

Sampling-Based Coverage Path Planning for Inspection of Complex Structures

Brendan Englot and **Franz S. Hover**

Massachusetts Institute of Technology
Department of Mechanical Engineering
Cambridge, MA, USA

Abstract

We present several new contributions in sampling-based coverage path planning, the task of finding feasible paths that give 100% sensor coverage of complex structures in obstacle-filled and visually occluded environments. First, we establish a framework for analyzing the probabilistic completeness of a sampling-based coverage algorithm, and derive results on the completeness and convergence of existing algorithms. Second, we introduce a new algorithm for the iterative improvement of a feasible coverage path; this relies on a sampling-based subroutine that makes asymptotically optimal local improvements to a feasible coverage path based on a strong generalization of the RRT* algorithm. We then apply the algorithm to the real-world task of autonomous in-water ship hull inspection. We use our improvement algorithm in conjunction with redundant roadmap coverage planning algorithm to produce paths that cover complex 3D environments with unprecedented efficiency.

Introduction

Coverage path planning enables fast and efficient task completion in applications that require an autonomous agent to sweep an end effector over some portion of its workspace, including sensing, cleaning, painting, and plowing (Choset 2001). Coverage path planning offers an advantage over greedy, next-best-view strategies when the area to be covered is expansive. Applications in 2D workspaces have utilized cellular decomposition methods (Choset and Pignon 1997) for open-area coverage, and boundary coverage has been achieved through both deterministic, Voronoi-based methods (Easton and Burdick 2005), and randomized, sampling-based methods (Danner and Kavraki 2000; Gonzalez-Baños and Latombe 2001). In 3D workspaces, the coverage task typically requires a full sweep of the interior or exterior boundary of a 3D structure embedded in the workspace. There are a variety of modular approaches which divide a workspace or structure into components and solve each component as a completely separate planning problem. This includes the 2.5D approach of dividing a workspace into 2D cross-sections, (Gonzalez-Baños and Latombe 2001), segmentation and cellular decomposition of 3D structures (Atkar et al. 2005), (Atkar et al. 2001), and

methods which combine multiple strategies (Cheng, Keller, and Kumar 2008).

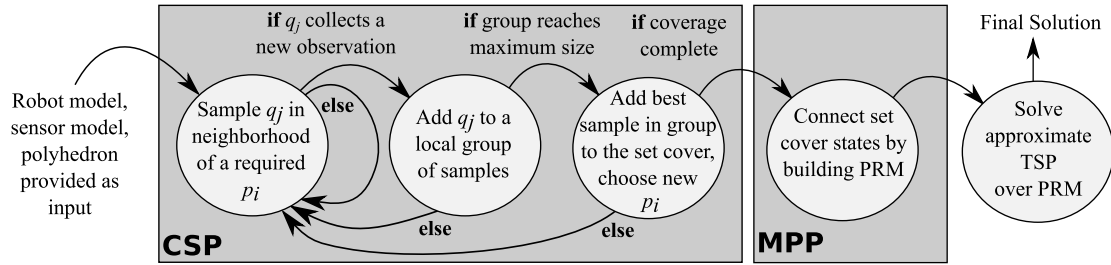
In confined areas where a robot cannot fit in the spaces between component structures, or occluded areas where visibility is blocked from all but a few vantage points, modular approaches are unsuitable. Global path planning strategies, utilizing sampling-based planning (Danner and Kavraki 2000), have been applied to this family of problems to find feasible paths that maneuver collision-free through confined areas and obtain occlusion-free views of the structure boundaries.

A desirable property for a sampling-based planning algorithm is *probabilistic completeness*. If a feasible solution exists for a given problem, then a probabilistically complete algorithm will find a solution with probability that tends to one as the number of random samples tends to infinity (Lamiroux and Laumond 1996). This property has been proven for a variety of sampling-based path planning algorithms, including the probabilistic roadmap (PRM) (Kavraki, Kolountzakis, and Latombe 1998) and the rapidly-exploring random tree (RRT) (LaValle and Kuffner 2001). Probabilistic completeness has not been explored, however, in the context of coverage path planning. Our first contribution in this paper is a framework for analyzing the probabilistic completeness of a sampling-based coverage path planning algorithm, and the extension of prior analyses to identify quantitative bounds on the probability of obtaining a feasible solution.

Since the existing algorithms for sampling-based coverage path planning produce feasible paths with no guarantee of optimality, it is desirable to improve the cost of an inspection path returned by one of these algorithms. Path planning algorithms such as PRM* and RRT* have been proven to adjust feasible paths into globally optimal paths as the number of samples approaches infinity (Karaman and Frazzoli 2011). We present an iterative procedure that significantly shortens coverage paths in practice, and we prove asymptotic optimality of the key subroutine in this procedure, employing a strong generalization of the RRT* algorithm. This is the second of our contributions.

After these analyses we consider the application of sampling-based coverage path planning to autonomous in-water ship hull inspection. This inspection task presents challenging complexity at the stern due to shafts, propellers, and rudders in close proximity to one another and to the

Watchman Route Algorithm using Dual Sampling, Danner and Kavraki, 2000



Redundant Roadmap Algorithm, Englot and Hover, 2011

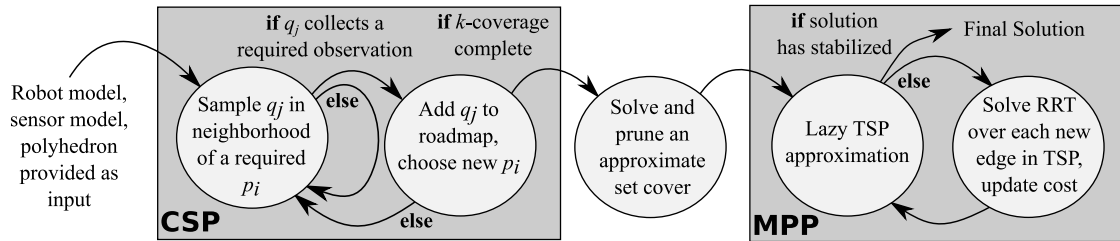


Figure 1: A stateflow diagram illustrating two algorithms for feasible sampling-based coverage path planning, highlighting the subroutines that solve the CSP and MPP subproblems.

hull. The Bluefin-MIT Hovering Autonomous Underwater Vehicle (Hover et al. 2007), pictured in Figure 4(d), is used to perform the inspection, but it cannot always fit into the spaces between component structures at the stern, and this demands a sampling-based coverage path planning strategy. We use the redundant roadmap algorithm and ship hull test cases of (Englot and Hover 2011) and apply our sampling-based improvement procedure, showing significant reductions in path length with reasonable computational effort.

