Leaders and Negotiators: An Influence-based Metric for Rank

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Abstract

We propose $influence$ as a measure of the centrality of nodes in a network. Influence takes into account not only direct links but also all paths between nodes. We parametrize the influence metric by a variable α that measures the strength of links. Variations in rankings as α changes provides a mechanism to identify the central nodes within communities (leaders), as well as nodes that act as bridges between communities (negotiators).

Introduction

Social scientists have long been interested in understanding how the structure of social networks affects the productivity and success of individuals. Simmel (Simmel 1950) considered the nature of ties in his analysis of the stability and dynamics of a network. A *dyadic* tie connects only two people, while a *triadic* tie is one of the three ties in a triad (or one of the $n(n-1)/2$ ties in an n-clique). Simmel argued that dyads are fundamentally different from triads, since they preserve the individuality of both members and allow them more bargaining power. Conflict, one the other hand, is more readily managed in a triad. According to Simmel, the cohesive forces of a triad contribute to a group's survival and preserve its identity at the expense of individuals. He also described two divisive forces: (i) two parties, both in a diadic relationship with a third, but not strongly tied to each other, enable the third party to take advantage of the situation, and (ii) the third party actively participates in separating the two parties to attain supremacy.

This motivates Granovetter's analysis of a network in terms of strong and weak ties. In a friendship network, "our acquaintances (weak ties) are less likely to be socially involved with one another than are our close friends (strong ties)" (Granovetter 1973). These "weak ties" serve as bridges between close-knit groups, facilitating the flow of information between groups, which explains their social importance. The social advantages conferred on weak ties arise from their diadic nature, allowing them to exploit the advantage given by their position within the network.

Burt (Burt 1992) also saw network structure as conferring social advantages to an individual. Burt's central theme was that an individual who is connected to people who are not connected to each other act as bridges ("structural holes") in the social system. These *negotiators* are more powerful and successful than others who are not in the bridging role. Bridges can be members of a triad (or n-clique), not only dyads. Burt's structural holes theory rests on Simmel's divisive forces. Unlike Granovetter, who focused on the $nature$ of a tie, according to which the weaker ties more likely to be bridges, Burt considers the strength of ties to be irrelevant. On the contrary, he claims that if the bridge is a stronger tie, it is likely to be more powerful.

At a macroscopic level, the balance between divisive and cohesive forces results a stratification of the network into communities. Divisive forces lead to formation of communities, while the cohesive forces stabilize them and give them distinct identity distinct from the identity of the individuals comprising them. The network could then be described by a two-tiered architecture, with individuals forming communities, and communities forming the network. Being central in a social network can have two distinct interpretations. First, a player could be central to one community. Such a player is 'close' to other community members and mediates communication among them. We call him the *leader*. While he has huge influence over community members, his actions are constrained by the rules of the community. Second, a player could be central to more than one community. While such a player could be peripheral to each community, he is 'closer' to members of several communities than other community members who are not connected to one another. This way, he can mediate communication, or act as a bridge, between the two communities. We call such a player the *negotiator*.

The traditional "weak ties" and "structural holes" theories focus on the microscopic structure of the network, thereby failing to take into account the macroscopic structure of the network that would allow us to detect central players. Global measures of node centrality or 'proximity' to other nodes do exist. One such global metric is betweenness centrality (Freeman 1979), which takes into account $paths$ and not just the $links$ or ties between nodes. Freeman proposed to measure centrality as the ratio of the number of shortest paths via the given node to the number of shortest paths in the network (Freeman 1979). PageRank (Page et al. 1998), which roughly gives the probability that a random walk initiated at one node will reach another, measures proximity between nodes. We present another measure, which we call *influence*, which uses all paths, not just the shortest, to measure centrality of nodes in a network. We define a ranking method based on influence. This metric provides a mechanism to find central nodes within communities(leaders) and bridges between communities (negotiators).

