SmartShift: Expanded Load Shifting Incentive Mechanism for Risk-Averse Consumers

Bochao Shen
CCIS
Northeastern University
Boston MA, 02115
ordinary@ccs.neu.edu

Balakrishnan Narayanaswamy
Computer Science and Engineering
University of California, San Diego
La Jolla CA, 92093
muralib@cs.ucsd.edu

Ravi Sundaram
CCIS
Northeastern University
Boston MA, 02115
koods@ccs.neu.edu

Abstract

Peak demand for electricity continues to surge around the world. The supply-demand imbalance manifests itself in many forms, from rolling brownouts in California to power cuts in India. It is often suggested that exposing consumers to real-time pricing, will incentivize them to change their usage and mitigate the problem - akin to increasing tolls at peak commute times. We show that risk-averse consumers of electricity react to price fluctuations by scaling back on their total demand, not just their peak demand, leading to the unintended consequence of an overall decrease in production/consumption and reduced economic efficiency. We propose a new scheme that allows homes to move their demands from peak hours in exchange for greater electricity consumption in non-peak hours - akin to how airlines incentivize a passenger to move from an over-booked flight in exchange for, say, two tickets in the future. We present a formal framework for the incentive model that is applicable to different forms of the electricity market. We show that our scheme not only enables increased consumption and consumer social welfare but also allows the distribution company to increase profits. This is achieved by allowing load to be shifted while insulating consumers from real-time price fluctuations. This win-win is important if these methods are to be embraced in practice.

Introduction

Power utilities worldwide face at least two major challenges. The first is *Peak Demand* - a period in which the demand for power is significantly higher than average. In order to satisfy a large peak demand, utilities (generation/distribution companies) have to make large capital investments including new ‘peaking’ generation stations, larger capacity lines, transformers, and operational expenditures including expensive purchases of electricity on the “spot market” (Harris 2006). For example, it is estimated that a 5% lowering of demand would have resulted in a 50% price reduction during the peak hours of the California electricity crisis in 2000-2001 (International Energy Agency 2003). As a result of the quadratic dependence between resistive losses and transmitted current, peaks also lead to substantial energy wastage. The second challenge faced by power utilities is that of *Supply-Demand* imbalance. (Borenstein 2002) states that ”the difficulties that have appeared in California and elsewhere are intrinsic to the design of current electricity markets, in which demand exhibits virtually no price responsiveness and supply faces strict production constraints”.

Research at the intersection of computer science and economics has established that incentives are a powerful way to allocate scarce resources (Courcoubetis and Weber 2003). Incentive mechanisms incorporating information exchange require back and forth communication between the consumers and the producers. Conventional electricity infrastructure is not designed to support such a dialog, instead relying on unilateral actions by the producers such as price-regulation and load-shedding. Fortunately, the introduction of new communication (Sood et al. 2009) and control (Farhangi 2010) infrastructure will allow increased consumer participation in the smart grid.

Traditionally, economists and electricity companies have focused on pricing as a mechanism to solve these two problems in a systematic way. However, homes and small businesses have an inherent and systemic requirement for stable electricity costs (Chao 2010). It is understood that ”Consumers generally shy away from markets when products are complicated, supply is uncertain, prices are volatile, and information is lacking” (Chao 2012). We demonstrate that exposing *risk-averse* consumers to the actual real-time costs of electricity production, will result in a reduction in aggregate demand leading to reduced revenue for the generators and distributors with potential knock-on effects for the economy at large. At the same time, the volatility of real-time pricing can have strong effects on grid stability (Roozbehani, Dahleh, and Mitter 2012). A second concern with real-time pricing, is that consumer prices may increase (Allcott 2009) or net electricity consumption may decrease (Allcott 2011), depending on the model. We initiate the study of a new model of incentives and utility-customer interaction. To reduce volatility, whilst yet accounting for end-user constraints (e.g., washer must be run only during the day), we introduce an inter-temporal characteristic to the customer ‘bidding’ language. More importantly, we ensure that the customer and the utility will both be no worse off under our scheme than under flat-rate pricing. Our mechanism, SmartShift, does this by rewarding consumers who shift consumption with increased allocations for the same cost. Consumers are paid in kind and not cash, advantaging both the consumer and the producer. For real-world adoption it is critical to devise mutually beneficial schemes, such as Smartshift, that increase the economic pie within the world of electrical power.