Sampling-Based Planning of Feasible Coverage Paths

Here we analyze the sampling-based solution of robot coverage path planning. We divide the solution into two phases, since two distinct sampling-based subroutines are used in the algorithms considered here. The first phase is comprised of sampling feasible robot configurations that together give 100% coverage of a structure boundary, which we term the *coverage sampling problem* (CSP). The CSP as we define it differs from the classical art gallery and fortress problems (O’Rourke 1987), as it does not require the selection of a minimum cardinality set, but merely the selection of a feasible covering set. After a set of configurations from the CSP is selected for traversal, the second phase requires the linking of these configurations with feasible paths, which we refer to as the *multi-goal planning problem* (MPP).

Algorithms for Sampling-Based Coverage

The analysis of probabilistic completeness in this section is designed for compatibility with two prior algorithms used for feasible coverage path planning, which are illustrated in Figure 1. In both algorithms, it is assumed that the robot col-

lects sensor information at the nodes of a graph, as opposed to the edges of a graph, and an inspection tour among the nodes is planned and executed.

Both the watchman route and redundant roadmap algorithms solve the CSP by randomly sampling configurations until the required structure is covered, although the latter algorithm does not terminate until coverage of multiplicity k is achieved among the configurations in its roadmap. In the analysis of this section, we will assume that k -coverage is required so the analysis will apply to both algorithms.

The two algorithms also differ in their solution of the MPP. The watchman route algorithm connects the nodes in the set cover using a PRM. The redundant roadmap algorithm employs an iterative solution of the RRT over all goal-to-goal paths in the tour. Our analysis of probabilistic completeness will address both methods for solution of the MPP, drawing largely on existing results on the completeness of the individual PRM and RRT.

Set Systems and the CSP

We will represent the coverage sampling problem using the *set system* (P, Q) , also known as a *range space* (Haussler and Welzl 1987; Isler, Kannan, and Daniilidis 2004). P is a finite set of geometric primitives p_i comprising a structure that that must be covered by the robot. Q is the robot configuration space. Every feasible configuration $q_j \in Q$ maps to a subset of P viewed by the robot’s sensor. These sets of observed primitives are known as *ranges*. Given a finite set of ranges from Q , the *set cover problem* calls for the minimum number of configurations q_j such that all elements $p_i \in P$ are covered.

The problem can also be modeled using the *dual set sys-*

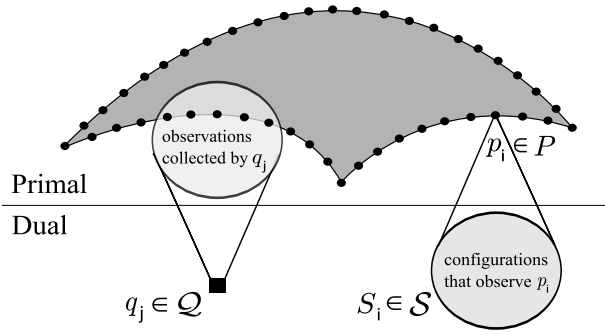


Figure 2: An illustration of the primal and dual set systems in the coverage sampling problem. Robot configurations q_j are used in both systems; the primal set cover problem employs the sensor observations collected at q_j and the dual hitting set problem employs the physical state of q_j . The primal (primitive) space is discrete and the dual (configuration) space is continuous.

tem (Q, S) , where $S_i \in S$ is the set of feasible robot configurations in Q that obtain views of the primitive $p_i \in P$. Given a finite set of robot configurations from Q , the *hitting set problem* calls for the minimum number of configurations q_j such that at least one configuration lies in every S_i for all $p_i \in P$. The structure of the primal and dual set systems for a robot coverage sampling problem is illustrated in Figure 2.

We now formally define the coverage sampling problem:

Definition 1 (Coverage Sampling Problem). *Let P be a finite set of discrete geometric primitives p_i comprising a structure to be inspected. Let the infinite set Q be the robot configuration space whose configurations $q_j \in Q$ map to observations of the Euclidean workspace which contains P . Let integer k be the number of times each $p_i \in P$ must be viewed. Find a finite set of feasible configurations $N \subset Q$ that obtains at least k distinct views of all $p_i \in P$.*

Let's now assume that an algorithm has been proposed for solution of the CSP using a random sampling scheme. We define the property of *probabilistic completeness* for a CSP algorithm as follows.

Definition 2 (Probabilistic Completeness of a CSP Algorithm). *Let CSA be a proposed coverage sampling algorithm for the CSP. Let (Q, S) be the dual set system over which the CSP is defined. Let $\delta = \min_{S_i \in S} \mu(S_i) / \mu(Q)$ be the volume fraction of the smallest range in S , where the measure μ represents the volume of the specified region of configuration space. If, when $\delta > 0$, the probability that at least k samples have landed in every $S_i \in S$ approaches one as the number of samples of Q drawn by CSA approaches infinity, then CSA is probabilistically complete.*

This definition implies that if a feasible CSP solution exists, a probabilistically complete CSP algorithm will find a feasible solution in the limit. In fact, we employ a rather strict definition of feasibility that deems a CSP to be feasible only if the smallest range in S has nonzero volume. This eliminates degenerate instances of the CSP from considera-

tion, in which some point $p_i \in P$ can only be viewed from a manifold in Q of lower dimension than Q itself.