Influence-based Ranking

In a previous work (Ghosh and Lerman 2008) we defined *influence* metric as the number of paths, of any length, that exist between two nodes in a network. This definition makes intuitive sense, because the greater the number of paths between nodes i and j, the more opportunities there are for i to affect (de Sola Pool and Kochen 1978 1979), or get a message to, node j.

The strength of an effect transmitted via longer paths is likely to be less than via shorter paths. We model the attenuation using parameters α_i where α_i $(0 \leq \alpha_i \leq 1)$ is the probability of transmission of effect, or message, in the (i-1)th hop. The probability of transmission along an n-hop path is $\Pi_{i=1}^{n+1} \alpha_i$. To simplify computation, we take $\alpha_1 = \beta$ (transmission to direct neighbors) and $\alpha_i = \alpha \forall i, i \neq 1$. α is the indirect attenuation factor, i.e., transmission via intermediaries.

A network can be represented by an adjacency matrix A, whose elements are defined as $A_{ij} = 1$ if $\exists$ a link from node i to j and $A_{ij} = 0$ otherwise. The capacity of i to influence j is given by the influence metric:

$$
\begin{aligned}
P &= (\beta A + \beta\alpha A^2 + \cdots + \beta\alpha^n A^{n-1} + \cdots) \\
&= \beta A (I - \alpha A)^{-1}
\end{aligned}
$$

The first term gives the number of (attenuated) paths of length one (direct links) between nodes, the second term the number of (attenuated, non-selfavoiding) paths of length two, etc., with last equation giving a closed-form solution of the geometric progression. This equation holds while $\alpha < 1/\lambda$, where λ is the largest characteristic root of A. In a previous work we used this influence metric to identify the stratification of the network into communities (Ghosh and Lerman 2008). In this paper we show that the same metric can be used to identify important nodes in a network. The ranking of node i is based on its influence score and is given by $R_i = \sum_j P_{ij}$. If $\alpha = \beta$ then this influence score reduces to the Katz score (Katz 1953). If $\beta = 1$, and instead of scalar α we have a vector $c \cdot D^{-1}$, where $D_{ij} = \sum_{j=1}^{n} A_{ij}$ if $i = j$ and 0 otherwise, then the influence metric reduces to PageRank (Page et al. 1998) algorithm for proximity scoring.

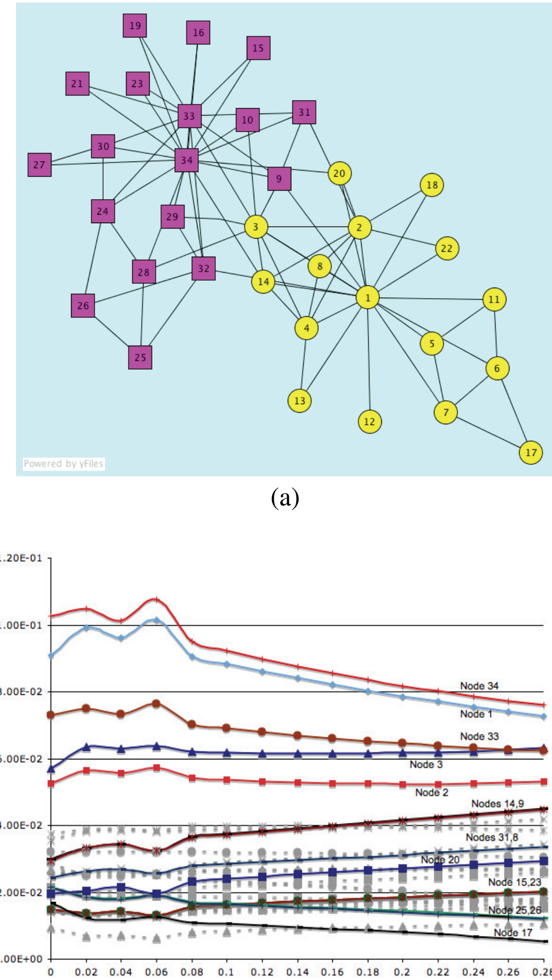
Influence metric P is parametrized by α, the strength of indirect connections[1] As we increase α from 0 to its maximum value, the change in the relative importance or ranking of nodes gives us insight into their roles within the network. As described previously, leaders are central nodes within their communities, but they may not be central to the network as a whole. Hence their influence score may decrease with increasing α. We may also find nodes whose influence rises significantly with increase in α. These nodes may be peripheral to any single community but are central to the entire network since they are connected to many different communities — they are the negotiators. There are also low ranking nodes which show a rise with the increase of α, but the final influence score reached is low. Such nodes may belong to a single community but they become more central to that community as α increases. Nodes that are peripheral to the community, as well to the network as the whole, have low ranking and their influence scores go on decreasing with increase in α. Broadly speaking, we can identify four categories: (i) High influence score, may decrease with increase in α – leaders; (ii) Moderate to high influence score, increase with α – negotiators or bridges between communities; (iii) Low influence score, increase with α — core community members; (iv) Low influence score, decrease with increasing α — peripheral community members not connected to any other community.

