On Modeling the Tactical Planning of Oil Pipeline Networks

Daniel Felix Ferber
PETROBRAS – Petróleo Brasileiro S/A
Av. Paulista, 901 – 01311-100, São Paulo – SP – Brazil
danielferber@yahoo.com.br

Abstract

This paper aims at incorporating tactical aspects of oil pipeline networks to the supply chain planning model. The strategic design of supply chains is covered in literature by well understood and recurring patterns such as multi-commodity networks, dynamic parameters over time, capacity on facilities, transportation capacity or facilities with demand, production and inventory. We consider the following characteristics: capacity for in-transit inventory, transit time and flow reversal. Our objective is a better estimate for resources required by the network and therewith allow a more precise optimization of their use. All aspects are modeled to be efficiently solved by linear programming algorithms.

Keywords: Pipeline, Supply Chain, Tactical Planning, Network Flow, Linear Programming

Introduction

Pipeline networks are preferred for transporting oil refined commodities. As mentioned in (Kennedy 1993), they require less operational resources, scale over large distances and present lower operational costs and less environmental risks. Pipeline networks incorporate additional aspects to the modeling of the supply chain planning problem. Flow capacity and transit time are two of the most important. A brief overview is found in (Mohitpour and Jenkins 2004).

The strategic planning of pipeline networks is related to the problem of facility location. It consists of deciding which facilities to open or to close as well as how to acquire, transform or distribute commodities. Such problem is well understood by network flow models that allow an efficient solution by linear programming algorithms according to (Dantzig 1998; Luenberger and Ye 2008). A comprehensive comparison of suggested approaches is found in (Melo, Nickel, and da Gama 2007; 2009; Farahani et al. 2011). They support multi-product, multi-period network, channel capacity, inventory on facilities, routing and transportation modes. Realistic issues are discussed in (Melo, Nickel, and da Gama 2003; 2006) such as echelons of facilities, external supplies, flexible inventory, storage limitations, availability for relocation and expansion or reduction of capacities. A deeper understanding of time influence and dynamic parameters is found in (Aronson 1989; Skutella 2009). As opposed to strategic planning, (Relvas et al. 2006; Moura et al. 2008a; 2008b; Lopes et al. 2010; Boschetto et al. 2010; MirHassani, Moradi, and Taghinezhad 2011) aimed at a detailed operational planning of the pipeline network. However, we consider them an overhead for the tactical planning.

We present a multi-product, multi-period network linear programming model that captures some important aspects of a pipeline network: capacity for in-transit inventory, transit time and flow reversal. We assume a network without layers of echelons and that facility location decisions were taken beforehand. Integer variables were avoided for an efficient solution on large instances. Compared to simple network flow models, our formulation provides more precise constraints to estimate flow capacity and resource allocation potentially closer to reality. The solution is a lower bound for resources used in operational planning.

The remainder of this paper is organized as follows. Next section defines the problem in the context of the oil industry. The main concepts for the approach are explained and followed by the formulation. Facts for experimental results are given and an example is dicussed. Finally, we present concluding remarks and suggest directions for future work.

Problem definition

The oil refined commodities supply chain consists of a pipeline network that connects facilities like refineries or distribution centers. The transportation of commodities such as gasoline, gas oil, diesel and kerosene usually shares the same pipeline, without any separator. The facilities are able to push commodity into a pipeline and to receive from it. They may even transship received commodity to another pipeline depending on their internal layout.

Within a pipeline, we call the amount of a certain commodity as *in-transit inventory*. As the pipeline must stay completely filled, an amount of commodity is pushed into one end to deliver the same amount on the other one. But portions of each in-transit inventory potentially change. Flow directions may change dynamically and force in-transit inventory to be pushed back. The flow rate depends mainly on product characteristics like viscosity and density, also on flow direction and pipeline attributes. If commodities share the same pipeline, then the one of slowest flow rate determines the resulting rate. Further, the flow rate of pipelines connected by transshipment is restricted by the slowest rate on any of those pipelines.