Probabilistic Completeness of the CSP

We can analyze probabilistic completeness by studying the simple event of whether a randomly-sampled configuration q_j lands in a particular range $S_i \in S$. We will assume throughout the analysis that some subset of the configuration space $A \subseteq Q$, which is relevant for the inspection task, is chosen for sampling. A is often comprised of the region of Q that is within sensor viewing range of the structure. The probability of a sample q_j landing in S_i is equivalent to the ratio $\mu(S_i \cap A) / \mu(A)$. Using these preliminaries, we give the following theorem on probabilistic completeness.

Theorem 1 (Completeness and Convergence of the Discrete CSP). *Any algorithm for the CSP that samples uniformly at random from an infinite subset $A \subseteq Q$ such that $\mu(S_i \cap A) / \mu(A) \geq \epsilon > 0 \forall S_i \in S$ is probabilistically complete. Additionally, the probability that a feasible solution has not been found after m samples is bounded such that*

$$Pr[FAILURE] < |P| \cdot \frac{e^k}{e^{m\epsilon/2}}, \quad (1)$$

where $|P|$ is the number of geometric primitives $p_i \in P$.

Proof. The probability of m samples producing a feasible CSP solution is equivalent to the probability that at least k random samples have landed in every range $S_i \in S$. This fails to occur if there is at least one S_i in which fewer than k samples have landed. To model this event, we define the binomial random variable $X_i = X_{i_1} + X_{i_2} + \dots + X_{i_m}$, which gives the number of samples that have successfully landed in S_i out of m total trials. We express the probability of CSP algorithm failure as follows:

$$\begin{aligned} Pr[FAILURE] &\leq Pr \left[\bigcup_{i=1}^{|P|} X_i < k \right] \\ &\leq \sum_{i=1}^{|P|} Pr[X_i < k] \\ &\leq |P| \cdot Pr[X_{i^*} < k] \end{aligned} \quad (2)$$

Using the union bound, the probability that $X_i < k$ for at least one S_i is bounded above by the sum of the probabilities of this event for all $S_i \in S$. This is further simplified by taking $Pr[X_{i^*} < k]$ as an upper bound on the failures of all X_i , where X_{i^*} is the binomial random variable corresponding to the range in S that minimizes $\mu(S_i \cap A) / \mu(A)$.

We next bound $Pr[X_{i^*} < k]$ using the Chernoff bound for the lower tail of a Poisson distribution, which accurately represents a binomial distribution for large numbers of samples:

$$Pr[X_{i^*} < \gamma \cdot \lambda] < e^{-\frac{(1-\gamma)^2}{2}\lambda}, \quad \gamma \in [0, 1) \quad (3)$$

The parameter $\lambda = m\epsilon$ is the expected number of Poisson successes and γ is a fractional coefficient of λ . If we choose $\gamma = k/m\epsilon$, this allows the product $\gamma \cdot \lambda$ to evaluate to k , the

exact number of successes we wish to model. We can now simplify (3).

$$Pr[X_{i^*} < k] < e^{-\frac{m\epsilon}{2} + k + \frac{k^2}{2m\epsilon}} \leq \frac{e^k}{e^{m\epsilon/2}} \quad (4)$$

Combining the result of (4) with (2), we obtain the desired relationship between m and the probability of failure:

$$Pr[FAILURE] < |P| \cdot \frac{e^k}{e^{m\epsilon/2}}, \quad \lim_{m \rightarrow \infty} |P| \cdot \frac{e^k}{e^{m\epsilon/2}} = 0 \quad (5)$$

Since $\mu(S_i \cap A)/\mu(A) > 0 \forall S_i \in \mathcal{S}$, $\epsilon > 0$ and the limit behaves as indicated in (5). \square

The bounding methods used in this analysis have been used previously in other probabilistic completeness proofs. The union bound was used previously in the proof of completeness of the PRM (Kavraki, Kolountzakis, and Latombe 1998), and the Chernoff bound was used in the proof of completeness of the RRT (LaValle and Kuffner 2001). Our analysis requires *both* of these tools since we need to reach every $S_i \in \mathcal{S}$ and we must do so at least k times.

Any algorithm that Theorem 1 applies to benefits from a probability of failure that decreases exponentially in the number of samples m . The theorem applies to both the redundant roadmap algorithm and the watchman route algorithm as long as A is selected to allow $\epsilon > 0$ whenever $\delta > 0$. Both algorithms sample from a subset $A \subseteq \mathcal{Q}$ that includes all areas where the robot's geometric sensor footprint intersects at least one p_i , so this condition will always be satisfied.

It is also true that poor selection of A can result in the failure of a CSP algorithm to attain probabilistic completeness. Consider an algorithm which chooses a manifold A of lower dimension than \mathcal{Q} , such as a set of cross-sections in \mathbb{R}^2 from a set $\mathcal{Q} \subseteq \mathbb{R}^3$, which is often the strategy of 2.5D coverage algorithms. Even though $\mu(S_i)/\mu(\mathcal{Q}) > 0 \forall S_i \in \mathcal{S}$, it may be possible that $\mu(S_i \cap A)/\mu(A) = 0 \exists S_i \in \mathcal{S}$ and a 2.5D algorithm does not achieve probabilistic completeness.

Attraction Sequences and the MPP

Next we analyze probabilistic completeness of the MPP phase of sampling-based coverage path planning. Once a covering subset of robot configurations is selected for traversal, these goal configurations must be connected by a system of feasible paths. We formally define the MPP as follows:

Definition 3 (Multigoal Planning Problem). *Let $G \subset \mathcal{Q}$ be a finite set of robot configurations which comprise the set of goals selected for traversal. Find a set of feasible paths in \mathcal{Q} that joins all goals into a single connected component.*

If the goals are joined into a single connected component, then a feasible closed walk of all goals in G exists, giving a feasible solution to the coverage path planning problem. Both coverage path planning algorithms depicted in Figure 1 generate a feasible inspection tour that is compatible with Definition 3, although the redundant roadmap method, after solving the MPP in its first iteration, adds to the connected

component in each subsequent iteration to shorten the inspection tour. We now define probabilistic completeness in the context of the MPP.

