Uncertainty and Dynamicity in Real-World Vehicle Routing (Student Abstract)

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Abstract
Interest in vehicle routing problems (VRP) with stochastic and dynamic elements has grown in the past decade. Despite numerous contributions in this area, the handling of uncertainties and dynamic changes in complex VRPs received little attention. Based on our experience from industrial practice, we discuss why accounting for uncertainties and dynamic changes is crucial for the applicability of the produced routing plans. Then, we first identify and justify the best-suited direction to address dynamicity and uncertainties in real-world VRPs. Second, we outline the key concepts and ideas of our approach to finally demonstrate that it is realistic to implement them efficiently.

Introduction
Vehicle routing problems (VRP) have been studied for decades for their apparent potential to address omnipresent challenges. To this date, the standard (static) VRP problem and its numerous variants are well-covered in the literature both in terms of solution approaches, constraints, and problem features (Vidal, Laporte, and Matl 2020). While concerns about uncertainties and dynamicity in real-world environments have been present for a long time, research in this direction did not become widespread until the past decade. The state-of-the-art on dynamic and stochastic VRPs addresses all the crucial aspects such as stochasticity of travel times, (un)loading times, customer demands, and dynamic introduction of new customers (Ojeda Rios et al. 2021). However, to the best of our knowledge, the current literature lacks both (1) works combining a wider range of stochastic and dynamic elements, and (2) a combination of real-world rich-featured VRP variants together with stochastic and dynamic elements.

Our main motivation is cooperation with the company Wereldo. We already designed a solver handling real-world features and constraints (Sassmann et al. 2023) and also adapted it for a more complex generalization of the real-world problem (Sobotka and Rudová 2023). The solver is used in everyday operations by the company, yet parts of its outputs are often considered unacceptable by human dispatchers. Based on our discussions, the main problem is that the solver addresses a static problem while its inputs are subject to uncertainties, and a non-trivial part of customers is not known upfront. In order to minimize necessary manual changes to the produced routing plans, the solver must explicitly incorporate risk-aversion and account for future accommodation of newly incoming customers during the day.

Applicable Methods
The problem at hand includes stochastic information (uncertainties in demands and time) as well as dynamicity (newly incoming customers). Thus, representing the problem as a Markov decision process (MDP) is a natural fit as argued in (Powell, Simao, and Bouzaïene-Ayari 2012). The states consist of the current routing plan and the event to be processed (incoming customer, realization of uncertain value). The actions in a state are all feasible routing plans covering the current instance together with the incoming event. The rewards are given by the change in objective function induced by the chosen action (new routing plan). The policy is any mechanism selecting an action for a given state.

For problems of this kind, four general classes of approaches exist (Powell, Simao, and Bouzaïene-Ayari 2012): reoptimizations (RO), look-aheads (LA), policy approximations (PA), and value function approximations (VFA). RO methods incorporate the incoming event and solve the resulting instance so that locally (near) optimal decisions are made. However, this leads to myopic decisions. LA methods select among actions by simulating possible future scenarios in Monte Carlo fashion. The concept of LA is a simple yet strong approach to promoting anticipatory behavior. Unfortunately, the large number of simulated scenarios needed to obtain sufficiently precise approximations proves prohibitive for complex VRPs. PA and VFA methods are closely linked to reinforcement learning. PA methods are applicable in cases when the functional form of the MDP policy is apparent and the main interest is in finding its best-suited parameters. Unfortunately, complex VRPs lack such a straightforward policy form. VFAs aim to learn the post-decision state value function in offline simulations to replace the costly explicit LA approximations. The key issue of VFAs is in the combinatorial size of state and action spaces as recently argued in (Hildebrandt, Thomas, and Ulmer 2023). Thus, our choice is to opt for ROs. As their main disadvantage is their myopic nature, our aim is to at
least push towards the construction of robust routes. This indirect handling of dynamicity and uncertainty will be performed by penalizing route failure risks and motivating dynamic customer anticipation within the objective.

**Proposed Approach**

Our aim is to extend the existing static solver. The first major extension is in promoting dynamic customer anticipation. The second is in quantifying and penalizing risks arising from uncertainties in demand and time.

**Dynamic Customers**

A conceptually simple and widely used approach is to artificially add dynamic customer requests into the instance. Then, the routes must incorporate even the artificial customers and the resulting routes are naturally constructed with anticipation of the potential dynamic changes to come. We plan to introduce artificial requests so that the routes serving them must maintain a favorable coverage of the whole service area over time. Serving the artificial requests will be optional, but the objective function will provide rewards for serving them. Importantly, we plan to draw inspiration from team orienteering problems (Gunawan, Lau, and Vansteenwegen 2016) as they share the common theme of optional customer service motivated by a reward.

**Risk-Aversion**

The major sources of route failure risks are uncertainties in demands and times. First, we must be able to quantify such risks in order to penalize them in the objective. Second, we must ensure that sufficiently representative values for the assessed risks may be calculated efficiently. For both demands and time, we are interested in finding the weakest point of a route where the probability of failure is the highest. We first demonstrate the concept on demand-related risks. Then, we explain the differences specific to time-related risks.

Given a route, the vehicle capacity limit, and a demand distribution for each of the requests served by the route, the goal is to identify the point in the route where the failure is the most likely. The severity of the weakest point will be penalized in the objective. We assume that the random variables representing the demands of different requests are independent. The planned load on the vehicle can be always represented as the sum of the loaded request demand random variables. With the independence assumption, the expectation and variance of this sum are equal to the sums of expectations and variances of the individual request demands. Consequently, these expectation and variance sums may be computed alongside the route in linear time. For arbitrary demand distributions, the risk of overreaching the capacity limit at a given location may be then bounded by, e.g., Chebyshev inequality. Also, if demands are modeled with (potentially skewed) normal distributions, the distributions may be still summed at the level of parameters, but the failure probabilities may be calculated more precisely based on tabular values. Since the probabilities may be calculated (and recalculated upon route changes) efficiently, it is possible to penalize capacity-wise risks directly in the objective.

Regarding time-related risks, the situation is conceptually similar but more complicated. If the independence assumption is acceptable, the uncertainties in travel times and (un)loading times, again, in principle sum alongside the route. In contrast to demands, however, these sums may be affected by waiting times. Since arrival at a location before its time window opening requires waiting, any delays may be fully or partially compensated. This adds a non-linear element as waiting may erase delays at best, but never make them negative. Consequently, the fast parameter summing demonstrated for the capacities becomes problematic. The distributions of delays alongside the route may be of course obtained via sampling or direct distribution summing for discrete distributions. Yet, heavily iterating any of these approaches within a search method would have severe performance implications. Consequently, we currently investigate how to approximate the impacts of waiting on delays while doing so fast, with sufficient precision and accent on preventing underestimation of the risks. The risk of failure at a location would be then calculated based on the arrival time, delay distribution, and end of the respective time window. Regarding efficient recalculation, we also analyzed the dynamics of arrival times and waiting upon route changes. With the understanding of the dynamics, we identified that changes to the maximum risk in the route can be recalculated with complexity limited by the delay distribution handling.

**References**


