Modeling Status Theory in Trust Prediction

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Abstract

With the pervasion of social media, trust has been playing more of an important role in helping online users collect reliable information. In reality, user-specified trust relations are often very sparse; hence, inferring unknown trust relations has attracted increasing attention in recent years. Social status is one of the most important concepts in trust, and status theory is developed to help us understand the important role of social status in the formation of trust relations. In this paper, we investigate how to exploit social status in trust prediction by modeling status theory. We first verify status theory in trust relations, then provide a principled way to model it mathematically, and propose a novel framework sTrust which incorporates status theory for trust prediction. Experimental results on real-world datasets demonstrate the effectiveness of the proposed framework. Further experiments are conducted to understand the importance of status theory in trust prediction.

Introduction

With the growing popularity of social media, trust, as a decision-support tool, dissects relevant and reliable information sources for online users (Gefen, Karahanna, and Straub 2003; McKnight, Choudhury, and Kacmar 2003; Ziegler and Golbeck 2007; Liu, Wang, and Orgun 2012). Trust has been extensively studied by the computer science community, which promotes many trust-related applications, such as trust-aware recommendation system (Golbeck 2009; Ma et al. 2011; Tang, Gao, and Liu 2012), high-quality user generated content finding (Lu et al. 2010) and viral marketing (Richardson and Domingos 2002). However, in reality, explicit trust relations are extremely sparse; hence, inferring unknown trust relations attracts increasing attention in recent years.

Existing trust prediction algorithms can be roughly categorized into two groups - supervised methods and unsupervised methods (Tang and Liu 2014). Supervised methods boil down the problem of trust prediction into a classification problem, which first represents each pair of users by extracting features from available sources and then treats the existence of trust as labels. Liu et al (Liu et al. 2008) propose a classification approach to address the trust prediction problem by developing a taxonomy to obtain an extensive set of relevant features derived from user attributes and user interactions in an online community. Korovaiko et al (Korovaiko and Thomo 2013) focus on a case where the background data are user ratings for online product reviews and show that the state-of-the-art classifiers can do an impressive job in predicting trust based on extracted features. Zolfaghar et al (Zolfaghar and Aghaie 2012) provide a framework of social trust-inducing factors that contribute in trust formation process using data mining and classification approaches, and then investigate the role of these factors in predicting trust between users by experimental study on real data from Epinions. While unsupervised methods try to infer trust relations based on some properties of trust networks such as trust propagation and low-rank representations. Liu et al (Liu, Wang, and Orgun 2012) propose a social context-aware trust network discovery algorithm by adopting the Monte Carlo method. Hyun-Kyo Oh et al (Oh et al. 2013) propose a probability-based trust prediction model based on trust-message passing which takes advantage of two kinds of information: an explicit information and an implicit information. Xiang et al (Xiang, Neville, and Rogati 2010) develop an unsupervised model to estimate relationship strength from interaction activity (e.g., communication, tagging) and user similarity with the goal of automatically distinguishing strong relationships from weak ones. However, the available trust relations may not be sufficient to guarantee the success of these methods, although there are several methods exploiting extra sources to mitigate the sparseness problem (Tang et al. 2013).

Social status is an important concept in trust, which refers to the position or rank of a user in a social community representing the degree of honor or prestige attached to the position of each individual (Giddens, Duneier, and Appelbaum 2012). Status theory is developed to explain how users trust each other based on their statuses (Leskovec, Huttenlocher, and Kleinberg 2010; Leskovec 2013), and indicates that a user is likely to trust users with higher statuses. Modeling status theory can potentially improve the performance, and bring about new opportunities for trust prediction.

In this paper, we study trust prediction with status theory. In essence, we investigate how to model status theory mathematically and how to incorporate it for trust prediction, which results in a novel unsupervised framework sTrust for trust prediction. Our contributions are summarized as follows,

- Verify status theory in trust relations;
- Provide an approach to model status theory mathematically via status regularization;
- Propose an unsupervised framework sTrust for trust prediction by incorporating status theory; and
- Evaluate sTrust on real-world datasets to understand the importance of status theory in trust prediction.

The rest of the paper is organized as follows. In Section 2, we validate status theory in trust relations. We introduce status regularization and the unsupervised framework sTrust in Section 3. In Section 4, we report experimental results on real-world datasets with discussions. We finally conclude the paper with future work in Section 5.