[1]The value of β does not affect results and without loss of generality, we set $\beta = 1$.

Ranking of Karate Club Members

We illustrate the influence-based ranking method on the friendship network of Zachary's karate club (Zachary 1977). During the course of a study by Zachary, the club divided into two factions, represented by circles and squares in Figure 1(a). We plot the influence scores for the players as α is increased from 0 to 0.28 (the upper bound of the reciprocal of the largest eigenvalue of the corresponding adjacency matrix) in Figure 1(b). As we can clearly see from the data, node 34, 1 have the highest influence scores for all values of α; therefore, they are the leaders. According to the original study, node 1 represents the administrator of the club and node 34 the instructor. A disagreement between these people led to the club's division, with one group following the instructor and the other the administrator. Hence nodes 1 and 34 are indeed leaders in their communities. Nodes 33 and 2 are similarly leaders of their communities. The influence scores for nodes 3, 14, 9, 31, 20, 8 increase with α from moderate to relatively high values. As we can clearly see from the figure, all of them (except 8) are bridges between the two communities and hence can act as negotiators. The 10 nodes mentioned above come among the topmost 14 nodes when the nodes are ranked according to influence score at maximum value of α. Nodes 15 and 23 have low influence score, but their influence score increases with α. Hence according to our hypothesis they are the core members of the community to which they belong. As we can see that both 15 and 23 belonging to Group1 are directly connected to the leaders (nodes 34 and 33) of that group. Hence it would not be too far-fetched to assume that they are the core members of the group. Nodes 25, 26 and 17 have low influence score and their influence score decreases with α. Hence according to our hypothesis they are peripheral to the groups to which they belong and are not connected to any other group. An eyeball inspection of the graph shows that this is indeed true.

The results on running the PageRank algorithm are somewhat similar to influence-based ranking at $\alpha = 0$. Betweenness centrality (BC) is 0 for nodes 8, 12, 13, 15, 16, 17,18,



(a)

(b)

Figure 1: Zachary's karate club: (a) network with the two communities formed , Group1(squares) and Group2(circles) and (b) node rankings vs. α

19, 21, 22, 23 and 27; hence we are unable to compare the centrality of these nodes with others and amongst themselves. Also both PageRank and BC give high rank to node 6, implying that node 6 is central to the network. On increasing α, the influence score of node 6 decreases and reaches a low value. Hence, our approach hypothesizes that node 6 peripheral not only with respect to the community but also with respect to the network on the whole. Observation of node 6 on the graph verifies this hypothesis.

Summary

We defined a social network having a two-tiered architecture. The first tier is composed of individuals who form communities, and the second tier comprises of communities which form the society. This architecture enables us to have a macroscopic view of social stratification. It also helps us to model the two kinds of central nodes, with distinct roles within a social network, namely, the leaders within communities and the negotiators between communities. We propose a new metric, influence, to measure centrality of nodes in a network and rank them. This metric takes into account the number of weighted paths between nodes. Examining how the ranking of nodes changes as the weighting parameter increases provides a mechanism for determining the role the nodes play in the network and community. We illustrated the influence-based ranking mechanism on the benchmark Karate Club network.

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