Efficiently Finding Optimal Winding-Constrained Loops in the Plane (Extended Abstract)*

Paul Vernaza
The Robotics Institute
Carnegie Mellon University
Pittsburgh, PA 19104

Venkatraman Narayanan
The Robotics Institute
Carnegie Mellon University
Pittsburgh, PA 19104

Maxim Likhachev
The Robotics Institute
Carnegie Mellon University
Pittsburgh, PA 19104

Abstract

We present a method to efficiently find winding-constrained loops in the plane that are optimal with respect to a minimum-cost objective and in the presence of obstacles. Our approach is similar to a typical graph-based search for an optimal path in the plane, but with an additional state variable that encodes information about path homotopy. Upon finding a loop, the value of this state corresponds to a line integral over the loop that indicates how many times it winds around each obstacle, enabling us to reduce the problem of finding paths satisfying winding constraints to that of searching for paths to suitable states in this augmented state space. We give an intuitive interpretation of the method based on fluid mechanics and show how this yields a way to perform the necessary calculations efficiently. Results are given in which we use our method to find optimal routes for autonomous surveillance and intruder containment.

Introduction

The subject of this work is finding optimal constrained loops in the plane. More specifically, out of all paths starting and ending at a specific location in the plane and satisfying certain *looping* constraints, we would like to find one such path that minimizes a given location-dependent cost accumulated along the path, while also avoiding some regions entirely. Instances of this problem of interest to us include planning for unmanned aerial vehicle (UAV) surveillance and planning optimal enclosures.

Our approach is inspired by the work of Bhattacharya et. al. (Bhattacharya, Kumar, and Likhachev 2010), who first proposed the general idea of performing graph search for navigation with a state vector augmented by information about homotopy. Although our work is based on the same framework, we are concerned principally with its application to planning optimal loops. Additionally, our method is equivalent but significantly simpler and motivated from an intuitive fluid analogy. Also notable is the work of (Gong et al. 2011), who applied similar ideas to the problem of vision-based tracking with occlusion.

*This paper is a rewritten, abridged version of a longer manuscript of the same name to be published at RSS 2012.

Method Overview

As in (Bhattacharya, Kumar, and Likhachev 2010; Gong et al. 2011), our method is very similar to any standard, graph-based search method for finding optimal point-to-point paths in the plane. In fact, the only significant difference is that the state is augmented with an additional variable that accumulates information about how the paths wind around each object of interest. This variable, which we refer to as the F-value, has a simple interpretation in terms of fluids: if we place a source of fluid within each object of interest, the F-value of a path is equal to the net rate at which fluid flows through the path. For a path that is a closed loop, the divergence theorem implies that the F-value of the loop is equal to the net rate at which fluid is produced by objects located within it. This property allows us to search for a loop enclosing a desired object set by searching for a state whose F-value is equal to the sum of the rates assigned to the objects in the set. More generally, we can in this way search for a loop that *winds* around objects (in the topological sense (Munkres 1975)) in any desired way.

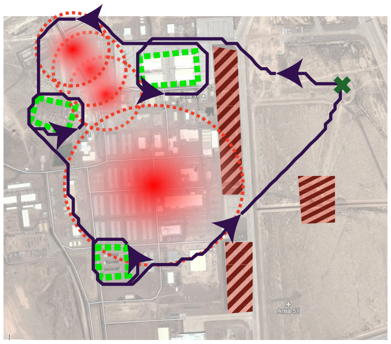
Formally, we assume we are given a description of the graph corresponding to a point-to-point navigation problem in the plane, consisting of a state space $\mathcal{X}$ and a successor function succ : $\mathcal{X} \to 2^{\mathcal{X}}$, where $2^{\mathcal{X}}$ is the power set of $\mathcal{X}$. We then define a new state space $\mathcal{X}' = \mathcal{X} \times \mathbb{R}$, with elements denoted by (x, f). A new successor function is then defined in the following way:

$$(x, f) \mapsto \{(x', f') \mid x' \in \text{succ}(x), f' = f + F(\rho(x, x'))\},$$