Status Theory in Trust Relations

In this section, we verify status theory by studying the correlation between trust relations and social statuses. In particular, we try to seek an answer for the question - are users with lower social statuses more likely to trust users with high statuses? We first introduce the datasets used in this paper.

To study the problem of trust prediction with status theory, we collect two publically available datasets, i.e., Epinions and Ciao (Tang et al. 2013). Users in Epinions [1] and Ciao [2] can trust other users and form their "circle of trust". Some statistics of these two datasets are demonstrated in Table 1. As suggested by the trust network density in the table, trust relations in both datasets are very sparse, less than 0.5%. Next, with these datasets, we will verify status theory in trust relations before modeling.

Table 1: Statistics of Epinions and Ciao

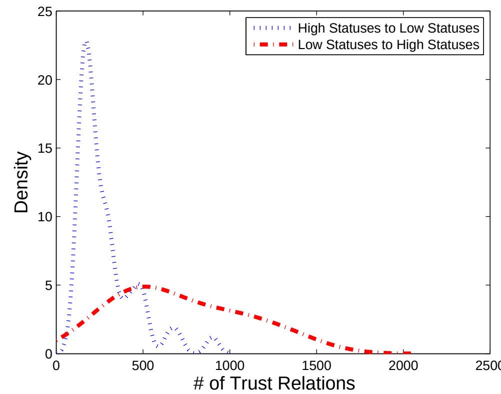
Datasets	Epinions	Ciao
Number Of Users	8,518	7,275
Number of Trust Relations	300,548	111,781
Max Number of Trustors	1,303	100
Max Number of Trustees	1,706	797
Trust Network Density	0.004	0.003
Clustering Coefficient	0.224	0.225

Social status refers to the position or ranking of a user in a social community (Giddens, Duneier, and Appelbaum 2012). Pagerank is one of the most popular ways to calculate the status scores for users in social networks (Page et al. 1999), and we choose Pagerank to calculate users' statuses in this verification. More status measurements will be discussed in the experiment section.

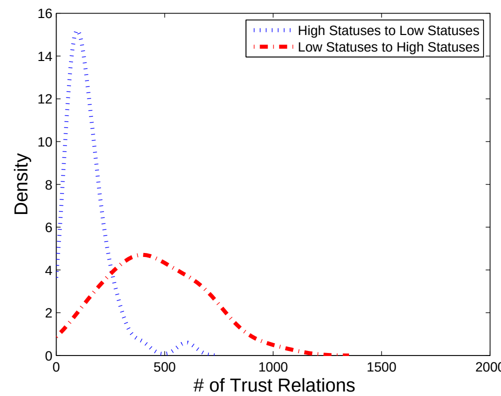
To answer the question, we sort all users according to their status rankings in a descending order and divide their social statuses into K levels with equal sizes denoted as $L = \{l_1, l_2, ..., l_K\}$. There are $K(K-1)$ pairs of $< l_i, l_j >$

[1]http://www.epinions.com

[2]http://www.ciao.com/



(a) Epinions



(b) Ciao

Figure 1: Density Estimate of Numbers of Trust Relations.

with $i \neq j$ including a set of $\frac{K(K-1)}{2}$ pairs HL with $i < j$, and a set of $\frac{K(K-1)}{2}$ pairs LH with $i > j$. For each pair $< l_i, l_j >$, we compute the number of trust relations n_{ij} from users in l_i to users in l_j. Therefore each n_{ij} for HL represents the number of relations from a high status l_i to a low status l_j, while each n_{ij} in LH denotes the number of relations from a low status l_i to a high status l_j. Then we have number vectors $\mathbf{n}_{HL}$ and $\mathbf{n}_{LH}$ for HL and LH, respectively. We conduct a two-sample t-test on $\mathbf{n}_{HL}$ and $\mathbf{n}_{LH}$; the null hypothesis is $H_0 : \mathbf{n}_{LH} \leq \mathbf{n}_{HL}$, and the alternative hypothesis is $H_1 : \mathbf{n}_{LH} > \mathbf{n}_{HL}$. When we choose $K = 10$, for both datasets, the null hypothesis is rejected at significance level 0.01 with p-values 7.34e-27 and 5.34e-14 in Epinions and Ciao, respectively. For a visual comparison, we demonstrate the Kernel smoothing density estimations based on the vectors $\mathbf{n}_{LH}$ and $\mathbf{n}_{HL}$ in Figures 1(a) and 1(b) for Epinions and Ciao, respectively. For both datasets, $\mathbf{n}_{LH}$s (i.e., low to high statuses) have larger concentrated numbers of trust relations compared with $\mathbf{n}_{HL}$s (i.e., high to low statuses).

