, T.; Soh, T.; and Tamura, N. 2023. Hamiltonian Cycle Reconfiguration with Answer Set Programming. In *Proc. JELIA 2023*, volume 14281 of *LNCS*, 262–277. Springer.

Ito, T.; Demaine, E. D.; Harvey, N. J. A.; Papadimitriou, C. H.; Sideri, M.; Uehara, R.; and Uno, Y. 2011. On the complexity of reconfiguration problems. *Theoretical Computer Science*, 412(12–14): 1054–1065.

Ito, T.; Kawahara, J.; Nakahata, Y.; Soh, T.; Suzuki, A.; Teruyama, J.; and Toda, T. 2023. ZDD-Based Algorithmic Framework for Solving Shortest Reconfiguration Problems. In *Proc. CPAIOR 2023*, LNCS 13884, 167–183. Springer.

Kamiński, M.; Medvedev, P.; and Milanič, M. 2012. Complexity of independent set reconfigurability problems. *Theoretical Computer Science*, 439: 9–15.

Nishimura, N. 2018. Introduction to Reconfiguration. *Algorithms*, 11(4): Paper id 52.

Toda, T.; Ito, T.; Kawahara, J.; Soh, T.; Suzuki, A.; and Teruyama, J. 2023. Solving Reconfiguration Problems of First-Order Expressible Properties of Graph Vertices with Boolean Satisfiability. In *Proc. ICTAI 2023*, 294–302. IEEE.

Yamada, Y.; Banbara, M.; Inoue, K.; Schaub, T.; and Uehara, R. 2024. Combinatorial Reconfiguration with Answer Set Programming: Algorithms, Encodings, and Empirical Analysis. In *Proc. WALCOM 2024*, LNCS 14549, 242–256. Springer.