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An Axiomatic Framework for Ex-Ante Dynamic Pricing Mechanisms in Smart Grid

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Abstract

In electricity markets, the choice of the right pricing regime is crucial for the utilities because the price they charge to their consumers, in anticipation of their demand in real-time, is a key determinant of their profits and ultimately their survival in competitive energy markets. Among the existing pricing regimes, in this paper, we consider ex-ante dynamic pricing schemes as (i) they help to address the peak demand problem (a crucial problem in smart grids), and (ii) they are transparent and fair to consumers as the cost of electricity can be calculated before the actual consumption. In particular, we propose an axiomatic framework that establishes the conceptual underpinnings of the class of ex-ante dynamic pricing schemes. We first propose five key axioms that reflect the criteria that are vital for energy utilities and their relationship with consumers. We then prove an impossibility theorem to show that there is no pricing regime that satisfies all the five axioms simultaneously. We also study multiple cost functions arising from various pricing regimes to examine the subset of axioms that they satisfy. We believe that our proposed framework in this paper is first of its kind to evaluate the class of ex-ante dynamic pricing schemes in a manner that can be operationalised by energy utilities.

Introduction

Meeting the growing demand for energy while mitigating the impact of fossil fuels on climate change is a major challenge for countries round the world. According to the U.S. EIA, energy demand is predicted to increase by 56% by 2040.[1] This challenge is exacerbated by the fact that demand for energy tends to peak at specific times of the day depending on the needs of industrial, commercial, and domestic consumers.[2] For example, domestic consumers tend to use more energy in the morning and evening than during the day. Instead, industrial users tend to use more energy during the day (DECC-UK 2014). Such peaks pose a number of challenges (Strbac 2008) as follows. First, they require heavy investment into 'peaking' power generation that is used for only a few hours a day. This, in turn, results in higher costs for end-users. Second, given that power plants need to ramp up during peak hours, they are typically powered by fossil fuels and therefore end up generating high volumes of greenhouse gases. Third, peak demand can cause instabilities in electricity networks if the required supply capacity is not available at the times peaks occur and may result in cascading blackouts (Nedic et al. 2006). In real terms, according to International Energy Agency 2003, the cost of energy could have been reduced by approximately 50% by lowering demand by 5% during the peak hours of the California electricity crisis in 2000/2001 (IEA 2003).

To alleviate such issues, a number of approaches have been proposed by researchers in an attempt to *diversify* consumption (Strbac 2008) (i.e., getting consumers to consume at different times such that the resulting aggregate actual consumption profile is 'diversified' or 'flattened'). Indeed, back in the 1980s, Schweppe and colleagues (Schweppe et al. 1988) proposed the idea of managing the demand of energy through incentives that vary over time (i.e., dynamic pricing). Thus, by adjusting prices according to demand, they aimed to get demand to follow supply as opposed to generation always ramping up when demand is higher. This approach is typically termed demand-side management (DSM) or demand response (DR)[3]. Thus, by applying demand-side management techniques, utilities have sought to encourage consumers to shift their consumption from peak hours to off-peak hours (Palensky and Dietrich 2011; Spees and Lave 2007). These pricing regimes tend to take a number of forms ranging from time-of-use pricing (TOU) whereby consumption at peak time is heavily penalised, to real-time pricing (RTP) whereby energy is priced according to demand at every half-hour. However, while a number of simulations have shown the pros and cons of such pricing regimes (Ramchurn et al. 2011), they have never been rigorously analysed to determine the trade-offs they create for utilities. Choosing the right pricing regime is crucial for utilities because the price they charge to their consumers, in anticipation of their demand in real-time, is a key determi-

[1]http://www.eia.gov/forecasts/ieo/.

[2]The US Energy Information Administration estimates that the peak to average ratio across the continental US is 1.7 and rising (see Fig. 1). This implies that utilities must have an additional 70% of their average generation capacity reserved for handling peak demand. This induces a huge stress on the stability and economic viability of the grid.

[3]Demand response is also specifically used in industry to describe the arbitrary curtailment of consumer demand.

nant of their profits, and ultimately their survival in competitive energy markets. Moreover DR pricing schemes may be susceptible to gaming, due to the presence of incentives (Borlick 2010).