We get similar results with various values of K. These results suggest a positive answer to the question: users with lower social statuses are more likely to trust users with high statuses. With this verification, in the following section, we will introduce our approach to model status theory for trust prediction.

Our Framework: sTrust

We first introduce notations used in this paper. Let $\mathcal{U} = \{u_1, u_2, \ldots, u_n\}$ be the set of users where n is the number of users. $\mathcal{R} = \{r_1, r_2, \ldots, r_n\}$ is the set of status scores for $\mathcal{U}$ where r_i denotes the status score of u_i. The larger r_i is, the higher the status of u_i is. *We further assume that users in $\mathcal{U}$ are sorted by their status scores $\mathcal{R}$ in a descending order.* That is to say, for u_i and u_j, if $i < j$, then $r_i > r_j$. We use $\mathbf{G} \in \mathbb{R}^{n \times n}$ to denote user-user trust relations where $\mathbf{G}_{ij} = 1$ if u_i trusts u_j, zero otherwise.

Before modeling status theory, we introduce the basic trust prediction algorithm we use in this paper. Previous work demonstrates that online trust has several properties, such as transitivity, asymmetry, and correlation with user preferences and multiple facets. In (Tang et al. 2013), the authors propose a trust prediction framework based on low-rank matrix factorization as,

$$\min_{\mathbf{U},\mathbf{H}} \ \|\mathbf{G} - \mathbf{U}\mathbf{H}\mathbf{U}^\top\|_F^2 + \alpha(\|\mathbf{U}\|_F^2 + \|\mathbf{H}\|_F^2), \quad s.t., \ \mathbf{U} \geq 0, \ \mathbf{V} \geq 0 \tag{1}$$

where $\mathbf{U} \in \mathbb{R}^{n \times d}$ is the user preference matrix and d is the number of facets of user preferences. $\mathbf{H} \in \mathbb{R}^{d \times d}$ captures the more compact correlations among $\mathbf{U}$. It is easy to verify that Eq. (1) can model the properties of trust mentioned above and performance improvement is reported by (Tang et al. 2013; Huang et al. 2012; 2013) in terms of trust prediction. Next we will introduce an approach to model status theory based on the matrix factorization method.

Modeling Status Theory

Status theory suggests that lower status users are more likely to trust higher status users. For a pair of users u_i and u_j, the likelihood of a trust relation established from u_i to u_j is calculated as $\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top$ under the framework with Eq. (1). To model status theory, we consider the following four cases for each pair u_i and u_j,

- *Case 1:* $r_i \geq r_j$ and $\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top > \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top$;
- *Case 2:* $r_i \geq r_j$ and $\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top \leq \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top$;
- *Case 3:* $r_i \leq r_j$ and $\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top \geq \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top$;
- *Case 4:* $r_i \leq r_j$ and $\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top < \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top$.

When $r_i \geq r_j$, status theory suggests that the likelihood of a trust relation from u_j to u_i should be no smaller than that of a trust relation from u_i to u_j, i.e., $\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top \leq \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top$. Similarly, when $r_i \leq r_j$, the likelihood of a trust relation from u_j to u_i should be no larger than that of a trust relation from u_i to u_j, i.e., $\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top \geq \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top$. Therefore, among above four cases, *Case 2* and *Case 3* satisfy status theory, while *Case 1* and *Case 4* contradict with status theory. Above analysis paves a way for us to model status theory.