Some tactical operations suggested in (Mula et al. 2010; Ferber 2011) were omitted for brevity as they fall back into special demand and production patterns: commodity *acquisition* or *discard* beyond network boundary, *substitution*, *degradation, blending* or *transformation* of commodities.

We look for a commodity flow assignment over time that opportunely drains excess of commodities from producing facilities towards deficiency on delivery locations. The assignment suggests which commodities to flow, the route between origin and destination and quantity. The primary goal is to minimize the effort to operate the pipeline network.

Problem formulation

The pipeline network is defined as a graph $G(N, A)$ and sets C, T and P. Set N comprises *nodes* representing facilities that may act as production or delivery locations, or just as transshipment. Set A represents the transportation infrastructure as *bidirectional arcs.* Each one is a tuple (n, m) of adjacent nodes n and m, called respectively arc *origin* and *destination*. The *sense* n to m is termed as *main sense* of the arc and m to n as its *inverse sense*. Set C enumerates all *commodities*. The ordered set $T{=}\{1, ..., k\}$ divides time as k continuous *time slices*, each $t \in T$ of duration $\mathcal{T}_t$. The existence of a route between two nodes characterizes a *path* that requires suitable switching capabilities on intermediary nodes. Set P contains those *paths* of interest, each one is a sequence of arcs $\{a_1, a_2, ..., a_k\}$, where arc a_i is written as $\overrightarrow{a_i}$ or $\overleftarrow{a_i}$ to emphasize respectively that the arc occurs with main or inverse sense. Let l_p be number of arcs on path $p \in P$ and for each arc $a \in A$ on p, let $k^a_p \in \{0, ..., l_p{-}1\}$ be its position of within the path.

Parameters Related to each node $n \in N$, commodity $c \in C$ and, in some cases, time slice $t \in T$:

- $\gamma_{nc0} \geq 0$: node inventory at beginning of first time slice;
- $\mathcal{E}_{nc} \geq 0$: effort to start a flow from the node;
- $\mathcal{P}_{nct} \geq 0$, $\mathcal{D}_{nct} \geq 0$: production and demand;
- $\mathcal{I}^{min}_{nct} \geq 0$ and $\mathcal{I}^{max}_{nct} \geq 0$: minimal and maximal node storage capacity;

Associated with each arc $a \in A$ and, in some cases, commodity $c \in C$ or time slice $t \in T$:

- $\mathcal{M}_{at} \in [0, 1]$: arc availability rate;
- $\overrightarrow{\mathcal{F}}^{max}_{ac} \geq 0$, $\overleftarrow{\mathcal{F}}^{max}_{ac} \geq 0$: maximal flow rate capacity in arc main and inverse sense.

And for each path $p \in P$, arc $a \in A$ on p, commodity $c \in C$:

- β^j_{pc0}: arc in-transit inventory at the beginning of the first time slice, where $j{=}k^a_p$.

Arc inventory relaxation On a simple network flow model, a commodity flow is simultaneous, unordered and instantaneous on each arc. For a better approximation of arc in-transit inventory aspects, we model arcs as fixed storage capacity with inbound and outbound connections to both adjacent nodes. The capacity is shared by any commodity and split into portions of commodity associated to each path that passes through the arc.

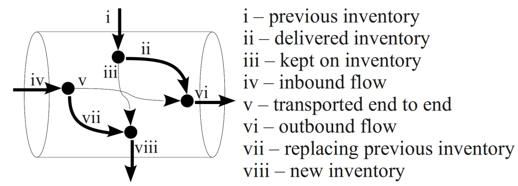

Figure 1: In-transit inventory relaxation network

An elementary control of in-transit inventory ordering is obtained by expanding the arc into the *in-transit inventory relaxation network*, depicted in figure 1. It approximates for each time slice and arc how a portion of certain commodity varies its in-transit inventory due to transportation on the arc. The connections of this network are handled as follows. For the *previous inventory* (i), a portion called *delivered inventory* (ii) reaches the *outbound flow* (vi). The remaining amount is *kept on inventory* (iii), assuming that the inbound flow did not push out all previous inventory. For an *inbound flow* (iv), the model adjusts the portion *transported end to end* (v) and the portion *replacing previous inventory* (vii). Without binary variables, the model does not guarantee that arc in-transit inventory is drained before further end to end transport, but the objective function will favor such solutions. The smaller we choose the time slices, the better gets the approximation and the larger becomes the formulation.