In wholesale power markets, the cost of electricity is composed of two parts: a forward market price and a balancing market price (Wang, Lin, and Pedram 2013). The first part is the cost of electricity based on the unit cost of electricity at that time and the electricity consumption estimated (potentially months ahead) for that time period. This component corresponds to the forward market price of electricity. The second part deals with the deviation of the consumer's actual consumption from her expected demand at that time. This component can be regarded as the penalty. This is the part that affects the amount bought in the (real-time) balancing market (or for single operator power systems, the generation costs).

Intuition for the Axiomatic Work

Against this background, in this paper, we propose an axiomatic framework to evaluate demand-side management schemes. In particular, we work with *ex-ante* dynamic pricing schemes where, given any value of actual consumption by the consumer (at any time slot), the consumer can calculate the cost of electricity. Ex-ante pricing schemes are transparent and fair to consumers as the cost of electricity can be calculated before the actual consumption, as opposed to what happens in ex-post pricing schemes (as stated in (Bandyopadhyay et al. 2015b)). We propose five key axioms that reflect the criteria that are vital for energy utilities and their relationship with the consumers. The following key principles form the basis for defining the set of axioms in this paper:

- The higher is the consumption of electricity by a customer, the higher is the cost she has to pay.
- The cost of electricity should be designed in such a way that it would encourage consumers to reduce consumption during peak demand hours. One intuitive approach can be to increase the price of energy at the peak hours.
- In order to reduce their costs on the wholesale market, utilities aim to incentivise their consumers to shift their consumption from peak to off-peak hours.
- The utility buys energy from the forward market by anticipating the energy demand from its consumers. Any unexpected change in the consumption pattern of its consumer can force the utility to buy energy from the real-time wholesale market, at much higher price. So the utility should also penalize consumers for any such unexpected consumption patterns.

Building upon these intuitions, we perform a thorough analysis of our proposed axiomatic framework and show that it is impossible for any cost function to satisfy all the axioms simultaneously. We then analyse multiple pricing regimes to determine the subset of axioms that they satisfy and draw conclusions as to how these mechanisms would work under different assumptions about the consumer behaviour. To our knowledge, this is the first formal framework that allows for an analytic evaluation of various pricing regimes which, in turn, informs the design of pricing regimes by energy utilities.

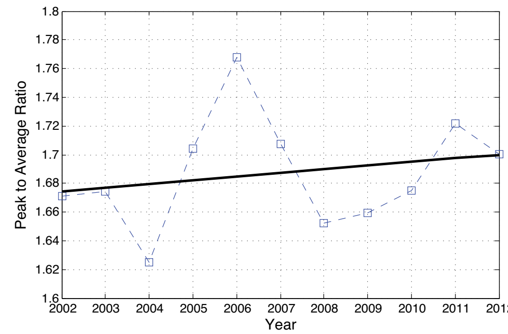

Figure 1: Peak-to-Average Energy Demand for Continental US. Courtesy US EIA (www.eia.gov/electricity/annual/).

Related Work

Axiomatic approaches have been applied to many areas of research in computer science and economics. In more detail, an axiomatic framework is a natural solution to decision problems for domains where the end goals are intuitively clear but not mathematically rigorous yet. In a typical axiomatic framework, intrinsic properties of the solution concept are stated in the form of axioms, and then the consistency of the axioms together in the form of some possibility or impossibility results are shown. For example, clustering has been studied extensively for the last few decades; however, there exists multiple definitions for clustering. Recently axiomatic approaches to clustering have been proposed (Kleinberg 2003; Zadeh and Ben-David 2009). Similarly, this approach has been adopted in many other domains such as social choice theory (Kelly 2014; Grandi 2011), and ranking & diversification (Gollapudi and Sharma 2009; Altman and Tennenholtz 2005). Following these lines, in electricity markets, we notice that the objectives of dynamic pricing are intuitively clear, but not yet mathematically rigorous. In this paper, we argue that an axiomatic approach provides a formal basis to construct and evaluate dynamic pricing mechanisms for Demand Response.

DR, in general, can be defined as the change of electricity usage by end consumers from their normal consumption pattern in response to some incentive based program (e.g., price or CO_2 emission reduction) implemented by the utility company (Albadi and El-Saadany 2007). DR programs help modulate the aggregate load to follow generation. A responsive consumer may be dispatched to supply "negawatts" of power by curtailing or shifting her load and DR programmes typically target specific consumers that are particularly high energy consumers.

