The Complexity of Optimal Monotonic Planning: The Bad, The Good, and The Causal Graph

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Abstract

For almost two decades, monotonic, or "delete free," relaxation has been one of the key auxiliary tools in the practice of domain-independent deterministic planning. In the particular contexts of both satisficing and optimal planning, it underlies most state-of-the-art heuristic functions. While satisficing planning for monotonic tasks is polynomial-time, optimal planning for monotonic tasks is NP-equivalent.

We took a step towards a fine-grained classification of worst-case time complexity of optimal monotonic planning, with a focus on "what gets harder" and "what gets easier" when switching from optimal planning to optimal relaxed planning, in the context of finite-domain planning task representations (FDR). Along the way, we established both negative and positive results on the complexity of some wide fragments of this problem, with the negative results emphasizing the role of the structure of state variable domains, and the positive results emphasizing the role of the causal graph topology.

Table lists our main results for optimal monotonic planning, contrasted with the complexity of the corresponding fragments of optimal (FDR) planning. The key conclusions are as follows.

(1) Optimal planning for monotonic relaxations is hard even if restricted to very simple causal graph structures, but the complexity there stems from the size of the state variable domains.

(2) Restricted to planning tasks with constant-bounded state variable domains, the problem becomes solvable in time exponential only in the tree-width of the causal graph, while it is known to be very much not so even for non-optimal regular planning.

(3) While the tree-width of digraphs is independent of the edge directions, exploiting the directed structure of the causal graph *together* with its tree-width allows the computational tractability to be expanded beyond fixed-size state variable domains.

$\mathbf{\Pi}$		FDR	MFDR
causal graph	extra condition	*in P?*	*in P?*
fixed size		Yes	No
$\omega = O(1)$	$\lvert\mathcal{D}(v)\rvert = O(1)$	No	Yes
$\omega = O(1)$ & DAG	$\lvert\mathcal{D}(v)\rvert = O(1)$	No	Yes
$\omega = O(1)$ & DAG	in-degree = $O(1)$	No	Yes, if M-unfoldable
$\omega = O(1)$ & DAG		No	Yes, if M-unfoldable & prevail decomposable

Table 1: $\mathbf{\Pi}$ is a fragment of FDR/MFDR planning, characterized in terms of the causal graph tree-width ω, causal graph in-degree, and upper bound $|\mathcal{D}(v)|$ on the size of the variable domains. M-unfoldability and prevail decomposability are two properties of MFDR tasks that have been introduced and exploited in this work.

References

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