Definition 4 (Probabilistic Completeness of a MPP Algorithm). *Let MPA be a proposed multigoal planning algorithm for the MPP. Let $G \subset \mathcal{Q}$ be the set of goals over which the MPP is defined. If, when a set of feasible paths in \mathcal{Q} exists that joins all goals into a single connected component, the probability that such a set is found by MPA approaches one as the number of samples of \mathcal{Q} drawn by MPA approaches infinity, then MPA is probabilistically complete.*

Proofs of completeness are straightforward for the MPP. For both the watchman route algorithm and the redundant roadmap algorithm, we utilize the notion of an *attraction sequence* (LaValle and Kuffner 2001). To connect a pair of goals $\{q_a, q_b\} \in G$ with a feasible path, an attraction sequence is a sequence of sets $\mathcal{A}_{a,b} = \{A_0, A_1, \dots, A_n\} \subseteq \mathcal{Q}$, where $A_0 = q_a$ and $A_n = q_b$, that bridge the gap between q_a and q_b . The defining property of an attraction sequence is the following: if a configuration q_{l-1} lies in A_{l-1} , and a sample q_l lands in A_l , then a PRM or RRT edge will be generated that connects q_{l-1} and q_l . In general it is desirable for an attraction sequence to have as few members A_l as possible, and so all A_l other than singletons A_0 and A_n should be as large in volume as possible.

We will use \mathcal{A}_{MPP} to refer to the set of all attraction sequences used in solving an instance of the multigoal planning problem, where $|\mathcal{A}_{MPP}|$ is the total number of sets A_l in \mathcal{A}_{MPP} . A worst-case analysis of the MPP will depend on both $|\mathcal{A}_{MPP}|$ and the volume fraction $\epsilon = \min_{A_l \in \mathcal{A}_{MPP}} \mu(A_l)/\mu(\mathcal{Q}_{free})$, where \mathcal{Q}_{free} is the obstacle-free portion of the configuration space. For both the watchman route algorithm and the redundant roadmap algorithm, a fast rate of decay of failure probability requires small $|\mathcal{A}_{MPP}|$ and large ϵ .

Probabilistic Completeness of the MPP

The watchman route algorithm solves the MPP by constructing a PRM that joins all goals into a single connected component. An all-pairs shortest paths algorithm can be used to determine the costs of all goal-to-goal paths, and a TSP algorithm can find a minimum-cost traversal. Unlike the typical use of the PRM, in which goal-to-goal queries are presented one at a time, the MPP presents a larger set of goals up-front and requires that all of these goals are connected to the roadmap. This can be handled easily by initializing the PRM so it contains the set of goals G . To show probabilistic completeness in this application we rely largely on prior analysis of the PRM (Kavraki, Kolountzakis, and Latombe 1998).

Theorem 2 (Completeness and Convergence of the PRM-Based Solution of MPP). *Constructing a PRM in \mathcal{Q} until the set of goals G belongs to a single connected component is a probabilistically complete algorithm for the MPP. Additionally, the probability that a feasible solution has not been found after m samples is bounded such that*

$$Pr[FAILURE] \leq \frac{|\mathcal{A}_{MPP}|}{e^{m\epsilon}}. \quad (6)$$

Proof. The analysis of the PRM in (Kavraki, Kolountzakis, and Latombe 1998) also applies to the use of a PRM for solution of the MPP. The key difference is that the standard PRM requires at least one sampled configuration to land in every set A_i in an attraction sequence $\mathcal{A}_{a,b}$, which is the attraction sequence for a single goal-to-goal path. The MPP requires at least one sampled configuration to land in every set A_i in the family of attraction sequences \mathcal{A}_{MPP} , and ϵ represents the smallest set in \mathcal{A}_{MPP} rather than the smallest set in $\mathcal{A}_{a,b}$. This difference in the analyses changes the numerator in (6) and the factor ϵ in the denominator of (6). In all feasible instances of the MPP, these quantities are finite and nonzero, respectively, and so the result of (Kavraki, Kolountzakis, and Latombe 1998) still applies. \square

In the case of the redundant roadmap algorithm, a revised ordering of the goals in G is determined in each iteration of the MPP procedure, and the RRT is subsequently called to find feasible goal-to-goal paths for all goal pairings in this ordering. In the absolute worst case, RRTs are constructed for all $O(n^2)$ possible goal-to-goal queries. To analyze this solution of the MPP, we will build on the analysis of RRT probabilistic completeness from LaValle and Kuffner (2001).

Theorem 3 (Completeness and Convergence of the RRT-Based Solution of MPP). *Iteratively connecting the goals in G by a sequence of RRTs is a probabilistically complete algorithm for the MPP. Additionally, the probability that a feasible solution has not been found after m samples is bounded such that*

$$Pr[FAILURE] \leq \frac{e^{|\mathcal{A}_{MPP}|}}{e^{m\epsilon/2}}. \quad (7)$$

Proof. The analysis of the RRT in (LaValle and Kuffner 2001) also applies to the use of RRTs for solution of the MPP. The key difference is that the standard RRT requires at least $|\mathcal{A}_{a,b}|$ successes in a series of m Bernoulli trials, in which $\mathcal{A}_{a,b}$ is an attraction sequence for a single goal-to-goal path. The MPP requires $|\mathcal{A}_{MPP}|$ successes instead, and ϵ represents the smallest set in \mathcal{A}_{MPP} rather than the smallest set in $\mathcal{A}_{a,b}$. This difference in the analyses changes the exponent in the numerator of (7) and the factor m in the denominator of (7). In all feasible instances of the MPP, these quantities are finite and nonzero, respectively, and so the result of (LaValle and Kuffner 2001) still applies. \square