A number of pricing mechanisms have been proposed for demand response in the literature. Game theory and mechanism design techniques are also used to address the problems in smart grids. For example, a novel multi-armed bandit incentive mechanism for demand response is proposed by (Jain, Narayanaswamy, and Narahari 2014). The formulation of an energy consumption scheduling game among users is carried out by (Mohsenian-Rad et al. 2010) and a game theoretic approach to optimize time-of-use pricing strategies is taken by (Yang, Tang, and Nehorai 2013). A non-cooperative game among all the consumers under some ex-post real-time pricing mechanisms is proposed and the characteristics of Nash equilibrium and socially efficient solutions are studied by (Bandyopadhyay et al. 2015b). However, so far, no analytical framework exists to help devise pricing mechanisms and analyze their performance in electricity markets. To the best of our knowledge, our work is the first approach to axiomatize the intrinsic and expected properties of electricity pricing schemes and analyze different pricing mechanisms based on this axiomatic framework.

Basic Definitions

Consider that the cost of electricity usage is computed on a daily[4] basis. Let a day be divided into T equal time slots. Energy consumed by individual consumers is recorded at each time slot and they are charged accordingly by the utility. At the start of each day, each consumer would reveal her expected demand (via a software agent residing on her smart meter to act on her behalf), d_i^t, $\forall i \in \{1, 2, \ldots, n\}$ and $\forall t \in \{1, 2, \ldots, T\}$. Otherwise, due to the availability of improved forecasting techniques (Bandyopadhyay et al. 2015a), the utility can as well suggest the consumers about their demand forecasts based on their past consumption history.

Note that the actual consumption of the consumers may vary from their expected demand. For each i and t, let $y_i^t \in Y_i$ be the actual consumption [5] for i-th consumer in the time slot t. Based on the consumers' revealed expected demand profile, the utility may be able to compute the aggregate level expected demand profile by all the consumers for the day. However, there can be a significant mismatch between the aggregate level actual consumption profile and the aggregate level expected demand profile, which may lead the utility to buy energy at a higher cost from the real-time market or be left with a significant amount of surplus energy, and it results in losses in both the cases (as explained in the first section). Our approach attempts to address this issue in this paper using an axiomatic framework.

A consumer needs to pay the cost to the utility based on her real-time electricity consumption. This cost would depend also on the deviation of the actual consumption y_i^t from the expected demand d_i^t for consumer i. We propose that the cost incurred by consumer i at time slot t is determined by a **cost function** C which can be defined as follows.

$$C : \mathbb{R}^3_{\geq 0} \to \mathbb{R}_{\geq 0}$$

It takes the following three parameters: (i) the total aggregated expected demand $d^t = \sum_{j=1}^{n} d_j^t$, at time t (ii) the expected demand d_i^t for the consumer i at time t; and (iii) the actual consumption y_i^t by consumer i at time t. The output of the cost function is a non-negative real number. So the total cost paid by consumer i in a day is given by:

$$C_i = \sum_{t=1}^{T} C(d^t, d_i^t, y_i^t), \quad \forall i = 1, 2, \cdots, n \qquad (1)$$

Note that, if the total expected aggregated demand vector $\mathbf{d}$ is revealed by the utility to all the consumers, for any given actual consumption value, any consumer can compute the cost of electricity just by knowing her own expected demand profile (which is reported to the utility at beginning of the day). Hence this cost function is ex-ante. The ex-ante cost functions (as defined above) would ensure the transparency of the cost incurred by consumers and also such functions do not require the individual consumers to reveal their actual consumption data (thus respecting the privacy concerns of the consumers). Table 1 summarizes the key notations used in the rest of this paper.

We now proceed to present an axiomatic framework wherein we enumerate a set of desirable properties that the cost function should satisfy. We also consider certain real-world cost functions to understand how far they satisfy the above set of desirable properties.[6]

Notations	Description
$N = \{1, \ldots, n\}$	Set of consumers
T	Number of discrete time slots in a day
i, j	Index for the consumers
t	Index for the time slots
y_i^t	Consumption of consumer i at time t
$y_i = (y_i^1, \ldots, y_i^T)$	Consumption profile of consumer i
d_i^t	Expected demand for consumer i at time t
$d_i = (d_i^1, \ldots, d_i^T)$	Demand profile for consumer i
$d^t = \sum_{i=1}^{n} d_i^t$	Aggregate demand at time t
$d = (d^1, \ldots, d^T)$	Aggregate demand profile by all consumers

Table 1: Notations used in this paper

An Axiomatic Framework

Here we formulate five axioms to capture the desirable properties (as discussed in the previous sections) of the ex-ante dynamic cost function and these are important from the perspective of both the consumers and the utilities.