Based on *Case 2* and *Case 3*, status theory suggests that $(r_i - r_j)(\mathbf{U}_i\mathbf{H}\mathbf{U}_j - \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top)$ should be no larger than 0. Therefore, we propose status regularization to model status theory as,

$$\sum_{i}^{n}\sum_{j\neq i}^{n}(\max\{0, f(r_i - r_j)(\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top - \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top)\})^2, \tag{2}$$

where $f(x)$ is a function which has the same sign of x. Next we will show that by minimizing Eq. (2), we can model status theory as the following,

- *Case 2* and *Case 3* satisfy status theory, where $f(r_i - r_j)(\mathbf{U}_i\mathbf{H}\mathbf{U}_j - \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top) \leq 0$. Therefore, status regularization is 0, which means that we should not add any penalty on these cases.
- *Case 1* and *Case 4* contradict status theory where $f(r_i - r_j)(\mathbf{U}_i\mathbf{H}\mathbf{U}_j - \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top) > 0$. Then, status regularization is $f(r_i - r_j)(\mathbf{U}_i\mathbf{H}\mathbf{U}_j - \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top)$, and minimizing this term will push $\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top$ close to $\mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top$ and force the likelihood of going from a high status user to a low status user to be no larger than that of going from a low status user to a high status user, which can mitigate *Case 1* and *Case 4*.

The above observations suggest that by minimizing status regularization in Eq. (2), we can model status theory.

Since $f(r_i - r_j)(\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top - \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top)$ is equivalent to $f(r_j - r_i)(\mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top - \mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top)$, the status regularization can be rewritten as,

$$\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\max\{0, g(r_i - r_j)(\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top - \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top)\})^2 \tag{3}$$

as mentioned above, we assume that users in $\mathcal{U}$ are sorted by their status scores in a descending order. Therefore $r_i - r_j$ in status regularization in Eq (3) is positive and we define a non-negative function $g(x)$ to replace $f(x)$. In this work, we empirically find that $g(x) = \frac{1}{1+log(x+1)}$ works well for sTrust.

Status Theory in Trust Prediction

With the introduction of status regularization to model status theory, our proposed framework sTrust is to minimize the following equation,

$$\begin{aligned}\min_{\mathbf{U},\mathbf{H}} \ & \|\mathbf{G} - \mathbf{U}\mathbf{H}\mathbf{U}^\top\|_F^2 + \lambda\sum_{i=1}^{n}\sum_{j=i+1}^{n} \\ & (\max\{0, f(r_i - r_j)(\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top - \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top)\})^2 \\ & + \alpha(\|\mathbf{U}\|_F^2 + \|\mathbf{H}\|_F^2), \\ & s.t., \ \mathbf{U} \geq 0, \ \mathbf{H} \geq 0\end{aligned} \tag{4}$$

where the second term is status regularization to model status theory and the parameter λ is introduced to control its contribution in trust prediction. The optimization problem in Eq. (4) is jointly convex with respect to $\mathbf{U}$ and $\mathbf{H}$; however, due to the $\max$ function in status regularization, there is no nice closed solution. Next, we will introduce a solution by an alternating optimization method.

We define $\mathbf{R}^k \in \mathbb{R}^{n\times n}$ in the k-th iteration as,

$$\mathbf{R}^k_{ij} = \begin{cases} \sqrt{g(r_i - r_j)} \text{ if } (j > i) \wedge (\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top - \mathbf{U}_j\mathbf{H}\mathbf{U}_i^\top) > 0 \\ 0 \qquad\qquad\quad \text{otherwise} \end{cases} . \tag{5}$$

Then the Lagrangian function of Eq. (4) in the k-th iteration can be written as,

$$\mathcal{L}_k = \|\mathbf{G} - \mathbf{U}\mathbf{H}\mathbf{U}^\top\|_F^2 + \lambda\|\mathbf{R}^k \odot (\mathbf{U}\mathbf{H}^\top\mathbf{U}^\top - \mathbf{U}\mathbf{H}\mathbf{U}^\top)\|_F^2 + \alpha Tr(\mathbf{U}^\top\mathbf{U} + \mathbf{H}^\top\mathbf{H}) - Tr(\Lambda^1\mathbf{U}) - Tr(\Lambda^2\mathbf{V}), \tag{6}$$

where Λ^1 and Λ^2 are Lagrangian multipliers for nonnegativity of $\mathbf{U}$ and $\mathbf{H}$, respectively. $\odot$ is the Hadamard product where $(\mathbf{A} \odot \mathbf{B})_{ij} = \mathbf{A}_{ij} \times \mathbf{B}_{ij}$ for any two matrices $\mathbf{A}$ and $\mathbf{B}$ with the same size.

