

# Planning with Multi-Valued Landmarks

**Lei Zhang\***, **Chong-jun Wang<sup>†</sup>**, **Jun Wu**, **Meilin Liu**, and **Jun-yuan Xie**

State Key Laboratory for Novel Software Technology,

Department of Computer Science & Technology,

Nanjing University, 210023, China

zhanglei.com@gmail.com, {chjwang,jyxie}@nju.edu.cn

## Abstract

Landmark heuristics are perhaps the most accurate current known admissible heuristics for optimal planning. A disjunctive action landmark can be seen a form of at-least-one constraint on the actions it contains. In many domains, some critical propositions have to be established for a number of times. Propositional landmarks are too weak to express this kind of constraints. In this paper, we propose to generalize landmarks to multi-valued landmarks to represent the more general cardinality constraints. We present a class of local multi-valued landmarks that can be efficiently extracted from propositional landmarks. By encoding multi-valued landmarks into CNF formulas, we can also use SAT solvers to systematically extract multi-valued landmarks. Experiment evaluations show that multi-valued landmark based heuristics are more close to  $h^*$  and compete favorably with the state-of-the-art of admissible landmark heuristics on benchmark domains.

The most accurate current admissible heuristics for optimal sequential planning are based on landmarks (Helmert and Domshlak 2009). A (disjunctive) action landmark for  $\Pi$  is a set of actions that has non-empty intersection with every solution plan for  $\Pi$ . Action landmarks can be seen as a simple form of cardinality constraints: at least one of its actions must be included in any plan. In many domains, propositions have to be established for many times, for example, in the blocks world domain, if the length of an optimal plan  $\pi^*$  for  $\Pi$  is  $n$ , the proposition HANDEMPTY will be established for at least  $\lfloor n/2 \rfloor$  times. Since only PUTDOWN and STACK actions have HANDEMPTY as add effects, the size of the action landmark that contains all PUTDOWN and STACK actions has a lower bound of  $\lfloor n/2 \rfloor$ . Propositional landmarks are too weak for these cases. In this paper, we introduce multi-valued landmarks to model these cardinality constraints.

**Definition 1.** A *multi-valued landmark*  $l$  for planning task  $\Pi$  is a set of actions  $\{a_1, \dots, a_k\}$  satisfying the constraint

\*Funded by Nanjing Univ. graduate school program(2011CL7).

<sup>†</sup>Funded by 973 program(2011 CB505300), NSFC(61021062, 61105069), Jiangsu Key R&D program(BE2011171, BE2012161). Copyright © 2013, Association for the Advancement of Artificial Intelligence (www.aaai.org). All rights reserved.

$lb \leq |l \cap \pi| \leq ub$  for any plan  $\pi$ . The  $lb$  and  $ub$  is referred to as  $l$ 's lower bound and upper bound respectively.

One particular example of multi-valued landmarks is the global multi-valued landmark  $L$  that consists of all actions in  $A$ :

$$L = \{a | a \in A\}.$$

A collection of action landmarks is complete for  $\Pi$  if the cost of a minimum-cost hitting set equals  $h^*(\Pi)$  (Bonet and Castillo 2011). The minimal cost hitting set is an optimal plan for  $\Pi$ . Similarly, a multi-valued landmark set  $L$  can also defined to be complete for  $\Pi$  if the tight lower bound induced by  $L$  is equal to  $h^*(\Pi)$ . It can be shown that the global multi-valued landmark set  $\{L\}$  for planning task  $\Pi$  is complete for its delete relaxation  $\Pi^+$ .

In planning tasks with delete effects, actions might be required more than once.  $\{L\}$  is not complete for these tasks. However, if we assume that the length of satisfying plan  $\pi$  is bounded, the number of each action also has a upper bound  $|\pi|$ . We can construct a complete multi-valued landmark set for  $\Pi$  by repeating each action for  $|\pi|$  times.

$$L' = \{a @ t | t \in [0, |\pi|], a \in A\}$$

It can also be easily shown that  $\{L'\}$  is complete for  $\Pi$ . Given a complete global landmark set  $\{L\}$  for  $\Pi$ , we can exploit  $L$  to find an optimal plan for  $\Pi$  by encoding  $L$  into CNF formula as an at-most-k constraint.

$$F(L) = CNF(\Pi) \wedge CNF(\sum_{a \in L} a \leq tb(L)) \quad (1)$$

where  $tb(L)$  is the tight lower bound of  $L$ . By solving  $F(L)$  using SAT solvers, we can get a valid plan  $\pi$  with cost less than or equal to  $tb(L)$ . Since  $tb(L)$  is a lower bound for any plan,  $\pi$  is optimal. The satisfying assignment of  $F(L)$  gives an optimal plan for  $\Pi$ . The At-Most-K cardinality constraints in  $F(L)$  can be encoded into CNF formulas compactly using various methods like sequential counters and sorting networks. Given a set of action  $l$ , we can also use general purpose SAT solvers to systematically extract its tight lower bound by testing the satisfiability of  $F(l, K) = CNF(\Pi) \wedge CNF(\sum_{a \in l} a \leq K)$ . Either linear or binary search can be performed on  $K$  to find the largest  $K$  that makes  $F(l, K)$  satisfiable, which is  $l$ 's tight lower bound  $tb(l)$ .