We also note that in spite of the watchman route algorithm and redundant roadmap algorithm possessing probabilistic completeness with respect to both the CSP and MPP subproblems, there exists a family of coverage path planning problems for which a feasible 100%-coverage inspection tour may exist and both algorithms might fail. These are problems that contain a “prison cell” in \mathcal{Q}_{free} from which a configuration can collect meaningful sensor information but there exists no feasible path from the cell to the rest of the configuration space. As long as prison cells are avoided, any feasible CSP solution will constitute a feasible MPP solution. A variety of measures can be taken to ensure this

Algorithm 1 $W'_G = ShortenInspection(G, W_G, P, Obst)$

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1:  $W'_G \leftarrow W_G$ ;
2: while  $TimeRemaining > 0$  do
3:    $q_j \leftarrow ChooseRandomGoal(G)$ ;
4:    $P_j \leftarrow UniquelyObservedPrimitives(q_j, G)$ ;
5:    $(q'_j, W'_{q_{j-1}, q_{j+1}}) \leftarrow RRT_{||}^*(q_{j-1}, q_{j+1}, P_j, Obst)$ ;
6:    $W'_G \leftarrow W'_G \setminus W'_{q_{j-1}, q_{j+1}}$ ;
7:    $W'_G \leftarrow W'_G \cup W'_{q_{j-1}, q_{j+1}}$ ;
8:    $G \leftarrow G \setminus q_j$ ;
9:    $G \leftarrow G \cup q'_j$ ;
10:   $UpdateCoverageTopology(G)$ ;
11: end while
12: return  $W'_G$ 

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problem does not occur in practice; our specific solution is to ensure that all configurations sampled in the CSP can be connected via feasible path to a common origin in the configuration space.

A Sampling-Based Improvement Procedure

As we have shown, existing algorithms for sampling-based coverage path planning do possess the property of probabilistic completeness as it applies to the CSP and MPP subroutines. Despite this fact, neither algorithm offers a means of smoothing or shortening a solution by changing the locations of the goals. Here we present a sampling-based improvement procedure which is compatible with both the watchman route algorithm and the redundant roadmap algorithm.

An Asymptotically Optimal Subroutine

We assume that a feasible inspection tour is provided as input to the improvement procedure. The inspection is described by the closed walk W_G of the set of goals G . W_G contains the precise sequence of nodes and edges that are traversed in the inspection, which begins and ends at the same goal. W_G may include intermediate nodes that obtain no sensor information, but are required to maneuver safely around obstacles. The improvement procedure is summarized in Algorithm 1 (where $Obst$ denotes any obstacles that must be avoided). As time for improvement allows, the algorithm iteratively selects a goal configuration $q_j \in G$ and tries to find a lower-cost configuration q'_j that observes all primitives in P that are uniquely observed by q_j . This is achieved by the subroutine $RRT_{||}^*$, an implementation of the RRT^* algorithm (Karaman and Frazzoli 2011) in which two problems are solved in parallel: an optimal collision-free path from q_{j-1} to q'_j , and an optimal collision-free path from q'_j to q_{j+1} . Solving these problems in parallel gives $W'_{q_{j-1}, q_{j+1}}$, the portion of the walk W_G that travels between goals q_{j-1} and q_{j+1} and includes the intermediate goal q'_j . We term this subproblem the *local coverage planning problem*.

Definition 5 (Local Coverage Planning Problem). *Let $W'_{q_{j-1}, q_{j+1}}$ be a feasible path on the inspection tour W_G in which a robot travels from goal configuration q_{j-1} to goal configuration q_j to goal configuration q_{j+1} . Let $S_{i \in q_j}$ be*

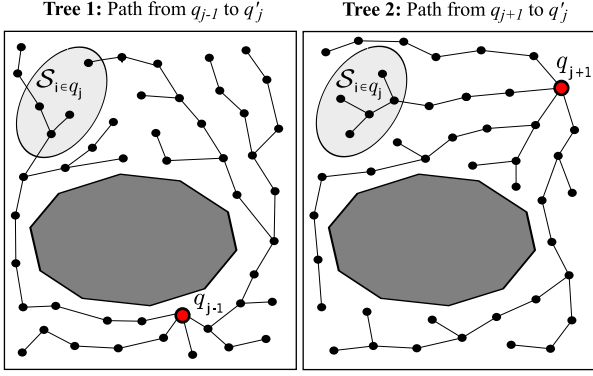


Figure 3: An illustration of the $RRT_{||}^*$ subroutine of the sampling-based improvement procedure.

the intersection of all ranges $S_i \in \mathcal{S}$ corresponding to the primitives $p_i \in P$ that are uniquely observed by goal configuration q_j . Find a replacement configuration q'_j that lies in $S_{i \in q_j}$ and a feasible path $W'_{q_{j-1}, q_{j+1}}$ such that the path is of minimum length over all possible choices of q'_j .

Definition 6 (Probabilistic Completeness of a Local Coverage Planning Algorithm). Let LCA be a proposed algorithm for the local coverage planning problem. If, when both a feasible path $W'_{q_{j-1}, q_{j+1}}$ exists such that $\mu(S_{i \in q_j}) / \mu(Q_{free}) \geq \epsilon > 0$ and there is non-degenerate clearance from obstacles along the full length of the path, the probability that such a path is found by LCA approaches one as the number of samples drawn from \mathcal{Q} approaches infinity, then LCA is probabilistically complete.

Definition 7 (Asymptotic Optimality of a Local Coverage Planning Algorithm). Let LCA be a probabilistically complete algorithm for the local coverage planning problem. If, when an optimal path $W^*_{q_{j-1}, q_{j+1}}$ exists with non-degenerate clearance from obstacles along the full length of the path, the length of the shortest path obtained by LCA approaches the optimal length $|W^*_{q_{j-1}, q_{j+1}}|$ as the number of samples drawn from \mathcal{Q} approaches infinity, then LCA is an asymptotically optimal algorithm.