By moving constants, $\mathcal{L}_k$ can be rewritten as,

$$\begin{aligned} \mathcal{L}_k &= Tr(-2\mathbf{G}^\top\mathbf{U}\mathbf{H}\mathbf{U}^\top + \mathbf{U}\mathbf{H}^\top\mathbf{U}^\top\mathbf{U}\mathbf{H}\mathbf{U}^\top) \\ &+ \lambda Tr\big(2\mathbf{U}\mathbf{H}^\top\mathbf{U}^\top(\mathbf{R}^k \odot \mathbf{R}^k \odot \mathbf{U}\mathbf{H}^\top\mathbf{U}^\top) \\ &- 2\mathbf{U}\mathbf{H}^\top\mathbf{U}^\top(\mathbf{R}^k \odot \mathbf{R}^k \odot \mathbf{U}\mathbf{H}\mathbf{U}^\top)\big) \\ &+ \alpha Tr(\mathbf{U}^\top\mathbf{U} + \mathbf{H}^\top\mathbf{H}) - Tr(\Lambda^1\mathbf{U}) - Tr(\Lambda^2\mathbf{V}) \end{aligned} \tag{7}$$

The partial deviations of $\mathcal{L}_k$ with respect to $\mathbf{U}$ and $\mathbf{H}$ are,

$$\begin{aligned} \frac{\partial \mathcal{L}_k}{\partial \mathbf{U}} &= \mathbf{B} - \mathbf{A} - (\Lambda^1)^\top, \\ \frac{\partial \mathcal{L}_k}{\partial \mathbf{H}} &= \mathbf{D} - \mathbf{C} - (\Lambda^2)^\top. \end{aligned} \tag{8}$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are defined as,

$$\begin{aligned} \mathbf{A} &= \mathbf{G}^\top\mathbf{U}\mathbf{H} + \lambda(\mathbf{R}^k \odot \mathbf{R}^k \odot \mathbf{U}\mathbf{H}\mathbf{U}^\top)\mathbf{U}\mathbf{H}^\top \\ &+ \lambda((\mathbf{R}^k)^\top \odot (\mathbf{R}^k)^\top \odot \mathbf{U}\mathbf{H}^\top\mathbf{U}^\top)\mathbf{U}\mathbf{H} \\ &+ \mathbf{G}\mathbf{U}\mathbf{H}^\top + \lambda(\mathbf{R}^k \odot \mathbf{R}^k \odot \mathbf{U}\mathbf{H}^\top\mathbf{U}^\top)\mathbf{U}\mathbf{H} \\ &+ \lambda((\mathbf{R}^k)^\top \odot (\mathbf{R}^k)^\top \odot \mathbf{U}\mathbf{H}\mathbf{U}^\top)\mathbf{U}\mathbf{H}^\top \\ \mathbf{B} &= \mathbf{U}\mathbf{H}^\top\mathbf{U}^\top\mathbf{U}\mathbf{H} + \mathbf{U}\mathbf{H}\mathbf{U}^\top\mathbf{U}\mathbf{H}^\top + \alpha\mathbf{U} \\ &+ 2\lambda((\mathbf{R}^k)^\top \odot (\mathbf{R}^k)^\top \odot \mathbf{U}\mathbf{H}\mathbf{U}^\top)\mathbf{U}\mathbf{H} \\ &+ 2\lambda(\mathbf{R}^k \odot \mathbf{R}^k \odot \mathbf{U}\mathbf{H}^\top\mathbf{U}^\top)\mathbf{U}\mathbf{H}^\top \\ \mathbf{C} &= \mathbf{U}^\top\mathbf{G}\mathbf{U} + \lambda\mathbf{U}^\top(\mathbf{R}^k \odot \mathbf{R}^k \odot \mathbf{U}\mathbf{H}\mathbf{U}^\top)\mathbf{U} \\ &+ \lambda\mathbf{U}^\top((\mathbf{R}^k)^\top \odot (\mathbf{R}^k)^\top \odot \mathbf{U}\mathbf{H}\mathbf{U}^\top)\mathbf{U} \\ \mathbf{D} &= \mathbf{U}^\top\mathbf{U}\mathbf{H}\mathbf{U}^\top\mathbf{U} + \alpha\mathbf{H} \\ &+ 2\lambda\mathbf{U}^\top(\mathbf{R}^k \odot \mathbf{R}^k \odot \mathbf{U}\mathbf{H}^\top\mathbf{U}^\top)\mathbf{U} \end{aligned} \tag{9}$$