Once a commodity is delivered at the end of a path, it must wait for the next time slice to move onwards. Else, the model could instantaneously reach impracticable long distances by always opting to the transported end to end connection.

Arc flow relaxation In order to enhance the transit time approximation, we consider some additional aspects on flow rates. Transportation on arcs is accounted as the amount of commodity on each path that passes through each extremity. We require first that the inbound and outbound total amounts equal from one end to another, regardless of their commodity or path. In case of a flow reversion, the inbound flow at the other end forces the previously inbound flow to move outbound again at the first end. Second, for a transshipment that connects two arcs on a path, the received and forwarded amount of a certain commodity are equal.

On both ends of an arc we apply maximal flow rate capacity restrictions. The total duration of inbound and outbound flows at each end must not exceed the time slice duration. A slow commodity entering or leaving the arc will hold a long time amount for itself at one arc extremity and less time, and thus less quantity, will remain for other faster commodities. As total quantity on both extremities is bound, those faster commodities will be forced to lower quantities as if they were slower. Considering this approach on each arc of the path, the entire flow will be driven by the tightest maximal flow rate restriction caused by the slowest commodity.

Decision variables Associated to each node $n \in N$, commodity $c \in C$ and time slice $t \in T$:

- $\gamma_{nct} \in [\mathcal{I}^{min}_{nct}, \mathcal{I}^{max}_{nct}]$: amount node inventory at the end of the time slice.
- $\gamma'_{nct} \geq 0$: amount from previous node inventory and production that is kept locally for future use.

Transshipment on a path just forwards the received amount. In such cases, the outbound amount of an arc will be represented by the same decision variable as for the inbound amount of the next arc on the path. Associated with each path $p \in P$, commodity $c \in C$ and time slice $t \in T$:

- $\alpha^0_{pct} \geq 0$: amount for withdrawal at path origin node into the first arc on path;
- $\alpha^{l_p}_{pct} \geq 0$: amount for receipt at path destination node from the last arc on path;
- $\alpha^j_{pct} \geq 0, j \in \{1, ..., l_p - 1\}$: amount for transshipment from the i-th arc on path into the next one.

The *in-transit inventory relaxation network* depicted in figure 1 is represented as follows, for path $p \in P$, arc $a \in A$ on p, commodity $c \in C$, time slice $t \in T$, where $j = k^a_p$ and α^j_{pct} are inbound or outbound flows (connections iv or vi):

- $\beta^j_{pct} \geq 0$: amount of in-transit inventory (i or viii);
- $\beta^{j,(ii)}_{pct} \geq 0$: amount of delivered inventory (ii);
- $\beta^{j,(iii)}_{pct} \geq 0$: amount kept on inventory (iii);
- $\beta^{j,(v)}_{pct} \geq 0$: amount transported end to end (v);
- $\beta^{j,(vii)}_{pct} \geq 0$: amount replacing previous inventory (vii).

Additional index sets Following index sets simplify the model description, considering node $n \in N$ and arc $a \in A$:

- $P^O_n, P^D_n \subseteq P$: paths with origin or destination at node n;
- $\overrightarrow{P}_a, \overleftarrow{P}_a \subseteq P$: paths having arc a, main and inverse sense.