Axiom 1 (Continuity of Cost Function) *The function $C(d^t, d_i^t, y_i^t)$ is continuous. Hence,* $\forall (\bar{d}^t, \bar{d}_i{}^t, \bar{y}_i{}^t) \in \mathbb{R}^3_{\geq 0}$,

$$\lim_{(d^t, d_i^t, y_i^t) \to (\bar{d}^t, \bar{d}_i{}^t, \bar{y}_i{}^t)} C(d^t, d_i^t, y_i^t) = C(\bar{d}^t, \bar{d}_i{}^t, \bar{y}_i{}^t).$$

This axiom ensures that there would not be any jump in the cost of electricity. If the cost function is not continuous, there can be sudden increase or decrease of cost even for a slight change of consumption (or expected demands), which would not be fair to the consumers.

[4]The same analysis can be done when the cost is calculated on a weekly or monthly basis.

[5]Similar types of set up have been used in (Jain, Narayanaswamy, and Narahari 2014; Vinyals et al. 2014)

[6]We chose cost functions that are typically used in open energy markets such as in the UK and the US.

Axiom 2 (Consumption Monotonicity) *The cost function $C(d^t, d_i^t, y_i^t)$ is strictly monotonically increasing with the actual consumption value y_i^t. That is, $\forall d^t, d_i^t$, and for any two y_i^t and $\bar{y}_i{}^t$ such that $y_i^t > \bar{y}_i{}^t$, then $C(d^t, d_i^t, y_i^t) > C(d^t, d_i^t, \bar{y}_i{}^t)$, $\forall i, t$.*

This axiom captures the fact that whenever a consumer increases her consumption at any particular point of time (assuming all other variables remain the same), then the consumer needs to pay more. This axiom is important because otherwise some consumer can adjust her expected demand at some point of time in such a way that she would get some incentive to increase her actual consumption at that point of time.

Axiom 3 (Peak Demand Compatibility) *The cost function $C(d^t, d_i^t, y_i^t)$ is strictly monotonically increasing with the aggregate demand value d^t. That is, $\forall d_i^t, y_i^t$, and for any two d^t and $\bar{d}^t$ such that $d^t > \bar{d}^t$, then $C(d^t, d_i^t, y_i^t) > C(\bar{d}^t, d_i^t, y_i^t)$, $\forall i, t$.*

This axiom captures the effect of *peak demand hours* on the cost incurred due to the consumption of electricity. Typically, the cost of electricity is high during peak demand hours and it is low during off-peak hours to encourage consumers to shift their consumption from peak to off-peak hours. Here the notion of *peak demand hours* is captured by the aggregated expected demand d^t at some time t. When the value of d^t is very high, we say that the time slot t is at peak demand. In an attempt to curtail demand, utilities tend to fix a high price for electricity at the time of peak demand. The above axiom essentially captures this requirement.

Axiom 4 (Dynamic Incentive) *Suppose the total demand at time t_1 and t_2 is d^{t_1} and d^{t_2} such that $d^{t_1} > d^{t_2}$. Consider any consumer $i \in N$ and her expected demands at these two time slots are $d_i^{t_1}$ and $d_i^{t_2}$ respectively and the actual consumption is $y_i^{t_1}$ and $y_i^{t_2}$ respectively. Then for $|d_i^{t_1} - y_i^{t_1}| = |d_i^{t_2} - y_i^{t_2}|$, it holds that $|C(d^{t_1}, d_i^{t_1}, d_i^{t_1}) - C(d^{t_1}, d_i^{t_1}, y_i^{t_1})| > |C(d^{t_2}, d_i^{t_2}, d_i^{t_2}) - C(d^{t_2}, d_i^{t_2}, y_i^{t_2})|$*

The above axiom captures the incentives (or penalties) a consumer receives by decreasing (or increasing) her actual consumption from her expected demand at a peak hour (t_1) to a non-peak hour (t_2). That is, when the aggregated demand is high at some point of time and a consumer reduces her consumption compared to her expected demand at that time, then the reduction of cost (incentive) for the consumer is more than when the aggregated demand is low at some point of time and that consumer reduces the same amount of consumption (compared to her expected demand). In the same vein, when a consumer increases her consumption from her expected demand at some time wherein the aggregated expected demand is high, then the consumer is penalized more than that when she increases the same amount of consumption wherein the expected aggregated demand is low. Thus this axiom would motivate consumers to reduce their consumption further during peak-demand hours.