We design consumer-supplier interaction protocols that respect the physical constraints and characteristics of the grid. Such protocols must account for realistic models of consumer behavior and requirements, while being computationally tractable to allow practical implementation in grids with millions of customers. Our main contributions are:

- SmartShift - a novel incentive mechanism, consistent with utilities' obligation to serve (Chao 2012), that allows risk-averse homes to move their demands from peak hours in exchange for greater electricity consumption in non-peak hours (SmartShift section).
- demonstration of the practical benefits of SmartShift through extensive simulations (Simulations section).
- characterization of the computational complexity of SmartShift (SmartShift section).
- formalization of the folklore that purely price driven approaches have negative consequences in a world with risk-averse consumers (SmartShift section).

Pricing in electricity markets - background

We study a market model with distribution companies (or utilities - the suppliers of electricity) and customers (consumers of electricity), each of whom is exposed to different prices, with associated risks and properties.

Prices seen by customers: *Flat-rate pricing* is the de-facto standard for retail electricity, where every unit costs the same fixed amount. Homes for example prefer this, particularly since, unlike businesses that may experience revenues in line with consumption growth, they have a strong inbuilt preference for stability in the costs. However, flat-rate pricing lacks any incentive for households to make rational usage decisions, especially during peak hours. We show that under flat-rate pricing risk-averse consumers will have high welfare due to lack of volatility but, (1) it cannot eliminate the peak load problem, since there is no slot-wise price differentiation, and as a result (2) the distribution company may face profit fluctuations.

With *real-time pricing*, peak reduction is achieved by charging consumers higher rates when the overall electricity load is high, encouraging them to reduce consumption (Mohsenian-Rad and Leon-Garcia 2010). However, as we will discuss in more detail, the fluctuating nature of market prices will discourage consumption by risk-averse consumers, while also being known to increase grid volatility (Roozbehani, Dahleh, and Mitter 2012). Real-time pricing will result in a decrease of the absolute total consumption of electricity (Di Cosmo, Lyons, and Nolan 2012) and consequently, total revenue of the distribution company (utility). Participation in most voluntary real-time pricing (RTP) programs has decreased, in particular "Between 2000 and 2003, half of all programs in existence prior to 2000 lost 25% or more of their participants, while only two programs saw participation increase" (Barbose, Goldman, and Neenan 2004), likely due to price volatility.

Prices seen by the distribution company: The time varying market, procurement or generation price of electricity can be modeled in different ways. The price can be modeled as *endogenous* or *exogenous*. We use the term *Endogenous Market Price* to denote a price that depends on the total electricity consumption at each time slot. This can be a useful model in situations where the number of consumers is relatively large, with their total consumption affecting market price. The resulting price is typically modeled as a quadratic function of the total electricity consumption; the quadratic or piecewise quadratic nature of generation and distribution cost curves is usually due to a multiplicity of fuel sources and generation modes. See (Wood and Wollenberg 2012; Harris 2006) for detailed justification of this common model.

Exogenous Market Prices are independent of the actions of the distribution company, its customers or their consumption. This is usually the case when the number of the responding consumers are relatively small, or if there is a large background electricity load which dominates an endogenous market price. In this case, we can just treat the real-time market price as a random variable independent of the decisions made by customers. See (Harris 2006; Chao 2012; Roozbehani, Dahleh, and Mitter 2012) for some justification and analysis of this stochastic pricing model.

Shortcoming of purely price driven approaches: We prove, in the exogenous price model, both that (i) risk-averse consumers have a lower expected social welfare and that (ii) the distribution company gets lower expected revenue under real-time pricing when compared to flat-rate pricing. On the other hand, we point out that the distribution company never makes a loss under real-time pricing, but can incur losses with some probability under flat-rate pricing. This leads us to develop a scheme with the advantages of both.