Objective function The cost is evaluated by expression 1. The effort is calculated proportional to the total amount transferred over the network. Minor penalties ρ_α and ρ_β apply to enable the *in-transit inventory relaxation*. Let o_p be the origin node of path $p \in P$:

$$\min \sum_{p \in P,\, c \in C,\, t \in T} \mathcal{E}_{o_p c}(\alpha^0_{pct}) + \sum_{\substack{p \in P,\, c \in C,\, t \in T \\ j \in [1..l_p - 1]}} (\rho_\alpha \alpha^j_{pct} + \rho_\beta \beta^{j,(iii)}_{pct}) \tag{1}$$

Constraints The typical conservation constraint has been split into two equations, connected by γ'_{nct}. This prevents an amount received at the destination node to move onwards within the same time slice. On constraint 2, production and previous node inventory are withdrawn or kept locally. On constraint 3, received and locally kept commodities are consumed or stored as new node inventory for the next time slice. For $n \in N$, $c \in C$, $t \in T$:

$$\mathcal{P}_{nct} + \gamma_{nc(t-1)} = \gamma'_{nct} + \sum_{p \in P^O_n} \alpha^0_{pct} \tag{2}$$

$$\mathcal{D}_{nct} + \gamma_{nct} = \gamma'_{nct} + \sum_{p \in P^D_n} \alpha^{l_p}_{pct} \tag{3}$$

The next constraints are related to the arc inventory relaxation and they implement the *in-transit inventory relaxation network* depicted in figure 1. For $p \in P$, $c \in C$, $t \in T$ and $j \in \{0, ..., l_p - 1\}$:

$$\beta^j_{pc(t-1)} = \beta^{j,(ii)}_{pct} + \beta^{j,(iii)}_{pct} \tag{4}$$

$$\beta^j_{pct} = \beta^{j,(vii)}_{pct} + \beta^{j,(iii)}_{pct} \tag{5}$$

$$\alpha^j_{pct} = \beta^{j,(vii)}_{pct} + \beta^{j,(v)}_{pct} \tag{6}$$

$$\alpha^{j+1}_{pct} = \beta^{j,(v)}_{pct} + \beta^{j,(ii)}_{pct} \tag{7}$$

The next constraints are related to the *arc flow relaxation*. Constraint 8 requires equal total inbound and outbound amount over all paths that pass the arc on its main sense. Constraint 9 is symmetric for the inverse sense. On transshipment, equality of received and forwarded amount is granted by construction. Constraint 10 and 11 limit the utilization of the arc at, respectively, origin and destination node extremity. For $a \in A$, $t \in T$, where a is written as $\overrightarrow{a}$ or $\overleftarrow{a}$ as a hint about the sense of the arc within the expression:

$$\sum_{p \in \overrightarrow{P}_{\overrightarrow{a}},\, c \in C} \alpha^{k^{\overrightarrow{a}}_p}_{pct} = \sum_{p \in \overrightarrow{P}_{\overrightarrow{a}},\, c \in C} \alpha^{k^{\overrightarrow{a}}_p + 1}_{pct} \tag{8}$$

$$\sum_{p \in \overleftarrow{P}_{\overleftarrow{a}},\, c \in C} \alpha^{k^{\overleftarrow{a}}_p}_{pct} = \sum_{p \in \overleftarrow{P}_{\overleftarrow{a}},\, c \in C} \alpha^{k^{\overleftarrow{a}}_p + 1}_{pct} \tag{9}$$

$$\sum_{p \in \overrightarrow{P}_{\overrightarrow{a}},\, c \in C} \frac{\alpha^{k^{\overrightarrow{a}}_p}_{pct}}{\overrightarrow{\mathcal{F}}^{max}_{\overrightarrow{a}c}} + \sum_{p \in \overleftarrow{P}_{\overleftarrow{a}},\, c \in C} \frac{\alpha^{k^{\overleftarrow{a}}_p + 1}_{pct}}{\overleftarrow{\mathcal{F}}^{max}_{\overleftarrow{a}c}} \leq \mathcal{T}_t \cdot \mathcal{M}_{at} \tag{10}$$

$$\sum_{p \in \overrightarrow{P}_{\overrightarrow{a}},\, c \in C} \frac{\alpha^{k^{\overrightarrow{a}}_p + 1}_{pct}}{\overrightarrow{\mathcal{F}}^{max}_{\overrightarrow{a}c}} + \sum_{p \in \overleftarrow{P}_{\overleftarrow{a}},\, c \in C} \frac{\alpha^{k^{\overleftarrow{a}}_p}_{pct}}{\overleftarrow{\mathcal{F}}^{max}_{\overleftarrow{a}c}} \leq \mathcal{T}_t \cdot \mathcal{M}_{at} \tag{11}$$

