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Branch-and-Bound for the Precedence Constrained Generalized Traveling Salesman Problem (Extended Abstract)

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An instance $\mathcal{P}$ of the Precedence Constrained Generalized Traveling Salesman Problem (PCGTSP) is defined by a directed graph $G = (V, A)$ where $V := \{1, \ldots, n\}$ is a set of vertices, $A \subseteq \{(i, j) : i, j \in V, i \neq j\}$ is a set of arcs, each of which has a cost $c_{ij} \geq 0$. We let $\{V_1, \ldots, V_m\}$ be a partition of V where V_p, $p \in M$, is called a *group*, and $M := \{1, \ldots, m\}$. We define the precedence constraints by an acyclic and transitive digraph $G' = (M, \Pi)$ so that if $(p, q) \in \Pi$ then group p must precede group q in a feasible tour. PCGTSP asks to find a cheapest closed tour that visits exactly one vertex per group and fulfills the precedence constraints. The tour must start and end in V_1. The precedence constraints apply only to the path without the last arc ending in V_1, but the cost of it is included.

PCGTSP combines features from the Sequential Ordering Problem (SOP) and the Generalized TSP (GTSP). Precedence constraints in SOP have been handled mostly by MILP models together with separation algorithms (Ascheuer, Jünger, and Reinelt 2000), (Balas, Fischetti, and Pulleyblank 1995). Some exact GTSP algorithms are based on branch-and-bound for the symmetric and asymmetric cases, utilizing integer relaxation or Lagrangian relaxation (Fischetti, Gonsalez, and Toth 1997), (Laporte, Mercure, and Nobert 1987), (Noon and Bean 1991). GTSP can be reduced to ATSP, to produce either optimal solutions (Noon and Bean 1993) or lower bounds (Noon and Bean 1991).

PCGTSP has recently attracted interest but is still largely unexplored. It arises, for example, in industrial processes where tasks which can be performed in different ways are to be sequenced with respect to some order to ensure the integrity of the process (Söderberg et al. 2017). Heuristics with a focus on industrial applications, namely coordinate measurement machines, may be found in (Salman et al. 2016). The present summary outlines the contributions of (Salman, Ekstedt, and Damaschke 2020) and discusses future plans.

We **branch** in such a way that we choose the group to visit next. Thus each tree node represents a partial group sequence $\sigma = (V_{p_1}, \ldots, V_{p_k})$ with $p_1 = 1$. Let $\mathcal{P}(\sigma)$ be the subproblem there – PCGTSP where any tour must begin with σ. By dynamic programming, the cheapest paths between any vertices in V_1 and V_{p_k} through σ are computed. Accordingly we define a reduced PCGTSP instance $\mathcal{P}_0(\sigma)$ on the subgraph of G without the inner groups of σ.

Let $C_{\min}(\mathcal{P})$ be the cost of an optimal solution, and $c_{\min}(\sigma)$ the cost of a shortest path through σ. It is easy to prove $C_{\min}(\mathcal{P}(\sigma)) \geq C_{\min}(\mathcal{P}_0(\sigma)) + c_{\min}(\sigma)$, which yields a lower bound for $\mathcal{P}(\sigma)$. For the reason for this approach, see the history utilization below.

We choose DFS as a branching strategy as this will rapidly deliver new upper bounds and is not memory intensive. When selecting the next branch at the same depth, we pick the group V_p with the largest successor set $\{q : (p, q) \in \Pi\}$ for obvious intuitive reasons. When a sequence with m groups is reached, the vertex choice is optimized by dynamic programming, and a special 3-opt heuristic (Gambardella and Dorigo 2000) is applied. The current best upper bound is updated when a lower value is obtained.

Similarly to (Shobaki and Jamal 2015) we use a simple **bounding** method where the precedence constraints are relaxed. The GTSP instance derived from a PCGTSP instance $\mathcal{P}$ by ignoring the precedence constraints is called the *weak version* of $\mathcal{P}$. Sophisticated steps for eliminating edges (Escudero, Guignard, and Malik 1994) (Ascheuer, Jünger, and Reinelt 2000) were discarded since initial experiments showed only marginal improvements.

To compute bounds for the remaining GTSP we look into two **relaxations** that have been used for ATSP (Williamson 1992): the Minumum Spanning Arborescence Problem (MSAP) and the Assignment Problem (AP). Since their generalized versions are NP-hard (Myung, Lee, and Tcha 1995), (Gutin and Yeo 2003), we have tried two different approaches: transform the GTSP to an equivalent ATSP instance $\mathrm{NB}(\mathcal{P})$ by the Noon-Bean transformation (Noon and Bean 1993), or to another ATSP instance $\mathrm{NC}(\mathcal{P})$ that relaxes the constraint to visit only one vertex per group. Relaxing the outdegree constraints in $\mathrm{NB}(\mathcal{P})$ or $\mathrm{NC}(\mathcal{P})$ results in an MSAP instance. To further tighten the bound we also add the cheapest incoming arc to the root, to get a 1-arborescence. By relaxing the subtour elimination constraints in $\mathrm{NC}(\mathcal{P})$ one obtains a Cycle Cover problem, which can be turned into an AP by a standard construction.

Our main idea is to extend parts of the **history utilization** technique from (Shobaki and Jamal 2015) to PCGTSP. It is applicable because identical bounding subproblems in the search tree are likely to appear, and the "principle of optimality" holds (sub-tours of optimal tours are optimal). We suitably define equivalence between tree nodes, and if it is not possible to prune a tree node based on already explored equivalent nodes we forgo solving the bounding subproblem and reuse information from an already explored tree node, using $\mathcal{P}_0(\sigma)$. There, the lower bound was separated in two parts, and the suffix part is equal for equivalent tree nodes.

The algorithm was run on 23 synthetic instances and 5 cases of coordinate measuring machines used for geometry assurance within the automotive industry. The synthetic instances were created from a library of SOP instances by duplicating vertices and edge costs, thus creating groups of identical vertices, and random perturbation to the costs. They have 13–70 groups of 1–9 vertices. The industrial cases have 12–173 groups. The AP lower bounds are almost consistently stronger than those of MSAP. This holds both for the end results and for subproblem nodes. Even though not all parts of the history utilization technique could be extended, we were able to solve the instances corresponding to the SOP instances solved in (Papapanagiotoua et al. 2015), indicating that the most important properties of this pruning technique carry over. For most unsolved instances we got solutions within 10% of optimum.

While pruning with history utilization is successful, we seem to produce very weak lower bounds, partly due to the DFS strategy (which only takes the bound at the root as the global lower bound) but mainly due to omission of precedence constraints. To incorporating them, we are considering relaxations using AP where costs are based on short valid paths rather than single arcs. These paths may go through spurious vertices making the lower bounds too small, but this might be avoided by modified branching rules.

One could also keep the precedence constraints in an ILP model. Tests in (Salman et al. 2016) indicate that solvers like CPLEX take far too long time for this and produce bounds which are not significantly stronger, indicating that some cutting plane augmentation is needed. However, this may be an issue with the used mixed ILP model. Another approach is to develop precedence constraint cuts for the AP or MSAP as in (Escudero, Guignard, and Malik 1994).

A better bound at the root is especially important. One could employ Lagrangian relaxation on the vertex choice or subtour elimination constraints. Some nested subgradient method which updates the multipliers for the dualized constraints could then strengthen the bound. One could also solve $\mathrm{NC}(\mathcal{P})$ or $\mathrm{NB}(\mathcal{P})$ by a TSP solver.

Other plans are to consider multi-agent versions and applications to item collection tours in warehouses.

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