Axiom 5 (Demand Consumption Mismatch) *For any fixed values of d^t and y_i^t, the cost function $C(d^t, d_i^t, y_i^t)$ is strictly monotonically increasing with $|d_i^t - y_i^t|$. That is, $\forall d^t, y_i^t$, and for any two d_i^t and $\bar{d}_i{}^t$ such that $|d_i^t - y_i^t| > |\bar{d}_i{}^t - y_i^t|$, then $C(d^t, d_i^t, y_i^t) > C(d^t, \bar{d}_i{}^t, y_i^t)$; and when $|d_i^t - y_i^t| = |\bar{d}_i{}^t - y_i^t|$, then $C(d^t, d_i^t, y_i^t) = C(d^t, \bar{d}_i{}^t, y_i^t)$.*

For a given value of actual consumption and the aggregate expected demand, this axiom ensures that the cost would increase with the increase in the difference between the expected demand and the actual consumption for any consumer. The above axiom penalizes the consumers as her expected demand deviates from her actual consumption. It is to be noted that the main purpose for the utility to collect the expected demand profiles from all consumers is to get an estimate on the actual consumption profiles for the individuals and also at the aggregate level. Thus it would help the utility to buy optimal amount of electricity from the electricity generators to avoid any shortage or wastage. So it is desirable that a consumer's expected demand on each time slot should be close to her actual consumption at that time slot.

An Impossibility Theorem

In this section, we present the key result of this paper. This result is in the form of an impossibility theorem, which states that there does not exist any cost function satisfying all the above five axioms simultaneously. To establish this result, we first prove a few supporting lemmas.

Lemma 1 *For any aggregate level expected demand d^t and $|d_i^{t_1} - y_i^{t_1}| = |d_i^{t_2} - y_i^{t_2}|$, it holds that $|C(d^t, d_i^{t_1}, d_i^{t_1}) - C(d^t, d_i^{t_1}, y_i^{t_1})| = |C(d^t, d_i^{t_2}, d_i^{t_2}) - C(d^t, d_i^{t_2}, y_i^{t_2})|$.*

Proof: Consider a function u such that $u(d^{t_1}, d_i^{t_1}, y_i^{t_1}) = |C(d^{t_1}, d_i^{t_1}, d_i^{t_1}) - C(d^{t_1}, d_i^{t_1}, y_i^{t_1})|$. Clearly the function u is continuous due to Axiom 1.

Now let $d^{t_1} = d^t + \epsilon$ and $d^{t_2} = d^t$. When $\epsilon > 0$, then Axiom 4 implies that $u(d^{t_1}, d_i^{t_1}, y_i^{t_1}) > u(d^{t_2}, d_i^{t_2}, y_i^{t_2})$. When $\epsilon < 0$, then Axiom 4 implies that $u(d^{t_1}, d_i^{t_1}, y_i^{t_1}) < u(d^{t_2}, d_i^{t_2}, y_i^{t_2})$. Since $u(.)$ is continuous at $(d^{t_2}, d_i^{t_2}, y_i^{t_2})$, we have that

$$\lim_{\epsilon \to 0} u(d^{t_1}, d_i^{t_1}, y_i^{t_1}) = u(d^{t_2}, d_i^{t_2}, y_i^{t_2}).$$

This further implies that $|C(d^t, d_i^{t_1}, d_i^{t_1}) - C(d^t, d_i^{t_1}, y_i^{t_1})| = |C(d^t, d_i^{t_2}, d_i^{t_2}) - C(d^t, d_i^{t_2}, y_i^{t_2})|$. ■

For notational convenience, for a fixed i, t, and d^t, let us assume that $C(d^t, d_i^t, y_i^t) = f_{d^t}(d_i^t, y_i^t) = f(d_i^t, y_i^t) = f(d, y)$. Using this notation, the above lemma can be interpreted as: $f(d, d+\delta) - f(d, d)$ is independent of d, i.e., the penalty for deviating from the expected demand is only dependent on the magnitude of deviation, but not on the actual consumption.

Lemma 2 *The function $f(\cdot)$ can be written as:*

$$f(d, y) = g(d) + h(y - d),$$

for some functions $g(\cdot)$ and $h(\cdot)$.