Model size The number of variables and constraints is bound by the product of number of commodities, time slices, paths and path maximal length. Real scenario sparsity leads to heuristics that limit the number of paths, their length and the combinations of commodities with nodes or arcs.

Experiments

Our real scenarios contain almost 75 classes of commodities, 25 nodes and 45 arcs. The model for two time slices produces about 100,000 variables and 50,000 constraints. It solves in less than a minute on a low end machine. With six more time slices, the model grows to nearly 300,000 variables and 200,000 constraints, reduced respectively to 150,000 and 65,000 after pre-processing. Execution required about ten minutes. Thus, the model remains tractable.

In order to highligh the benefits from the tactical aspects proposed in this work, we present an unit test as illustrative scenario whose initial network configuration is shown in figure 2. It contains three nodes: refineries A and C and distribution center B, all three allow storage for the two commodities that are H and L, repectively shown as gray and

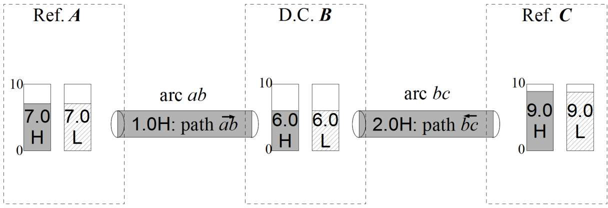

Figure 2: Initial network configuration.

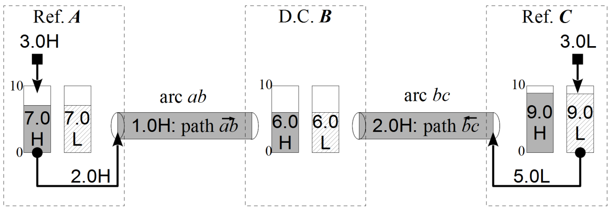

Figure 3: Network snapshot at beginning of time slice 1.

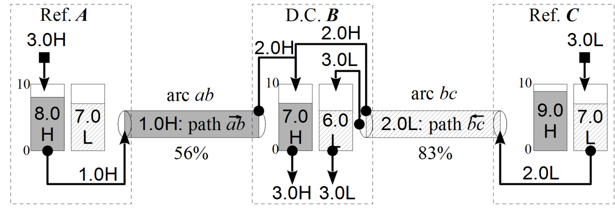

Figure 4: Network snapshot between time slice 1 and 2, with arc utilization rates of time slice 1.

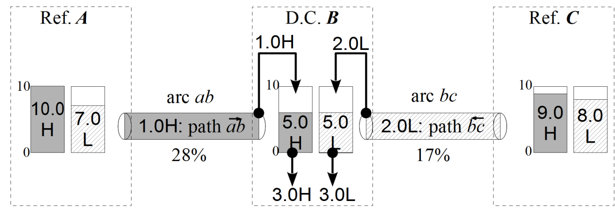

Figure 5: Network snapshot at end of time slice 2, with arc utilization rates of time slice 2.

hatched. All minimal and maximal storage capacities are always 5.0 and 10.0, respectively. There are two arcs, ab between A and B and bc between B and C. Six paths are available: $\{\overrightarrow{ab}\}$, $\{\overleftarrow{ab}\}$, $\{\overrightarrow{bc}\}$, $\{\overleftarrow{bc}\}$, $\{\overrightarrow{ab}, \overrightarrow{bc}\}$ and $\{\overleftarrow{ab}, \overleftarrow{bc}\}$. Arc ab is initially completely filled with 1.0 unit of H on path $\{\overrightarrow{ab}\}$. Arc bc is completely filled with 2.0 units of H on path $\{\overleftarrow{bc}\}$. Time is divided in two time slices of 15 days. Production and demand are respectively represented in figures 3, 4 and 5 as inbound arrows at the beginning of time slice and outbound arrows at the end of time slice.