The KKT complementary condition is,

$$\begin{aligned} &\mathbf{U}_{ik}\Lambda^1_{ik} = 0, \\ &\mathbf{H}_{ik}\Lambda^2_{ik} = 0, \;\; \forall\, i \in [1, n],\; k \in [1, d]. \end{aligned} \tag{10}$$

Setting $\frac{\partial \mathcal{L}_k}{\partial \mathbf{U}} = 0$ and $\frac{\partial \mathcal{L}_k}{\partial \mathbf{V}} = 0$, and using the KKT complementary condition in Eq. (10), we have the following,

$$\begin{aligned} \mathbf{U}_{ik} &\leftarrow \mathbf{U}_{ik}\sqrt{\frac{\mathbf{A}_{ik}}{\mathbf{B}_{ik}}}, \\ \mathbf{H}_{ik} &\leftarrow \mathbf{H}_{ik}\sqrt{\frac{\mathbf{C}_{ik}}{\mathbf{D}_{ik}}}, \end{aligned} \tag{11}$$

We can verify that the updating rules in Eq. (11) satisfy the above KKT condition. Since all matrices in Eq. (11) are nonnegative, $\mathbf{U}$ and $\mathbf{H}$ are nonnegative during the updating process. We also can prove that the updating rules in Eq. (11) are guaranteed to converge. Since the proof process is similar to that in (Ding, Li, and Jordan 2008), we omit the details due to space limitation.

With the updating rules for $\mathbf{U}$ and $\mathbf{H}$, the detailed algorithm for sTrust is demonstrated in Algorithm 1. Next we briefly review Algorithm 1. In line 1, we calculate users' social statuses, and from line 4 to line 17, we update $\mathbf{U}$ and $\mathbf{H}$ until convergence. After learned $\mathbf{U}$ and $\mathbf{H}$, sTrust suggests the likelihood of a trust relation established from u_i to u_j as $\mathbf{U}_i\mathbf{H}\mathbf{U}_j^\top$.

Algorithm 1 The Proposed Framework sTrust with Status Theory.

Input: Trust relations $\mathbf{G}$ and λ
Output: Ranking list of pairs of users

1: Calculate user statuses $\{r_i\}_{i=1}^n$
2: Initialize $\mathbf{U}$ randomly
3: Initialize $\mathbf{H}$ randomly
4: **while** Not convergent **do**
5:   Calculate $\mathbf{R}^k$ according to Eq. (5)
6:   Construct $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ based on Eq. (9)
7:   **for** $i = 1$ to n **do**
8:     **for** $k = 1$ to d **do**
9:       Update $\mathbf{U}_{ik} \leftarrow \mathbf{U}_{ik}\sqrt{\frac{\mathbf{A}_{ik}}{\mathbf{B}_{ik}}}$
10:     **end for**
11:   **end for**
12:   **for** $i = 1$ to d **do**
13:     **for** $k = 1$ to d **do**
14:       Update $\mathbf{H}_{ik} \leftarrow \mathbf{H}_{ik}\sqrt{\frac{\mathbf{C}_{ik}}{\mathbf{D}_{ik}}}$
15:     **end for**
16:   **end for**
17: **end while**
18: Set $\tilde{\mathbf{G}} = \mathbf{U}\mathbf{H}\mathbf{U}^\top$
19: Ranking pairs of users (e.g., $\langle u_i, u_j\rangle$) according to $\tilde{\mathbf{G}}$ (e.g., $\tilde{\mathbf{G}}_{ij}$) in a descending order.

Experiments

In this section, we conduct experiments to assess the effectiveness of the proposed framework sTrust. Through the experiments, we aim to answer the following two questions:

- Does status theory improve the performance of trust prediction?
- How does status regularization affect the proposed framework sTrust?