Proof: From Lemma 1, $f(d_1, d_1 + \delta) - f(d_1, d_1) = f(d_2, d_2 + \delta) - f(d_2, d_2)$, $\forall d_1, d_2$, and δ is any real number consistent with the function definition. That is, this difference should be independent of d_1 and d_2, and depends only on δ. Hence, $f(d, d+\delta) - f(d, d) = h(\delta)$, for some function

h, with $h(0) = 0$. Now,

$$f(d,y) - f(d,d) = h(y-d)$$

$$\Rightarrow f(d,y) = f(d,d) + h(y-d) = g(d) + h(y-d)$$

for some function $g(.)$. ■

Lemma 3 *The function $h(.)$ is an odd function. That is, $h(\delta) = -h(-\delta), \forall\delta$.*

Proof: From Lemma 1, we have that

$$f(d, d+\delta) - f(d,d) = f(d,d) - f(d, d-\delta).$$

Then, using Lemma 2, we get

$$g(d) + h(\delta) - g(d) - h(0) = g(d) + h(0) - g(d) - h(-\delta)$$

$$\Rightarrow h(\delta) = -h(\delta).$$ ■

Lemma 4 *The function $g(.)$ is linear; i.e., $g(d) = ad + b$ for any arbitrary constants a and b.*

Proof: We get from Axiom 5 that

$$f(d-\delta, d) = f(d+\delta, d).$$

Then, using Lemma 2, we get

$$g(d-\delta) + h(\delta) = g(d+\delta) + h(-\delta)$$

$$\Rightarrow g(d+\delta) - g(d-\delta) = 2h(\delta) \quad \text{[From lemma 3]}.$$

This proves that $g(.)$ is a linear function on the variable d. ■

The above results state that any cost function satisfying the axioms *must* be such that for a given aggregate expected demand, it decomposes additively into a linear function which varies with the expected demand and an odd function that varies with the difference between expected demand and actual consumption. We now formally state and prove the impossibility theorem.

Theorem 1 *There does not exist any cost function that simultaneously satisfies all the five axioms.*

Proof: Let us assume that there exists a cost function $C(.)$ that satisfies all the five axioms. We prove this theorem by establishing a contradiction to this assumption. Towards this end, we choose any arbitrary value and fix it for the variable d^t for the rest of this proof. Consider the function $f(d,y)$ as stated above wherein d and y are the expected demand and actual consumption respectively. We omit the subscript i and the superscript t (as before) for this analysis.

Case 1: Let us consider two different values of expected demand d_1 and d_2 such that $d_1 < d_2 < y$, where y is the actual consumption. Clearly from Axiom 5, we get

$$f(d_1, y) > f(d_2, y)$$

$$ad_1 + b + h(y-d_1) > ad_2 + b + h(y-d_2)$$

[From Lemma 2 and Lemma 4]

$$\therefore h(x_1) - h(x_2) > a(x_1 - x_2), \tag{2}$$

where $x_1 = y - d_1$ and $x_2 = y - d_2$.

Case 2: Consider that $d_1 = y + \delta$, and $d_2 = y - \delta$. Then,

$$f(d_1, y) = f(d_2, y) \quad \text{[From Axiom 5]}$$

$$a(y+\delta) + b + h(-\delta) = a(y-\delta) + b + h(\delta) \quad \text{[Lemma 4]}$$

$$a\delta - h(\delta) = -a\delta + h(\delta) \quad \text{[From Lemma 3]}$$

$$\Rightarrow a\delta = h(\delta)$$

Hence for any choice of x_1 and x_2, $h(x_1) - h(x_2) = a(x_1 - x_2)$, which is a contradiction to Equation (2). Hence proved. ■

Existence of Cost Functions Satisfying Subsets of Axioms

In this section, we consider meaningful subsets of axioms and check if there exists any cost function satisfying each such subset of axioms.

Theorem 2 *There exists a cost function that satisfies Axioms 1, 2, 3, and 4 simultaneously.*

Proof: Consider the following cost function:

$$C(d^t, d_i^t, y_i^t) = A d^t y_i^t \tag{3}$$

where $A > 0$ is an arbitrary constant. Clearly, this cost function satisfies Axiom 1. The cost would definitely increase with unilateral (keeping other variables fixed) increase of both d^t and y_i^t respectively. Thus, this cost function satisfies Axiom 2 and Axiom 3.