The solution requires node A to deliver commodity H and node C to deliver commodity L to node B. Figure 3 shows the events at the beginning of the first time slice. Nodes A and C respectively receive a production of 3.0 units of H and L. The model decides to push 2.0 units of H from A into arc ab. Thus, this arc delivers 1.0 unit of in-transit inventory and transports 1.0 unit end to end. The arc gets filled with new in-transit inventory of the same commodity. Meanwhile, arc bc is found occupied with 2.0 units of in-transit inventory H that must be first transferred somewhere else. The model pushes 5.0 units of L from node C into arc bc, causing 2.0 units of delivered inventory of H and 3.0 units transported end to end of L. The arc gets filled with new inventory L. Node B then consumes 3.0 units of each H and L, as shown at the end of the first time slice in figure 4. At the start of the second time slice, nodes A and C produce again 3.0 units of, respectively, H and L. Since arcs ab and bc are already filled with adequate inventory, the model now decides to just push 1.0 unit of H from A into ab and 2.0 units of L from C to bc. Figure 5 shows the final state where node B consumes 3.0 units of each H and L.

The importance of the arc inventory relaxation and the arc flow relaxation becomes evident when we analyze the arc utilization rate within each time slice. Arc ab transfers only a small amount, but due to the low flow rate, that takes about 56% of the first time slice. On the second time slice, arc ab transfers half amount and arc utilization decreases proportionally. On arc bc, however, arc occupation reached a very high value: 86%. An investigation reveals that it takes 56% of the first time slice just to transfer the in-transit inventory out the arc with a slow flow rate. Then it takes the remaining 27% to transfer a larger amount of L. The model decided to transfer less commodity from A to B since the delivered inventory of arc bc was already suitable for the demand of H on B. A model without our proposed aspects would estimate a utilization of 27% instead of 86% for this example, falsely believing that the arc would still admit transferring a large amount of commodities. A better prediction of occupation adjusts the amount transported and may even lead to an assignment with alternative paths.

Conclusion

We described a network flow linear programming model that captures important aspects for the tactical planning of oil pipeline networks: capacity for in-transit inventory, transit time and flow reversal. We have shown that the *arc inventory relaxation* and the *arc flow relaxation* allow incorporating such features without introducing binary decision variables. By restricting the approach to a linear programming model, it is possible to solve large instances in reasonable time. Previous literature focused mostly on mixed integer programming models or opted to disregard detailed aspects of tactical planning.

The commodities with a slow flow rate and the existing in-transit commodities may have a strong influence on the pipeline utilization rate. If the pipeline availability gets tighter, a network model without our considered aspects might underestimate utilization rates and run the risk of producing a flow assignment exceeding the real network operational capacity. Our model may permit a closer approximation to the reality and provides a potentially better starting point for operational planning.

Many important aspects were left out, such as commodity *acquisition* or *discard* beyond network boundary, *substitution*, *degradation, blending* or *transformation* just to mention a few. As a future work we plan to revise the arc inventory relaxation for a better formulation of the flow reversion on in-transit inventory. Further work is foreseen to improve the arc flow relaxation to enable a dynamic flow rate calculation. Finally, the recommended time slicing is still open. Preliminary experiments suggest there is a time slice granularity from which the approximation improvement becomes irrelevant. The enhancements suggested for the problem provide additional challenges for a efficient and more realistic tactical planning for the pipeline network.

Acknowledgments

The author gratefully acknowledges the company under study for authorizing the publication of the information here present. Furthermore, the opinions and concepts presented are the sole responsibility of the author.