Our approach

We propose *SmartShift*, an expanded load shifting mechanism, where consumers are incentivized to shift their electricity load from peak-time to non-peak-time in exchange for an expanded electricity load, while still incurring the same cost. Since consumers self select, their social welfare (of consuming electricity) is non-decreasing under this scheme. Observe that from a pragmatic stand-point the unit of a home allows for the effective use of an in-kind incentive as a tool of behavior change. A single appliance/user may not be able/willing to consume the incentive, but an aggregate of a household will be able to more effectively consume the incentive via coordination among the household members (de Vries 2008).

In addition to the development of a new in-kind incentive model, our work is distinguished by a focus on computational issues. We prove that, in the *endogenous price* model, calculating the optimal shifted load is NP-complete for both real-time and flat-rate pricing. In contrast, we show in the *exogenous price* case that we can calculate the optimal shifted load in polynomial time.

We prove that employing *SmartShift* increases the expected profit of the distribution company both under real-time pricing and flat-rate pricing. We employ numerical experiments to show that it also decreases the probability of scenarios where the distribution company incurs losses, i.e. its risk. Thus, in the exogenous price model the use of *SmartShift* with flat-rate pricing provides a solution to the problems of *Peak Load* and *Supply-Demand* imbalance; the resulting electricity load allocation simultaneously increases consumers' social welfare, and distribution companies' profits and thus is a win-win solution.

Related Work

Evidence from the use of sophisticated usage-based pricing schemes (Courcoubetis and Weber 2003) indicates there is a strong case to be made for managing a grid via incentive schemes. Price based congestion control schemes in the power systems literature (often based on successful protocols developed for internet congestion control (Keshav and Rosenberg 2011)) that discourage consumption when the grid is loaded, fall into four basic categories: 1. day ahead or time-of-use (TOU) pricing, 2. dynamic pricing, 3. back-off strategies and 4. tâtonnement (or negotiation).

TOU pricing (particularly when users have significant storage, patience or flexibility) leads to the herding problem, where large amounts of consumption are shifted to low price regions creating new peaks (Carpenter et al. 2012). Dynamic pricing, as we discuss, can lead to uncertainity and reduced consumption by risk-averse customers. Randomized back-off type approaches, where users back-off on consumption when the grid is loaded (motivated by CSMA protocols developed for wireless communication), require trust that the agent will respond as expected to signals. This is not incentive compatible (Basar et al. 1995) as the response strategies are unverifiable and it is in the agents' interest to choose small back-off windows which allow them to consume electricity with minimum inconvenience. In Tâtonnement (Walker 1987), consumption agents and the distribution company exchange both price information and consumption profile information until convergence. While they were not explicitly categorized as such, tâtonnement forms the basis of much work in this area including (Mohsenian-Rad et al. 2010; Vytelingum et al. 2010; Li, Chen, and Low 2011; Ilic, Xie, and Joo 2011). Since typical distribution grids could have millions of users and tens of millions of devices, computational complexity is of paramount importance (Jain et al. 2013). In this context, the use of in-kind incentives and attention to computational complexity and risk-aversion makes our model and analysis novel.

Market Model Description

We now describe the overall electricity market model, explain the roles of the generation factory, the distribution company and the end consumers. We describe the two pricing mechanisms of the distribution company and then explain the two models for the market price of electricity: (1) Endogenous Market Price; (2) Exogenous Market Price.

Generation companies generate electricity, which is purchased by distribution companies. They then transfer it to the (n) end consumers, and charge for this service. The distribution company payes a price per unit of $p_m^{(t)}$ in time slot t, and must decide how to charge users - i.e. decide on a pricing mechanism.

We assume that both the distribution company and the end consumers know the statistical properties of the market price $p_m^{(t)}$. For the gaussian model we use, the mean μ_m and the standard deviation σ_m suffice. This can be learned, for example, based on the long run price history. In the short term, i.e. over the period of a single billing cycle, say a day, only the distribution company is assumed to know the market price $p_m^{(t)}$. A billing cycle is assumed to consist of k slots and without loss of generality we index t in the next billing cycle as $1 \leq t \leq k$. In this work, we study the market behavior (i.e. social welfare, revenue, etc) in the short term given the known long term statistical information.