We intend to show that the $RRT_{||}^*$ subroutine possesses both probabilistic completeness and asymptotic optimality. Figure 3 shows q_{j-1} , q_{j+1} , and $S_{i \in q_j}$ in the context of $RRT_{||}^*$. Tree 1 is rooted at q_{j-1} and Tree 2 is rooted at q_{j+1} . Both of these trees share $S_{i \in q_j}$ as a goal region. The two trees, unlike two completely separate instances of RRT^* , share the same sampling process. Every randomly sampled configuration must be introduced into the tree rooted at q_{j-1} and the tree rooted at q_{j+1} . When this occurs, the nearest node in each tree will attempt to “steer” toward the sample, and the tree will directly connect to the sample if this connection is collision-free and spans a distance less than the designated growth distance η . We now state the probabilistic completeness and optimality properties of $RRT_{||}^*$.

Theorem 4 (Probabilistic Completeness of $RRT_{||}^*$). $RRT_{||}^*$

is a probabilistically complete algorithm for the local coverage planning problem.

Proof. From the properties of RRT^* , we know that Tree 1 and Tree 2 will reach their respective goal regions in probability. We must also show, however, that they will have some identical nodes in their goal regions so that a feasible path $W'_{q_{j-1}, q_{j+1}}$ will be produced. Due to the condition on $S_{i \in q_j}$ in Definition 6, there is a nonzero probability that random samples will land in $S_{i \in q_j}$. The samples that land in $S_{i \in q_j}$ will be added as nodes to both Tree 1 and Tree 2 if they land within a distance η of existing nodes in both trees. We know this does occur because:

- The samples in an RRT^* tree converge to the uniform distribution over Q_{free} (Kuffner and LaValle 2000; Karman and Frazzoli 2011)
- The dispersion of the uniform distribution, which varies as $O((\log(m)/m)^{1/d})$ in the number of samples of a d -dimensional space (Niederreiter 1992), will eventually reach η as the number of samples increases

After enough samples are drawn, all new samples will lie within a distance η of multiple tree nodes, and samples landing in $S_{i \in q_j}$ will be directly connected to both trees. \square

This result is important because it demonstrates the key factors that will allow a feasible solution $W'_{q_{j-1}, q_{j+1}}$ to be obtained in finite time: the ease with which Trees 1 and 2 reach $S_{i \in q_j}$, and time required for the sampling sequence to achieve a dispersion of η . We now give the result on asymptotic optimality:

Theorem 5 (Asymptotic Optimality of $RRT_{||}^*$). $RRT_{||}^*$ is an asymptotically optimal algorithm for the local coverage planning problem.

Proof. $S_{i \in q_j}$ is the “goal region” of each tree in $RRT_{||}^*$, and in the limit, we will obtain the set of asymptotically optimal paths from q_{j-1} and q_{j+1} to the goal region, by the properties of RRT^* . By choosing the node $q'_j \in S_{i \in q_j}$ that minimizes the sum of distances to q_{j-1} in Tree 1 and q_{j+1} in Tree 2, we obtain the optimal path $W^*_{q_{j-1}, q_{j+1}}$. \square

Our improvement procedure is designed to extract the maximal benefit from recent results on asymptotically optimal sampling-based planning while avoiding non-trivial combinatorial optimization. If we added just one additional degree of freedom and tried to design $W_{q_{j-1}, q_{j+2}}$ optimally (which requires hitting the two sets $S_{i \in q_j}$ and $S_{i \in q_{j+1}}$ and connecting them with an optimal path from q_{j-1} to q_{j+2}) we could not do so by building trees. The much denser PRM^* would be required to find an optimal path between the infinite-set goal regions $S_{i \in q_j}$ and $S_{i \in q_{j+1}}$, and choosing optimal states q'_j and q'_{j+1} and the order in which to visit them would amount to solving the NP-hard generalized traveling salesman problem (Fischetti, Gonzalez, and Toth 1997) over the PRM^* roadmap.

Modifications for Autonomous Ship Hull Inspection

We now discuss the application of the improvement procedure to the problem of autonomous ship hull inspection. This is a unique challenge for coverage path planning in which the structure to be inspected is comprised of one large, contiguous piece, and the robot’s sensor footprint is small relative to the size of the structure. In turn, the set of goals required for 100% coverage is numerous, and every goal will be in close proximity to several others.

As a result, intermediate configurations are rarely needed between goal configurations in the inspection tour, as evidenced by prior computational work on this application (Englot and Hover 2011). This allows for a simplification of the improvement procedure, and the $RRT_{||}^*$ algorithm does not need to be used in its entirety. Instead, the algorithm will be used as a selection mechanism for goal-to-goal paths that have no intermediate nodes between q_{j-1} , q_j , and q_{j+1} . Sampling will occur only in $\mathcal{S}_{i \in q_j}$ (specifically, in a larger region of \mathcal{Q} known to contain $\mathcal{S}_{i \in q_j}$), and if a single graph edge cannot be built from each tree root to the sample q_j , sampling continues until either this task is achieved or the maximum number of samples is reached and we move to a different goal in the inspection.

A benefit of this simplification is that we need not wait until samples land near the optimal location in $\mathcal{S}_{i \in q_j}$; we can project samples toward this location. Because we are looking for solutions in which the goal q_j is connected directly to q_{j-1} and q_{j+1} by straight-line paths in \mathcal{Q}_{free} , we can move the individual samples from their random locations in $\mathcal{S}_{i \in q_j}$ to locations of improved cost, knowing that the path $W'_{q_{j-1}, q_{j+2}}$ also improves in cost. We do this using a growth distance ρ , by which we incrementally push a sample toward the optimal-cost frontier (a straight-line path connecting q_{j-1} and q_{j+1}) until a collision is detected or we cross the boundary of $\mathcal{S}_{i \in q_j}$. Many fewer samples need to be drawn to propagate new goals toward optimal-cost locations.

When the opposite situation occurs, and a structure to be inspected is comprised of separate pieces which may be far from one another, the benefits of $RRT_{||}^*$ can be fully realized and the algorithm will be needed in its entirety to connect goal configurations with high-quality feasible paths.