References

Aronson, J. 1989. A survey of dynamic network flows. *Annals of Operations Research* 20(1):1–66.

Boschetto, S. N.; Magatão, L.; Brondani, W. M.; Neves-Jr, F.; Arruda, L. V. R.; Barbosa-Póvoa, A. P. F. D.; and Relvas, S. 2010. An operational scheduling model to product distribution through a pipeline network. *Industrial & Engineering Chemistry Research* 49(12):5661–5682.

Dantzig, G. 1998. *Linear programming and extensions*. Landmarks in Physics and Mathematics. Princeton University Press.

Farahani, R. Z.; Asgari, N.; Heidari, N.; Hoseininia, M.; and Goh, M. 2011. Covering problems in facility location: A review. *Computers & Industrial Engineering*.

Ferber, D. F. 2011. Modelos de programação linear para planejamento tático da distribuição de suprimentos. In *XLIII SBPO - Simpósio Brasileiro de Pesquisa Operacional*.

Kennedy, J. L. 1993. *Oil and gas pipeline fundamentals*. PennWell Nontechnical Series. PennWell Books.

Lopes, T.; Ciré, A.; de Souza, C.; and Moura, A. 2010. A hybrid model for a multiproduct pipeline planning and scheduling problem. *Constraints* 15:151–189.

Luenberger, D., and Ye, Y. 2008. *Linear and nonlinear programming*. International series in operations research & management science. Springer.

Melo, M.; Nickel, S.; and da Gama, F. S. 2006. Dynamic multi-commodity capacitated facility location: a mathematical modeling framework for strategic supply chain planning. *Computers & Operations Research* 33(1):181 – 208.

Melo, T.; Nickel, S.; and da Gama, F. S. 2007. Facility location and supply chain management – a comprehensive review. Technical Report 130, Mathematik.

Melo, M.; Nickel, S.; and da Gama, F. S. 2009. Facility location and supply chain management - a review. *European Journal of Operational Research* 196(2):401 – 412.

Melo, M.; Nickel, S.; and da Gama, F. S. 2003. *Large-scale models for dynamic multi-commodity capacitated facility location*. Berichte des Fraunhofer ITWM. ITWM.

MirHassani, S. A.; Moradi, S.; and Taghinezhad, N. 2011. Algorithm for long-term scheduling of multiproduct pipelines. *Industrial & Engineering Chemistry Research* 50(24):13899–13910.

Mohitpour, M., and Jenkins, A. 2004. Pipeline egineering. In *Encyclopedia of Life Support Systems (EOLSS)*. EOLSS Publishers.

Moura, A.; de Souza, C.; Ciré, A.; and Lopes, T. 2008a. Planning and scheduling the operation of a very large oil pipeline network. In Stuckey, P., ed., *Principles and Practice of Constraint Programming*, volume 5202 of *Lecture Notes in Computer Science*. Springer Berlin / Heidelberg. 36–51.

Moura, A. V.; Souza, C. C. d.; Cire, A. A.; and Lopes, T. M. T. 2008b. Heuristics and constraint programming hybridizations for a real pipeline planning and scheduling problem. In *Proceedings of the 2008 11th IEEE International Conference on Computational Science and Engineering*, 455–462. IEEE Computer Society.

Mula, J.; Peidro, D.; Diaz-Madronero, M.; and Vicens, E. 2010. Mathematical programming models for supply chain production and transport planning. *European Journal of Operational Research* 204(3):377 – 390.

Relvas, S.; Matos, H. A.; Barbosa-Póvoa, A. P. F. D.; Fialho, J. a.; and Pinheiro, A. S. 2006. Pipeline scheduling and inventory management of a multiproduct distribution oil system. *Industrial & Engineering Chemistry Research* 45(23):7841–7855.

Skutella, M. 2009. An introduction to network flows over time. In *Research Trends in Combinatorial Optimization*. Springer-Verlag. 451–482.