At the beginning of each (short term) billing cycle, the distribution company will anounce the specific pricing mechanism. The prices seen by the consumers, $p_c^{(t)}$ will then be (1) *real-time pricing*: $p_c^{(t)} = p_m^{(t)}$; (2) *Flat-rate pricing*: $p_c^{(t)} = \mu_m$, where μ_m is the mean of $p_m^{(t)}$.

Risk-averse consumers: utility, valuation: Each consumer i has a valuation function $V_i^{(t)} : x_i^{(t)} \rightarrow R$ which reflects the value consumer i receives when she consumes $x_i^{(t)}$ units of electricity at time slot t. In general, the valuation function could be any concave increasing function and all the results in this paper continue to hold at a qualitative level. For sake of concreteness, we assume that consumers' valuation function $V_i^{(t)}(\cdot)$ has the form $V_i^{(t)} = \alpha_i^{(t)} \log x_i^{(t)}$.

Intuitively, the marginal value of consuming a additional unit of electricity will increase more slowly given the increased consumption of electricity. $\alpha_i^{(t)}$ can be seen as a parameter specific to consumer i that pegs the logarithmically increasing valuation curve to the value that consumer i puts on consuming $x_i^{(t)}$ units of electricity at time slot t.

Under flat rate pricing, consumer i estimates her net utility $U_i^{(t)}$ of consuming $x_i^{(t)}$ units of electricity at time slot t by $U_i^{(t)} = V_i^{(t)} - \mu_c x_i^{(t)}$.

As explained earlier, under uncertain real time pricing, consumers tend to be *risk-averse*. Risk-averse consumers will take the additional risk of price fluctuation into consideration when they estimate their net valuation, i.e., the net utility function for risk-averse consumers should be revised as $U_i^{(t)} = V_i^{(t)} - (\mu_c + \lambda_i \sigma_c) x_i^{(t)}$; σ_c is the standard deviation of the price $p_c^{(t)}$ and μ_c is it's average over the k time slots. σ_c represents the degree of price fluctuation; λ_i ($\lambda_i \geq 0$) quantifies the level of risk-aversion of consumer i. Specifically, $\lambda_i = 0$ captures the situation where consumer i is risk-neutral while $\lambda_i < 0$ models the risk-averse consumer ($\lambda_i > 0$ models the risk-seeking consumer, a type rarely seen in markets).

Electricity consumption, social welfare and profit: We apply our incentive mechanism to the two different pricing mechanisms - real-time and flat-rate - of the distribution company and analyze the resulting market behavior.

Real-time pricing: The distribution company charges the consumers a time varying price of $p_c^{(t)} = p_m^{(t)}$ over the time slots of the billing cycle. Clearly, $\mu_c = \mu_m$ and $\sigma_c = \sigma_m$. With predictions (possibly gleaned from long run observations) of the mean and variance of the real-time electricity price, a rational consumer will consume $x_i^{(t)}$ units of electricity so as to maximize the net utility $U_i^{(t)}$. By the first order derivative condition $\mathrm{d}U_i^{(t)}/\mathrm{d}x_i^{(t)} = 0$, the net utility $U_i^{(t)}$ at time slot t is maximized when $x_i^{(t)*} = \alpha_i^{(t)}/(\mu_m + \lambda_i \sigma_m)$. Thus, risk-averse consumers will consume less electricity when the real-time market price has a large standard deviation (large fluctuations and uncertainty in price).

Flat-rate pricing: The distribution company charges the end consumers a fixed price $p_c^{(t)} = \mu_m$ over the time slots of the billing cycle. Since the consumers know that they will be charged a flat-rate price equal to the mean of the market price they will infer that $\mu_c = \mu_m$ and $\sigma_c = 0$ and will respond by consuming the amount $x_i^{(t)*} = \alpha_i^{(t)}/\mu_m$ (even though they are risk-averse).

Social welfare of consumers: After each consumer is charged price $p_c^{(t)}$ at time slot t, the actual net utility she receives is $U_i^{(t)\prime} = V_i^{(t)} - p_c^{(t)} x_i^{(t)}$. And, the social welfare W of all the consumers is $W = \sum_{t=1}^{k} \sum_{i=1}^{n} U_i^{(t)\prime}$.