Updating the Coverage Topology

Algorithm 1 contains another important subroutine, $UpdateCoverageTopology(G)$, that we now address briefly. As goal configurations q_j are replaced by new goals q_j' that shrink the length of an inspection tour, the coverage topology among the goals changes and occasionally a goal in G becomes obsolete, contributing no unique sensor observations to the inspection. When this occurs, the obsolete goal is removed from the tour, and an attempt is made to connect q_{j-1} and q_{j+1} using a shorter path than the path through obsolete q_j . Sometimes, the two goals can be bridged by a single straight-line path, and other times intermediate nodes are needed, which are found using the RRT-Connect algorithm (Kuffner and LaValle 2000). Occasionally, a path shorter than the route through q_j cannot

be found, and q_j remains in the tour as an intermediate node, but is no longer a member of the goal set G .

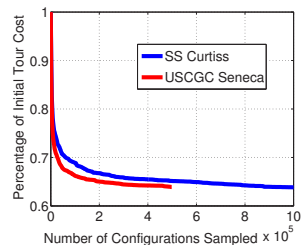
Computational Results

We now present results in which ship hull inspection paths are computed using the redundant roadmap algorithm of (Englot and Hover 2011) and iteratively shortened using our proposed improvement procedure. We use the HAUV configuration space and sensor model from this prior work, which, due to holonomic, fully-actuated dynamics and dominant hydrodynamic drag, models HAUV inspection planning as a purely geometric problem. The HAUV has a four degree-of-freedom state comprised of x , y , z , and yaw angle θ . We assume the sonar is operated at a sensor viewing range of 1-3m, giving high-resolution range scans within ± 15 degrees of vehicle-relative bearing, pitched up and down through ± 90 degrees at each configuration.

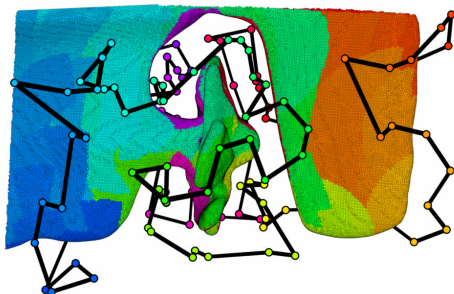
Paths are planned over two naval ships whose models were constructed from HAUV acoustic range data, the *SS Curtiss* and *USCGC Seneca*. These models are finely discretized such that a 10cm object will be detected on the hull if all mesh vertices are observed by the HAUV. The *SS Curtiss* model, a ship with a single seven-meter-diameter propeller, has 107,712 vertices to be covered in the inspection. The *USCGC Seneca*, a ship with two two-and-a-half-meter-diameter propellers, has 131,657 vertices. Initial, feasible paths for 100% coverage of the meshes are computed using roadmaps of redundancy ten, giving inspection tours whose hundred-or-so nodes are chosen from a one- to two-thousand node instance of the set cover.

Two hours were allotted in each problem instance for the computation of a feasible path and implementation of the improvement procedure. The improvement procedure was implemented in C++ and used the OMPL (<http://ompl.kavrakilab.org>), OpenSceneGraph (<http://www.openscenegraph.org/projects/osg>), and FLANN (<http://people.cs.ubc.ca/~mariusm/index.php/FLANN/FLANN>) libraries. Ten two-hour test cases were run for each mesh on a computer with a 3.20 GHz processor and 24 GB RAM running the Linux operating system.

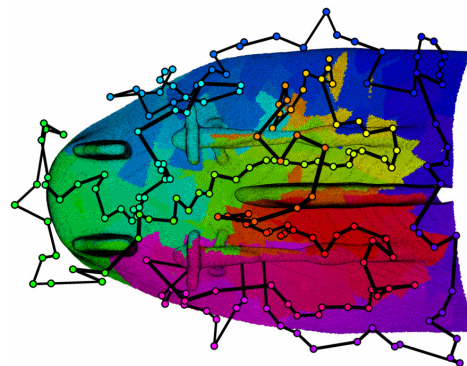
Computation of the initial feasible path required no more than twenty minutes in any problem instance. This initial step was solved faster for the *Curtiss*, which required a maximum of four minutes in any problem instance. Figure 4(a) illustrates the average shortening of the inspection tours as a function of the number of samples drawn by the improvement procedure. We show the total number of samples drawn, which includes samples found to be in collision with the mesh. The *Seneca* test cases each achieved at least a half million samples in the allotted time, while the *Curtiss* cases achieved at least six times this amount. Only the first one million *Curtiss* samples are pictured in 4(a), after which further improvements are minimal by comparison. The *Seneca* mesh contains more protruding, nonconvex structure, requiring more ray shooting computations per individual sample. Ray shooting checks the clearance of the line of sight between the robot sensor and a vertex of the mesh, and is the major added computational expense associated with coverage path planning.



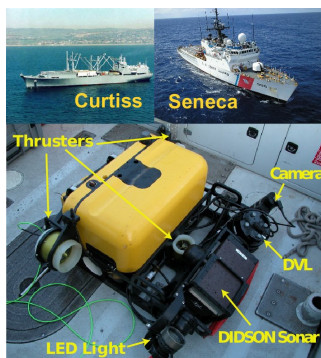
(a) Average improvement in tour length as a function of the number of configurations sampled.



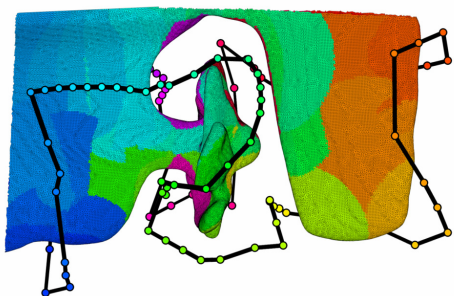
(b) Feasible tour for full coverage of *SS Curtiss* running gear. The tour is 176 m in length and contains 121 configurations.