Experimental Settings

We divide each dataset into two parts $\mathcal{A}$ and $\mathcal{B}$, where $\mathcal{A}$ is the set of user pairs with trust relations and $\mathcal{B}$ is the set of user pairs without trust relations. User pairs in $\mathcal{A}$ are sorted in a chronological order in terms of the time when they established trust relations. We choose $x\%$ of $\mathcal{A}$ as old trust relations $\mathcal{C}$ and the remaining $1 - x\%$ as new trust relations $\mathcal{D}$ to predict. x is varied as $\{50, 55, 60, 65, 70, 80, 90\}$ in this paper and for each x, we repeat the experiments 5 times and report the average performance.

We follow a common metric for unsupervised trust prediction in (Liben-Nowell and Kleinberg 2007) to evaluate the performance of trust prediction. In detail, trust predictor ranks user pairs in $\mathcal{D} \cup \mathcal{B}$ in a decreasing order of confidence, and we take the first $|\mathcal{D}|$ pairs as the set of predicted trust relations, denoted as $\mathcal{E}$. Then, the trust prediction accuracy $TP_{accuracy}$ can be calculated in Eq. (12),

$$TP_{accuracy} = \frac{|\mathcal{D} \cap \mathcal{E}|}{|\mathcal{D}|} \tag{12}$$

Performance Comparisons with Different Trust Predictors

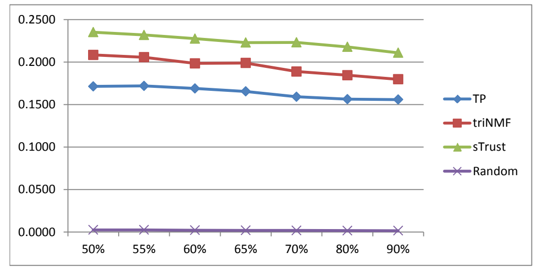
To answer the first question, we compare the proposed framework sTrust with the following representative methods,

- TP: it utilizes four types of atomic propagations, i.e., direct propagation, co-citation, transpose trust, and trust coupling, to predict trust relations (Guha et al. 2004);
- triMF: it performs a low-rank matrix factorization on the user-user trust relation matrix as shown in Eq. (1); and
- Random: it randomly suggests trust relations to pairs of users.

Since our proposed framework sTrust is unsupervised method, we do not compare with these supervised methods (Liu et al. 2008; Korovaiko and Thomo 2013; Nguyen et al. 2009). Also we do not compare with unsupervised methods like (Oh et al. 2013; Tang et al. 2013), since they use extra sources in addition to existing trust relations. All parameters of trust predictors are determined via cross-validation. For sTrust, we set λ to 0.7 and 0.5 for Epinions and Ciao, respectively. We empirically set $\alpha = 0.1$ and $d = 100$. More details about parameter analysis for sTrust will be discussed in the following subsections. The experiment results are shown in Figure 2.

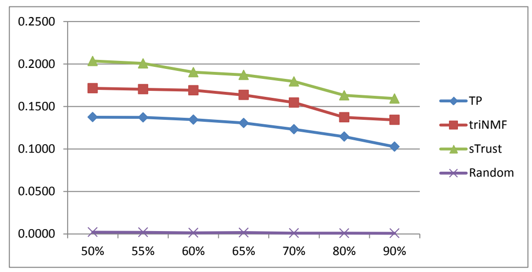
We draw following observations,

- With the increase of x, the performance of all methods decreases. With the increase of x, the number of new trust relations $(1 - x\%)$ $\mathcal{D}$ decreases. Since $\mathcal{B}$ is fixed, it becomes more difficult to predict $\mathcal{D}$ from $\mathcal{D} \cup \mathcal{B}$.
- The performance of TP, triNMF and sTrust is much better than that of Random, which supports that modeling trust properties can improve the performance significantly.
- Our proposed framework sTrust always outperforms all baseline methods. sTrust is based on triNMF by incorporating status theory. These results directly demonstrate



	50%	55%	60%	65%	70%	80%	90%
TP	0.1713	0.1719	0.1689	0.1654	0.1591	0.1563	0.1558
triNMF	0.2083	0.2056	0.1984	0.1989	0.1888	0.1845	0.1797
sTrust	0.2351	0.2318	0.2275	0.2229	0.2231	0.2178	0.2109
Random	0.0025	0.0025	0.0022	0.0020	0.0019	0.0017	0.0016