The **profit** η that the distribution company gains is $\eta = \sum_{t=1}^{k} \sum_{i=1}^{n} (p_c^{(t)} - p_m^{(t)}) x_i^{(t)}$. $\eta < 0$ means that the distribution company incurs losses with respect to its expected margins; $\eta > 0$ means that the distribution company profits. Note that in general, in a monopoly, a company can ensure profits by overcharging (increasing $p_c^{(t)}$ arbitrarily. In that case η quantifies the difference to expected profits. In addition, in most markets, electricity companies are at minimum semi-regulated and required to fix an operating margin. We avoid these complications by setting $p_c^{(t)} = p_m^{(t)}$.

Recall that the market price $p_m^{(t)}$ can be either endogenous or exogenous.

Endogenous market price: When the total consumption of a group of end consumers is large enough to dominate the overall electricity load of the grid, then the cost of generating electricity depends on those end consumers' total consumption. So does the market price $p_m^{(t)}$. Concretely, we assume $p_m^{(t)}$ has the form: $p_m^{(t)} = Q(\sum_{i=1}^{n} x_i^{(t)})$, and in particular that $Q(\cdot)$ has a quadratic form, i.e. $Q(x) = ax^2 + bx + c$. In this case, due to the convex nature of the problem, the market equilibrium is the fixed point of the interaction between the consumers and the supplier. It can hence be computed iteratively: starting with an initial estimate for mean μ_m and standard deviation σ_m, the end consumers respond with their electricity demand $\{x_i^{(t)}\}$. Based on this response, i.e. $\{x_i^{(t)}\}$, the endogenous market prices are generated by the quadratic function $Q(\sum_{i=1}^{n} x_i^{(t)})$. These prices are then used as input for the next round of computation until convergence is achieved.

Exogenous market price: When the total consumption of a group of end consumers is too small to meaningfully affect the overall electricity in the grid, then the market price $p_m^{(t)}$ can be assumed to be independent of consumers' consumption. Under the exogenous model, we assume $p_m^{(t)}$ $(1 \leq t \leq k)$ are independent random variables drawn from a normal distribution $p_m^{(t)} \sim N(\mu_m, \sigma_m^2)$.

SmartShift

SmartShift is an alternative, expanded load shifting, incentive mechanism that addresses the challenges of peak demand and the supply-demand imbalance detailed in the Introduction. In order to guide the design of our mechanism, we first ask ourselves two questions: (1) What incentivizes the consumers to shift their electricity load? (2) What goal will the distribution company achieve after it offers the incentivies to the consumers? The answer to these two questions will provide a win-win solution to both consumers and the distribution company. We then study how to achieve the optimal solution under (1) Endogenous (2) Exogenous market price.

What is the incentive for consumers to shift load?: Each rational consumer i will only agree to shift her consumption $x_i^{(s)}$ from time slot s to time slot t, if her valuation does not decrease. This gives us the following result,

Theorem 1. *With the same payment p_0, consumer i will have no decrease in utility, if she shifts $x_i^{(s)}$ from time slot s to time slot t with expansion, which results in an expanded load of $m_i^{(s\to t)} x_i^{(s)}$ at time slot t. The necessary and sufficient condition, therefore is for the expansion ratio $m_i^{(s\to t)}$ to satisfy the following:*

$$m_i^{(s\to t)} \geq \max\{\exp(\frac{(\alpha_i^{(s)} - \alpha_i^{(t)}) \log x_i^{(s)}}{\alpha_i^{(t)}}), 1\}. \qquad (1)$$

The proofs of all the theorems in this paper can be found in (Shen, Narayanaswamy, and Sundaram 2014). This gives us a lower bound on the expansion ratio of a consumer. How close a user's expansion ratio is to her baseline is a measure of her *tolerance* or flexibility. The smaller the required expansion ratio, the greater the tolerance. Conversely, the larger the expansion ratio the more intolerant is the consumer of the slot change.