To show that it satisfies Axiom 4, let us consider two aggregate expected demands d^{t_1} and d^{t^2} such that $d^{t_1} > d^{t_2}$. Also, consider that $|d_i^{t_1} - y_i^{t_1}| = |d_i^{t_2} - y_i^{t_2}|$. Now,

$$|C(d^{t_1}, d_i^{t_1}, d_i^{t_1}) - C(d^{t_1}, d_i^{t_1}, y_i^{t_1})|$$

$$= |Ad^{t_1}(d_i^{t_1} - y_i^{t_1})|$$

$$= |Ad^{t_1}(d_i^{t_2} - y_i^{t_2})| \quad [\text{as } |d_i^{t_1} - y_i^{t_1}| = |d_i^{t_2} - y_i^{t_2}|]$$

$$> |Ad^{t_2}(d_i^{t_2} - y_i^{t_2})| \quad [\text{as } d^{t_1} > d^{t_2}]$$

$$|C(d^{t_2}, d_i^{t_2}, d_i^{t_2}) - C(d^{t_2}, d_i^{t_2}, y_i^{t_2})|$$

Hence it satisfies Axiom 4. ■

Theorem 3 *There exists a cost function that satisfies Axioms 1, 2, 3, and 5 simultaneously.*

Proof: Consider the following cost function:

$$C(d^t, d_i^t, y_i^t) = Ad^t + By_i^t + D|d_i^t - y_i^t| \tag{4}$$

where $A, B, C > 0$ and $B > C$ are arbitrary constants. It is not difficult to see that this cost function satisfies Axiom 1, as all the components are continuous in nature.

To prove that it satisfies Axiom 2, we select two actual consumption values y_i^t and $\bar{y}_i{}^t$ such that $\bar{y}_i{}^t - y_i^t = \epsilon > 0$. Now,

$$C(d^t, d_i^t, \bar{y}_i{}^t) - C(d^t, d_i^t, y_i^t)$$

$$= B(\bar{y}_i{}^t - y_i^t) + D(|d_i^t - \bar{y}_i{}^t| - |d_i^t - y_i^t|)$$

Now if $d_i^t < y_i^t$, then $(|d_i^t - \bar{y}_i{}^t| - |d_i^t - y_i^t|) > 0$. And if $d_i^t > y_i^t$, then $|d_i^t - \bar{y}_i{}^t| \geq |d_i^t - y_i^t| - \epsilon$, and so $(|d_i^t - \bar{y}_i{}^t| - |d_i^t - y_i^t|) \geq -\epsilon$. Hence,

$$C(d^t, d_i^t, \bar{y}_i{}^t) - C(d^t, d_i^t, y_i^t)$$

$$\geq B\epsilon - D\epsilon \quad [\because \bar{y}_i{}^t - y_i^t = \epsilon]$$

$$> 0 \quad [\because B > D]$$

That is, the cost is increasing with the actual consumption while other variables are fixed. Hence it satisfies Axiom 2.

Now the cost also increases when d^t increases, keeping all other variables fixed, and hence it satisfies Axiom 3. Since the cost function increases with the term $|d_i^t - y_i^t|$ while d^t and y_i^t are fixed, it satisfies Axiom 5. ■

So we have shown that, though it is not possible to satisfy all the five axioms simultaneously, there exist simple and intuitive cost functions that satisfy two different subsets, each having 4 axioms of this framework. We will discuss more on this trade-off in the last section.

Formulation and Evaluation of Various Cost Functions

In this section, we evaluate different electricity tariffs and cost functions with respect to our axiomatic framework. Specifically, we would like to see how different types of cost functions satisfy different subsets of axioms. Towards this end, we consider different cost functions which are currently implemented by the utilities or exists in the literature or very relevant to our framework. Along with their mathematical properties, we also give intuition about their possible impact on addressing different problems of smart grid.

Flat Rate Cost: The total cost to the consumers is calculated on a flat rate over the total consumption over T time slots. If the cost of a unit of electricity is u at any time $t = 1, 2, \cdots, n$; then the cost to any consumer i at time t is:

$$C(y_i^t) = uy_i^t \tag{5}$$

So total cost over T time slots is, $C_i = u \sum_{t=1}^{T} y_i^t$. In this set up, typically consumers do not need to report their expected demands d_i^t at the beginning of the day. To fit this into our framework, we assume that the cost function is invariant to expected demand of the consumer and aggregate level expected demand.

It is easy to see that this cost function satisfies Axioms 1 and 2. As the cost does not depend on aggregate demand d^t, and expected demand d_i^t, it does not satisfy Axioms 3, 4, and 5. In general, this type of cost functions have been used widely in different parts of the world mainly because they are very simple and easy to understand. However, such cost functions typically result in uncontrollable peaks in demand.