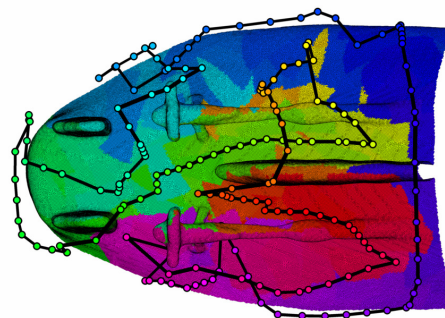
(c) Feasible tour for full coverage of *USCGC Seneca* running gear. The tour is 246 m in length and contains 192 configurations.



(d) Photo of Bluefin-MIT HAUV, *SS Curtiss*, and *USCGC Seneca*.



(e) Shortening the tour of (b) using improvement procedure. The shortened tour is 102 m in length and contains 97 configurations.



(f) Shortening the tour of (c) using improvement procedure. The shortened tour is 157 m in length and contains 169 configurations.

Figure 4: Representative full-coverage inspection paths before (top) and after (bottom) the improvement procedure. Each plotted point represents a position and orientation of the robot at which required information is collected. Robot configurations along each tour are color-coded and correspond to the colored patches of sensor information projected onto each ship model. The changes in color occur gradually and follow the sequence of the inspection tour. The thickness of each line segment along the path corresponds to the relative distance of that segment from the viewer’s perspective.

Although diminishing returns can be observed in 4(a) as cost improvements are made, the representative inspection tours plotted in Figure 4 show that significant simplification and shortening has occurred in the time allotted for improvement.

Conclusion

We have given an analysis of sampling-based coverage path planning, and proposed an iterative procedure for shortening feasible paths over complex structures. This method makes asymptotically optimal local improvements to an inspection, the best possible without invoking an NP-hard combinatorial optimization problem.

As is generally the case in the iterative improvement of paths in obstacle-filled environments, a larger investment is required to achieve an optimal (or near-optimal) solution than to simply construct a feasible solution. This investment is characterized by a diminishing returns relationship, but

it is worth pursuing when significant mission time can be saved as a result.

We have extended the work on this subject from traditional path planning to coverage path planning, in which not only is obstacle avoidance required, but also the observation of thousands of geometric primitives by the robot sensor. This is a challenging task for which sampling-based planning tools continue to be well-suited.

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References

Atkar, P.; Choset, H.; Rizzi, A.; and Acar, E. 2001. Exact cellular decomposition of closed orientable surfaces embedded in \mathbb{R}^3 . In

Proceedings of the IEEE International Conference on Robotics and Automation, volume 1, 699–704.

Atkar, P.; Greenfield, A.; Conner, D.; Choset, H.; and Rizzi, A. 2005. Hierarchical segmentation of surfaces embedded in \mathbb{R}^3 for auto-body painting. In *Proceedings of the IEEE International Conference on Robotics and Automation*, 572–577.

Cheng, P.; Keller, J.; and Kumar, V. 2008. Time-optimal uav trajectory planning for 3d urban structure coverage. In *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2750–2757.

Choset, H., and Pignon, P. 1997. Coverage path planning: the boustrophedon decomposition. In *Proceedings of the International Conference on Field and Service Robotics*.

Choset, H. 2001. Coverage for robotics - a survey of recent results. *Annals of Mathematics and Artificial Intelligence* 31:113–126.

Danner, T., and Kavraki, L. 2000. Randomized planning for short inspection paths. In *Proceedings of the IEEE International Conference on Robotics and Automation*, volume 2, 971–976.

Easton, K., and Burdick, J. 2005. A coverage algorithm for multi-robot boundary inspection. In *Proceedings of the IEEE International Conference on Robotics and Automation*, 727–734.

Englot, B., and Hover, F. 2011. Planning complex inspection tasks using redundant roadmaps. In *Proceedings of the International Symposium on Robotics Research*.

Fischetti, M.; Gonzalez, J.; and Toth, P. 1997. A branch-and-cut algorithm for the symmetric generalized traveling salesman problem. *Operations Research* 45(3):378–394.

Gonzalez-Baños, H., and Latombe, J.-C. 2001. A randomized art gallery algorithm for sensor placement. In *Proceedings of the 17th Annual ACM Symposium on Computational Geometry*, 232–240.

Hausser, D., and Welzl, E. 1987. Epsilon-nets and simplex range queries. *Discrete and Computational Geometry* 2(1):127–151.

Hover, F.; Vaganay, J.; Elkins, M.; Willcox, S.; Polidoro, V.; Morash, J.; Damus, R.; and Desset, S. 2007. A vehicle system for autonomous relative survey of in-water ships. *Marine Technology Society Journal* 41(2):44–55.

Isler, V.; Kannan, S.; and Daniilidis, K. 2004. Sampling based sensor-network deployment. In *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, 1780–1785.

Karaman, S., and Frazzoli, E. 2011. Sampling-based algorithms for optimal motion planning. *International Journal of Robotics Research* 30(7):846–894.

Kavraki, L.; Kolountzakis, M.; and Latombe, J.-C. 1998. Analysis of probabilistic roadmaps for path planning. *IEEE Transactions on Robotics and Automation* 14(1):166–171.

Kuffner, J., and LaValle, S. 2000. RRT-connect: An efficient approach to single-query path planning. In *Proceedings of the IEEE International Conference on Robotics and Automation*, volume 2, 995–1001.

Lamiroux, F., and Laumond, J. 1996. On the expected complexity of random path planning. In *Proceedings of the IEEE International Conference on Robotics and Automation*, volume 4, 3014–3019.

LaValle, S., and Kuffner, J. 2001. Randomized kinodynamic planning. *International Journal of Robotics Research* 20(5):378–400.

Niederreiter, H. 1992. *Random Number Generation and Quasi-Monte Carlo Methods*. Philadelphia, PA: Society for Industrial and Applied Mathematics.

O’Rourke, J. 1987. *Art gallery theorems and algorithms*. Oxford, UK: Oxford University Press.