What is the benefit to the distribution company?: The distribution company will agree to shift consumer i's load $x_i^{(s)}$ from time slot s to slot t with an expansion ratio $m_i^{(s\to t)}$ for the same payment only if it benefits the distribution company to do so. For load $x_i^{(s)}$, the distribution company obtains a profit $\eta = (p_c^{(s)} - p_m^{(s)}) x_i^{(s)}$. After the expanded load shifting to time slot t, the profit becomes $\eta' = p_c^{(s)} x_i^{(s)} - p_m^{(t)} m_i^{(s\to t)} x_i^{(s)}$. So long as $p_m^{(s)} > p_m^{(t)} \cdot m_i^{(s\to t)}$, the distribution company stands to increase its profit ($\eta' - \eta > 0$) and hence will agree to the expanded load shifting.

Our expanded load shifting incentive mechanism, SmartShift, meets both these requirements and is supported

Protocol 1 The message exchange protocol for SmartShift

STEP 1: The distribution company and the end consumers exchange information, or observe common historical data, to achieve consensus on the long term mean μ_m and standard deviation σ_m of the market price.
STEP 2: The n end consumers communicate their consumption $\{x_i^{(t)}|1 \le i \le n, 1 \le t \le k\}$ over the next period of k time slots, and their expansion ratios (which must be satisified if their loads would be shifted) $\{m_i^{(s\to t)}|1 \le i \le n, 1 \le s \le k, 1 \le t \le k\}$ to the distribution company.
STEP 3: The distribution company computes the optimal load shifting and communicates the resulting load allocation to the end consumers.

Algorithm 2 Optimal load shifting under exogenous market price

Input: $\{x_i^{(t)}|1 \le i \le n, 1 \le t \le k\}$, $\{m_i^{(s\to t)}|1 \le i \le n, 1 \le s \le k, 1 \le t \le k\}$.
Output: $\{z_i^{(t)}|1 \le i \le n, 1 \le t \le k\}$.

for each consumer i **do**
  for each time slot t **do**
    $z_i^{(t)} \leftarrow \arg\min_{1 \le t' \le k}\{m_i^{(t\to t')} p_m^{(t')}\}$;
  end for
end for

by Protocol 1 that exchanges messages between the distribution company and the end consumers.

We now study how to achieve optimal load shifting in both cases - endogenous and exogenous prices.

Endogenous market price: optimal load shifting

Theorem 2. *Computing optimal load shifting under endogenous market price is (weakly) NP-complete.*

We make a further comment, for the special setting where the consumption of electricity is discrete, i.e. users may shift entire appliances or units but not arbitrary amounts. In this setting it is not hard to create examples, where an equilibrium (in terms of prices and load distribution) may not exist. Suppose utilities and prices are such that, if a user doesn't consume a unit of electricity then the resultant lower price makes it favorable for her to in fact consume the unit; but, in doing so, the load is driven up and along with it the price (which is dependent quadratically on the load) thus making it unfavorable to consume the additional unit. Thus, determining endogenous prices with all or nothing or discrete consumption is an inherently hard problem where an equilibrium may not exist.

Exogenous market price: optimal load shifting In the exogenous market price case, the load shifting will never affect the market price $\{p_m^{(t)}\}$. To achieve the optimal load shifting, for each consumer i, her load $x_i^{(s)}$ should be shifted to the time slot $t^* = z_i^{(t)}$ at which $p_m^{(t^*)} \cdot m_i^{(s\to t^*)}$ is the minimum over all time slots $\{t^*\}$. Computing optimal load shifting under exogenous market price, as Alg. 2, runs in $O(nk^2)$ time.

Exogenous market price: real-time vs flat-rate: We now study how the two pricing mechanisms (real-time pricing and flat-rate pricing) have different performance on social welfare, revenue and profit in the model of exogenous market price. Specifically, we have the following theorems:

Theorem 3. *Given exogenous market prices, the expected social welfare W in real-time pricing is less than the expected social welfare W' in flat-rate pricing.*

Theorem 4. *Given exogenous market prices, the expected revenue R_{RT} in real-time pricing is less than the expected revenue R_{FR} in flat-rate pricing.*

Theorem 5. *Given exogenous market prices, real-time pricing always leads to a zero profit ($\eta_{RT} = 0$), while flat-rate pricing leads to a zero profit in expectation ($E(\eta_{FR}) = 0$).*

(Recall that profit and loss refer to money gained or lost beyond the expected or pre-set margins.)