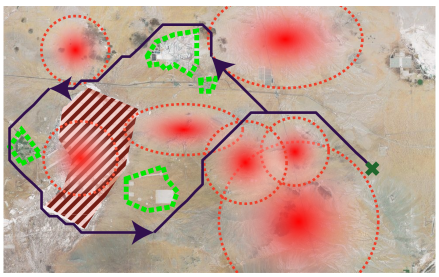
where $\rho(x, x')$ denotes a path corresponding to a transition between locations x and x', and $F(\rho(x, x'))$ computes the net flow through this segment. If $\rho(x, x')$ consists of a straight line, then it can be shown that

$$F(\rho(x, x')) = \sum_i s_i \frac{r_i}{2\pi} \arccos \left\langle \frac{x' - o_i}{\|x' - o_i\|}, \frac{x - o_i}{\|x - o_i\|} \right\rangle,$$

where o_i is the location of the ith object of interest, r_i is its flow rate, and the sign $s_i \in \{-1, 1\}$ is determined by whether o_i falls on the left or right side of the directed segment $x \to x'$, according to the desired convention. The start state for graph search is defined by $(x_0, 0)$, where x_0 is the start state in the pre-augmented graph. The goal state is defined as $(x_0, \sum_i W_i \log z_i)$, where W_i is the desired



(a)



(b)

Figure 1: Optimal plans (solid lines) for hypothetical UAV surveillance missions. The UAV is constrained to take off and return to a particular location, *winding* around ROIs in desired ways. ROIs are designated by polygonal dashed lines. Red, striped polygons denote high-traffic regions that the UAV must avoid. Circular dashed lines denote location and ranges of radar installations, with radar power decreasing with distance to center.

winding number of the solution path around the ith object of interest and z_i is the ith prime number. This assumes $r_i = \log z_i$; setting the flow rates in this way ensures that the map from winding winding configurations to F-values is invertible, which can be proved by the fundamental theorem of arithmetic.

Experiments

UAV surveillance

In this set of experiments, we applied our method to the problem of finding optimal looping routes for a UAV tasked with obtaining 360-degree views of regions of interest (ROIs). We simulated hypothetical UAV missions by annotating aerial imagery of military bases with ROIs, hypothetical radar installations, and regions of excessive risk to be avoided entirely. Radar installations were modeled as overlapping ellipses, each containing a cost attaining its maximum value at the center of the ellipse and diminishing gradually to zero at the boundary of the ellipse. Our objective was to find minimum-cost loops subject to winding constraints and entirely avoiding regions of excessive risk. Results are shown in Fig. 1.

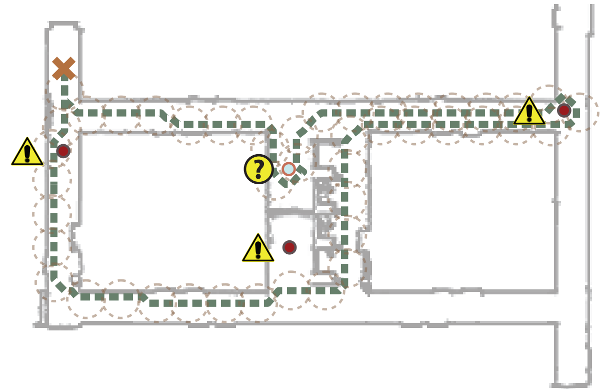


Figure 2: Results of intruder confinement experiments. Locations of intruders are marked by circles beside exclamation symbols. Locations of bystanders are marked by circles beside question marks. Paths generated by the method are shown superimposed on floor plan of environment. Robot deployments and sensing/communications ranges indicated by dashed circles.

Intruder confinement

We also studied the application of our method to a problem we refer to as *intruder confinement*. This problem consists of finding an optimal way to confine several intruders in a maze-like environment, subject to the constraint that innocent bystanders should not be confined. Confinement is achieved by surrounding the intruders with a team of robots that deploy from a central location. A feasible deployment strategy consists of a loop that winds around each intruder exactly once without winding around a bystander. Fig. 2 demonstrates that the method is able to find optimal enclosures in highly irregular, maze-like environments.

Discussion

A particular challenge for the method as implemented up to this point is the case where many winding constraints are to be enforced, in which case the the method may systematically explore different combination of windings until it reaches the goal state. In the near-term, we anticipate that employing straightforward heuristics will greatly enhance the ability of the method to solve problems of this type.

Acknowledgments

This research was sponsored by the ONR DR-IRIS MURI project grant N00014-09-1-1052.

References

Bhattacharya, S.; Kumar, V.; and Likhachev, M. 2010. Search-based path planning with homotopy class constraints. In *SOCS*.

Gong, H.; Sim, J.; Likhachev, M.; and Shi, J. 2011. Multi-hypothesis motion planning for visual object tracking. In *ICCV*.

Munkres, J. 1975. *Topology: a first course*.