(a) Epinions



	50%	55%	60%	65%	70%	80%	90%
TP	0.1374	0.1371	0.1345	0.1306	0.1231	0.1145	0.1027
triNMF	0.1714	0.1703	0.1691	0.1635	0.1546	0.1372	0.1342
sTrust	0.2035	0.2006	0.1903	0.1871	0.1794	0.1631	0.1593
Random	0.0022	0.0020	0.0014	0.0017	0.0010	0.0009	0.0008

(b) Ciao

Figure 2: Performance Comparisons in Epinions and Ciao.

that status regularization can improve the performance of trust prediction. More discussions about the effects of status theory in the proposed framework will be presented in the following subsections.

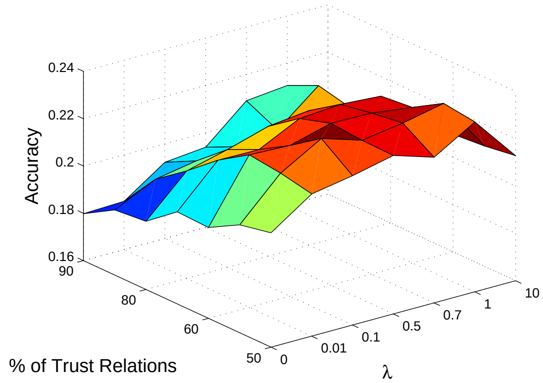
We perform t-test on all comparisons and the t-test results suggest that all improvement is significant. With the help of status regularization, the proposed framework sTrust gains significant improvement over representative baseline methods, which can answer the first question.

Impact of Status Theory in sTrust

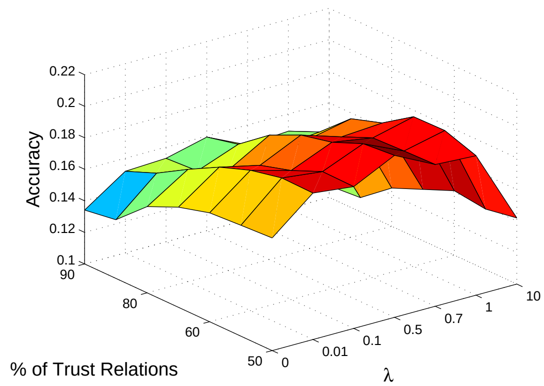
In this subsection, we study the effects of status regularization in the proposed framework sTrust and accordingly answer the second question. The parameter λ is introduced to control the contribution from status theory for the proposed framework sTrust; hence, we investigate the impact of status regularization by analyzing how the changes of λ affect the performance of sTrust in terms of trust prediction accuracy. In this experiment, λ is varied as$\{0, 0.01, 0.1, 0.5, 0.7, 1, 10\}$, and the results are shown in Figures 3(a) and 3(b) for Epinions and Ciao, respectively.

It can be observed,

- In general, with the increase of λ, the performance in both Epinions and Ciao shows similar patterns: first increasing, reaching peak value, and then degrading rapidly.
- When $\lambda = 0$, we eliminate the contribution from status



(a) Epinions



(b) Ciao

Figure 3: The Impact of Status Theory in the Proposed Framework sTrust.

regularization, and the accuracy is much lower than the peak performance. When λ is from 0 to 0.01, the performance improves greatly, suggesting that status regularization can significantly improve the performance of trust prediction.

- After a certain value of λ, continuing to increase λ will result in performance reduction. When λ is very large, status regularization dominates the learning process and the learned $\mathbf{U}$ and $\mathbf{H}$ may be inaccurate.

These results further demonstrate the importance of modeling status theory in trust prediction, which correspondingly answers the second question.

Conclusion

In this paper, we exploit status theory for trust prediction under the trust prediction framework based on low-rank matrix factorization. Experimental results on real-world datasets demonstrate that the proposed framework sTrust can significantly improve trust prediction performance.

There are several interesting directions we will investigate in further work. In our current work, we intend to investigate how to exploit status theory in other unsupervised and supervised methods. Currently, we only use the network information to calculate social statuses of users. Incorporating multiple sources to obtain robust status scores will be another interesting direction.

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