Theorem 5 shows that under real-time pricing the distribution company always achieves the anticipated profit per electricity unit; under flat-rate pricing the distribution company achieves the anticipated profit per electricity unit on average (in expectation). In other words, under flat-rate pricing the distribution company will sometimes incur a loss. This actually happens roughly about 50% of time, if the exogenous market price is assumed to be drawn from a normal distribution. In summary, flat-rate pricing mechanism has the advantage of encouraging consumers to consume the electricity (larger social welfare) they need without being constrained by considerations of price risk, but this risk is then shifted to the distribution company which could incur losses, i.e. $\eta < 0$, in some cases. However, we show, using simulations, that employing our expanded load shifting incentive mechanism, SmartShift, with flat-rate pricing reduces the probability of such (loss-incurring) scenarios.

Simulations

We evaluate SmartShift using a real-time electricity load data set from *Smart** (Barker et al. 2012). Through numerical experiments, we then study how our expanded load shifting mechanism can improve the performance over flat-rate pricing while providing a win-win solution to both the distribution company as well as the consumer.

We obtain **real-time electricity load** $\{x_i^{(t)}\}$ from the *Microgrid Data Set* (Barker et al. 2012) which includes average real power usage (kilowatts) at one minute sampling rate from 400 homes for 24 hours ($k = 24$). We average the (per minute) sampled data points per hour to get the hourly averaged electricity usage. We filter out households with zero power usage resulting in $n = 395$ profiles.

We simulate the **real-time market price** $\{p_m^{(t)}\}$ using a normal distribution $N(\mu_m, \sigma_m^2)$ motivated by the model of (Roozbehani, Dahleh, and Mitter 2010). We obtained similar results for other ditributions. We fix μ_m and vary σ_m to study how the fluctuation of the price will affect the market. We generate **valuation coefficients** $\{\alpha_i\}$ as follows: We assume that the electricity load profiles are generated under a flat-rate pricing model with mean price $\mu_p = 50$. Thus, $\alpha_i^{(t)} = x_i^{(t)} \cdot \mu_p$. Similar results were obtained with other

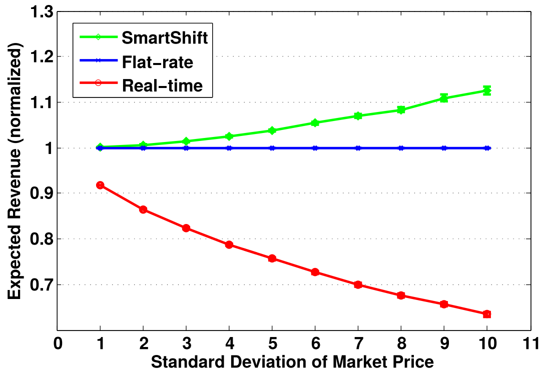

Figure 1: Normalized revenue vs price volatility

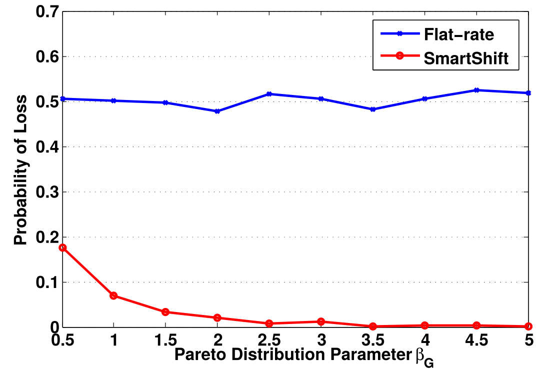

Figure 2: Loss probability vs consumer tolerance

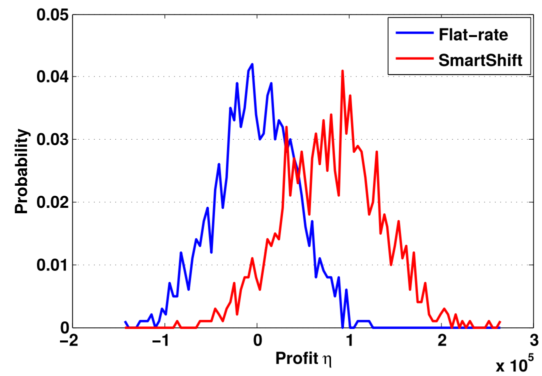

Figure 3: Probability density function of profit

settings. We draw the **risk-aversion parameter** $\{\lambda_i\}$, from a Pareto distribution as follows:

$$f_{pdf}(\lambda_i) = \frac{\beta_R \cdot \lambda_{min}}{\lambda_i^{\beta_R+1}} \tag{2}$$

where f_{pdf} gives the probability density function for the Pareto distribution; λ_{min} sets the minimum value for λ_i; β_R controls how much of the probability mass is close to λ_{min}, when β_R is larger, there is a larger probability mass of λ_i that is close to λ_{min}. To generate the **expansion ratio** $\{m_i^{(s\to t)}\}$, we first use Eq. 1 to calculate the minimum value $m_{i,min}^{(s\to t)}$. Then, we draw $m_i^{(s\to t)}$ from a Pareto distribution:

$$f_{pdf}(m_i^{(s\to t)}) = \frac{\beta_G \cdot m_{i,min}^{(s\to t)}}{(m_i^{(s\to t)})^{\beta_G+1}} \tag{3}$$

As before, the larger β_G is, the larger the probability that $m_i^{(s\to t)}$ is close to the minimum expansion ratio $m_{i,min}^{(s\to t)}$. Thus, β_G controls the dispersion in the tolerance of risk-averse consumers; the larger β_G is the more tightly concentrated is the population around a high tolerance level.

Risk-aversion and pricing mechanism: In all our simulations, each point is averaged over 100 repeated experiments. 95% confidence interval is also shown. In Figure 1, we draw the risk-aversion parameter $\{\lambda_i\}$ from the Pareto distribution, Eq. (2), by setting $\beta_R = 1$ and $\lambda_{min} = 1$. These are fixed for subsequent repetitions. Figure 1 shows that, while real-time pricing induces risk-averse consumers to contract usage with increased volatility (Theorem 4), SmartShift increases total revenue by simultaneously buffering consumers from price shocks and exploiting the increased volatility to more optimally shift the load.

In Figure 2, we fix the fluctation of the price by setting $\mu_m = 50, \sigma_m = 5$, but vary the tolerance of the consumers by varying the β_G from 0.5 to 5 with step size of 0.5. For each β_G, we sample a set of prices from normal distribution $N(\mu_m, \sigma_m^2)$ where $\mu = 50, \sigma_m = 5$, then sample the $\{m_i^{(s\to t)}\}$ from Eq. (3) with the fixed $\{m_{i,min}^{(s\to t)}\}$ and the varying β_G. We show that the larger β_G is (i.e. the more tightly clustered consumers are around a high tolerance level) the less the probability of the distribution company incurring a loss $\eta < 0$. This relationship follows from the fact that when the desired expansion ratio $m_i^{(s\to t)}$ is small, it is more probable for the expanded load shift to be satisfied.

In Figure 3, we fix the price fluctuation, risk-aversion and tolerance of the consumers. We sample prices from $N(\mu_m, \sigma_m^2)$ where $\mu_m = 50, \sigma_m = 5$, risk-aversion from Eq. (2) where $\beta_R = 1, \lambda_{min} = 1$, and tolerance from Eq. (3) where $\beta_G = 1$ and $m_{i,min}^{(s\to t)}$ computed by Eq. (1). By sampling 1000 sets of prices, we compute the profit η distribution. Figure 3 shows that the probability mass moves to the right after applying SmartShift. This also demonstrates that SmartShift reduces the probability of the distribution company incurring a loss ($\eta < 0$).

Conclusion

We have presented a general incentive-based mechanism, SmartShift, for reducing the load on the electricity grid. Our scheme grants users increased consumption in exchange for reducing their usage in peak periods. We have shown analytically that SmartShift under flat-rate pricing is a win-win for both consumers (increased social welfare) and producers (enhanced profits). SmartShift has elements of algorithms for iterative price setting (Mohsenian-Rad et al. 2010; Jain et al. 2013) with the added features of in-kind incentives and a slot-pairwise bidding language. A separate study of each these effects is of interest.

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