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The Application of Pareto Local Search to the Single-Objective Quadratic Assignment Problem

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Introduction

Pareto optimization, that includes the simultaneous optimization of multiple conflict objectives, has been employed as a high level strategy to reduce the effect of local optima (Segura et al. 2013). This approach was first introduced in (Louis and Rawlins 1993), and later reinvestigated and termed as *multi-objectivization* in (Knowles, Watson, and Corne 2001). Since then it has been studied by many researchers actively. The idea of multi-objectivization is to translate the target single-objective optimization problem into a multi-objective one, and then to solve the later using a Preto optimization technique.

There have been several studies on multi-objectivization with various applications (Segura et al. 2013), resulting in two main groups of multi-objectivization: methods that decompose the primary objective into multiple conflicting objectives (Knowles, Watson, and Corne 2001), and methods that optimizes at least one additional "helper" objective simultaneously with the primary objective (Jensen 2004). Both approaches rely on devising new effective problem-dependent objectives, which is normally a tedious task.

This paper briefly presents the application of Pareto local search (PLS) (Paquete, Chiarandini, and Stützle 2004), as a Pareto optimization technique, to the single-objective quadratic assignment problem. The idea is to use PLS instead of local search, to optimize the primary objective together with an additional augmented function. The augmented objective function is defined using a general penalty-based approach, an idea that comes from Guided Local Search (GLS) (Voudouris, Tsang, and Alsheddy 2010). This results in a multi-objectivization approach that is simple and general.

The Proposed Approach

We will begin with a brief overview of PLS and GLS, and then describe the proposed penalty-based multi-objectivization approach.

PLS is a direct generalization of local search to handle more than one objective. Generally, PLS maintains a set of potentially efficient solutions, called $archive$, and iteratively improves this set by exploring all or part of its solutions' neighbourhood. The acceptance criterion in PLS depends on the notion of Pareto optimality. PLS stops when the neighbourhoods of all solutions in the archive have been explored, i.e. reaching a Pareto local optimum set.

In **GLS**, the primary objective of local search is augmented with penalties to direct the search away from local optima. Every time the local search settles in a local optimum, GLS penalizes a selected feature of the candidate solution. Therefore, to apply GLS, the solution features need to be defined in order to distinguish between solutions with different characteristics. Each feature is associated with a cost to help GLS choose bad features to penalize them. GLS replaces the main objective function $g(s)$ with an augmented function $h(s)$:

$$h(s) = g(s) + \lambda \sum_{i \in F} p_i * I_i(s) \tag{1}$$

In this formula, λ is a parameter of GLS, F is the set of all features, p_i is the penalty of feature i, and $I_i(s)$ is equal to 1 only if s exhibits feature i; 0 otherwise.

Assuming a single-objective optimization problem with an objective $g(s)$, we propose multi-objectivizing by adding a helper objective that is defined using the GLS's augmented objective function $h(s)$ (Equation 1). PLS is applied to optimize $h(s)$ together with $g(s)$. Every time PLS reaches a local Pareto optimum set, $h(s)$ will be updated by the penalization phase, and PLS is restarted again. We name this algorithm as Guided Multi-objectivized Local Search (GMLS). GMLS modifies the penalization scheme of GLS by penalizing a set of K features exhibited by any solution in the $archive$.

GMLS for the QAP

The QAP (Burkard, Karisch, and Rendl 1997), in its simplest form, is described as follows. Given a set $N = 1, 2, ..., n$ and $n \times n$ matrices A= $[a_{ij}]$ and B = $[b_{kl}]$:

$$minimize f(\pi) = \sum_{i=1}^{N} \sum_{j=1}^{N} A_{ij} \cdot B_{\pi_i \pi_j} \tag{2}$$

where π is a permutation of N.

The implementation of GLS for the QAP was reported in (Mills, Tsang, and Ford 2003), we follow the same approach

Table 1: The performance of GMLS, GLS and RMLS on QAPLIB instances

Instance	GMLS			GLS			RMLS		
	δ	R_{opt}	$time_s$	δ	R_{opt}	$time_s$	δ	R_{opt}	$time_s$
els19	0.00	10	7.8	0.00	10	4.4	0.00	10	0.1
nug30	0.00	10	2.1	0.00	10	2.8	0.20	3	29.9
kra30a	0.00	10	6.4	0.00	10	1.9	0.30	8	18.4
tho30	0.00	10	7.0	0.07	7	13.0	0.24	3	29.3
ste36a	0.07	6	19.7	0.18	2	35.9	1.30	0	40.0
sko42	0.00	10	34.6	0.02	3	72.4	0.50	0	100.0
sko49	0.05	2	90.1	0.12	0	100.0	0.56	0	100.0
sko56	0.08	0	100.0	0.12	0	100.0	0.68	0	100.0
tai60a	1.03	0	150.0	1.68	0	150.0	2.71	0	150.0
sko64	0.09	1	99.1	0.13	0	100.0	0.82	0	100.0
tai80a	1.14	0	240.0	1.55	0	240.0	2.62	0	240.0
sko100a	0.20	0	400.0	0.26	0	400.0	0.74	0	400.0
tai100a	1.18	0	400.0	1.58	0	400.0	2.58	0	400.0
tho150	0.68	0	800.0	0.69	0	800.0	1.16	0	800.0
bur26a	0.00	10	0.3	0.00	10	0.3	0.00	10	1.3

to implement GMLS for the QAP. QAP solutions are represented by permutations. The neighbourhood operator used for the problem is simply to exchange (i.e. swap) the contents of two permutation positions. The features used for the QAP were all the possible location facility pairs.

The performance of GMLS is examined on a set of QAPLIB test instances[1], and compared to GLS as a single-objective optimizer, and a PLS algorithm, referred to as RMLS, that uses a random value as the additional helper objective as another multi-objectivization approach.

The results are given in Table 1 that gives, for each combination of algorithm and problem instance, the average best solution as a percentage over the best known solution (δ), the optimal runs (i.e. runs where the algorithm obtains an optimal solution) out of ten (R_{opt}), and the average time in seconds ($Time_s$). The results show that GMLS obtains better results than GLS on 11 instances out of 15 QAPLIB instances. The differences are statistically significant on three out of the 11 instances. The results of GLS are (insignificantly) better than that of GMLS in three instances, whereas both algorithms always obtain the best known value in four instances. In addition, GMLS finds the optimal solution, with respect to the best known value, in at least one run over the 10 runs for 9 instances, compared to 8 instances for GLS. The results also confirm the outperformance of GMLS over RMLS. GMLS obtains better results than RMLS on 13 instances, 11 of which are statistically significant. Both algorithms always obtain the best known value in the other two instances.

Conclusion

This paper briefly describes a study on the application of PLS, which is a straightforward extension to local search, to tackle the single-objective quadratic assignment problem. This is achieved by adopting a new multi-objectivization method, that is problem-dependant, yet simple and general. It employs the GLS strategy, which is the augmentation of the primary objective by penalties associated to solution features, in defining the additional helper objective. Then, PLS is applied to optimize both the primary objective and the additional augmented objective function simultaneously. Preliminary results on the QAP confirm the effectiveness and potential of the proposed multi-objectivization approach.

[1] https://www.seas.upenn.edu/qaplib/

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