Time-of-Use Pricing: In this scheme, the unit price of electricity at some point of time t is predetermined at the beginning of the day. Suppose this cost per unit is $u^t > 0$ at time t. Hence the cost to any consumer i at time t is $C(y_i^t) = u^t y_i^t$. Here also we assume that the cost function is invariant to aggregate level and expected demands.

It is clear that this cost function satisfies Axioms 1 and 2. As the cost does not depend on d^t and d_i^t, it does not satisfy Axioms 3, 4, and 5.

Time-of-use pricing is popular in different parts of Europe and USA. In such schemes, the price (unit cost of electricity u^t) is high when the demand is high. But they calculate the peak hours roughly based on historical demands, and this remains fixed for long period of time (may be for few months or even years). This is why they are not able to capture the daily behavioral changes of the consumers in electricity consumption. Instead, in our framework, peak hours are determined based on the aggregate level expected demand profile, which is calculated at the beginning of the day or so. Hence our framework should be more efficient in addressing peak-energy demand problem in smart grids.

Prediction-of-Use Tariff: Prediction-of-Use Tariff has been discussed in Vinyals (Vinyals et al. 2014). Their set up is similar to ours in terms of reporting (predicting) the expected demand and then have an actual consumption in real time. The tariff in Prediction-of-Use is defined as follows:

$$C(d_i^t, y_i^t) = \begin{cases} py_i^t + \bar{p}(y_i^t - d_i^t), & if \;\; d_i^t \leq y_i^t \\ py_i^t + \underline{p}(d_i^t - y_i^t), & otherwise \end{cases} \tag{6}$$

Here, $p, \bar{p} > 0$ and $0 < \underline{p} \leq p$. As aggregate level expected demand d^t is not present in the function definition, we assume that the function is invariant over d^t. This cost function satisfies Axiom 1. As $C_i^t(d_i^t, y_i^t) \geq C_i^t(d_i^t, \bar{y_i}^t)$ for any $y_i^t > \bar{y_i}^t$ (Vinyals et al. 2014), it satisfies Axiom 2. As this cost function does not consider the total expected demand at any point of time to determine the cost at that time, it does not satisfy Axioms 3 and 4. It is also evident that this cost function can satisfy Axiom 5 only when $\bar{p} = \underline{p}$.

Clearly, this cost function can penalize a consumer for over or underestimating her consumption. But it cannot solve the peak energy demand problem as the price of energy is invariant to aggregate level (expected) demand.

The summary of this whole evaluation is given in Table 2. We discuss the significance of this evaluation along with other theoretical results in the next section.

Cost Function	Axiom 1	Axiom 2	Axiom 3	Axiom 4	Axiom 5
Flat Rate Cost	✓	✓	✗	✗	✗
Time-of-Use Pricing	✓	✓	✗	✗	✗
Prediction-of-Use Tariffs	✓	✓	✗	✗	✓[7]
Cost Function in Eq. 3	✓	✓	✓	✓	✗
Cost Function in Eq. 4	✓	✓	✓	✗	✓

Table 2: Different cost functions satisfying subsets of axioms of the proposed framework

Discussion and Conclusions

We have proposed an axiomatic framework in this paper, which gives a formal basis to construct and compare different cost functions based on the axioms they satisfy. We have justified the need of the axioms. We have addressed different issues of smart grids such as the peak-demand problem or incentivizing (or penalizing) consumers based on their shift of load during peak-hours. We have shown that it is impossible to satisfy all the axioms of our framework simultaneously. However, we have also presented different cost functions which satisfy different subsets of axioms in the framework.

Based on their own requirements, utilities need to select a cost function that satisfies the axioms they need the most. For example, Axiom 5 talks about penalizing a consumer for over or under estimating her demand. This would also encourage consumers to reveal their consumption truthfully at the beginning of the day. Instead, in a different setting, when the utility predicts the expected demand of individual consumers rather than consumers reporting the demands (as in (Vinyals et al. 2014)), Axiom 5 becomes less relevant to the utility than the other axioms. So the utility may go for a cost function as stated in equation 3, which satisfies all the axioms except Axiom 5 (see theorem 2). Further in Table 2, we have actually shown different possibility results through different cost functions, along with their practical interpretation and possible impact on the smart grid. We have not considered the impact of distributed and renewable generation in this framework. So an interesting future direction

[7] Assuming $\bar{p} = \underline{p}$ in equation 6

to extend the proposed axiomatic framework is to consider their effect into this framework. In conclusion, we hope that our axiomatic framework helps to advance the mathematical understanding of pricing regimes as well as to design new pricing mechanisms.

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