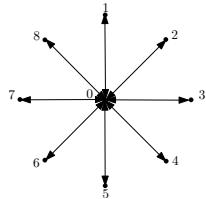


Figure 1: A visitall planning task  $\Pi$ :  $P = \{At(i), Visited(i) | i \in [0, 8]\}$ ,  $I = \{At(0), Visited(0)\}$ ,  $G = \{Visited(i) | i \in [0, 8]\}$ . For every position  $i$  and position  $j$  that are adjacent, there is an action  $Move(i, j)$  that deletes  $At(i)$  and adds  $At(j)$  and  $Visited(j)$ .

Global landmarks are computationally hard to extract and exploit in practice. Instead of using one global landmark with only one global lower and upper bound, we can maintain smaller lower and upper bounds for many **local** multi-valued landmarks that contains only a small number of actions. An interesting class of local multi-valued landmarks can be efficiently extracted from propositional landmarks. It's obvious that for each fact  $p$ , the number of times it is added must be equal to the number of times it is deleted:

$$\sum_{a_i \in add(p)} a_i + |p \in I| = \sum_{a_i \in del(p)} a_i + |p \in G|, \forall p \in P \quad (2)$$

where  $add(p)$  is the set of actions that have  $p$  as add effect and  $del(p)$  is the set of actions that have  $p$  as delete effect. Note that both the left hand side and right hand size of equation 2 can be seen as a local multi-valued landmark. New non-trivial local landmarks can be obtained by iteratively applying equation 2 to existing landmark set  $L_0$ .  $L_0$  can be initialized by propositional landmarks.

Once we have a collection  $C$  of local landmarks, there are many ways to exploit them. For example, if landmarks in  $C$  are disjoint, we can obtain a heuristic by taking the sum of their lower bounds:  $h = \sum_{l \in C} lb(l)$ . More informative heuristics can be obtained by solving the following integer program defined by local landmarks set  $C$ :

$$M(C) = \min\{\sum_{a \in C} a | \forall c \in C, lb(c) \leq \sum_{a \in c} a \leq ub(c)\} \quad (3)$$

As an example, consider the planning task  $\Pi$  shown in figure 1. Initially, we are at position 0. The goal is to visit all position from 0 to 8. By iteratively cutting the justification graph, LM-cut will give the following landmark set  $L$ :

$$L = \{\{Move(0, i)\} | i \in [1, 8]\} \quad (4)$$

The heuristic value given by LM-cut is 8, which is equal to  $h^+(\Pi)$ . If we apply equation 2 to  $At(0)$ , we can obtain:

$$1 + \sum_{i \in [1, 8]} Move(i, 0) = |At(0) \in G| + \sum_{i \in [1, 8]} Move(0, i) \quad (5)$$

Since actions  $Move(0, i)$  for  $i = 1..8$  are all unit landmarks, the rhs of equation 5 is greater than or equal to 8. Therefore, we obtain a local multi-valued landmark  $l' = \{Move(i, 0) | i \in [1, 8]\}$  with lower bound 7. Since  $l'$  is

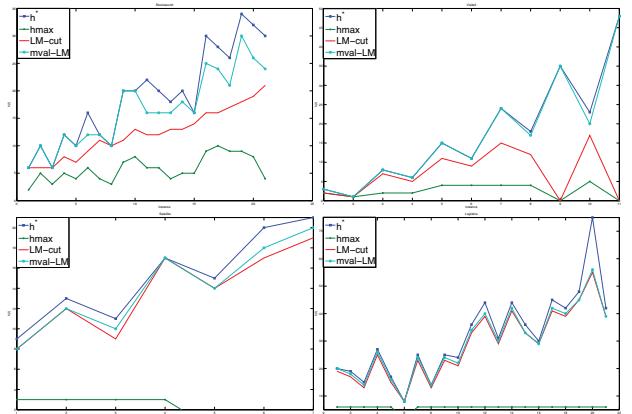


Figure 2: Comparison of heuristic values for initial state  $I$  in the blocksworld, visitall, SAT, satellite and logistics domains

disjoint with landmarks in  $L$ , we can obtain a global lower bound by adding their costs:  $8 + 7 = 15$ , which is exactly the cost of  $\Pi$ 's optimal plan.

We can also extract new local landmarks from an existing local landmark set  $C$ . By solving the integer program  $M(C)$ , we can get a plan  $\pi$ . Let  $F = CNF(\Pi) \wedge CNF(\{\neg a | a \notin \pi\})$ , we can solve  $F$  using SAT solvers. Modern SAT solvers can give an unsatisfiable core of  $F$  if  $F$  is unsatisfiable. The actions in the unsatisfiable core will constitute a new landmark. This process can be repeated until the local landmark set  $C$  is complete, and we will get an optimal plan for  $\Pi$ .

We study the accuracy and performance of multi-valued heuristics on 8 domains from recent international planning competitions. We also use a satisfiability domain(SAT) that are compiled to PDDL from random satisfiability instances. The platform used for all experiments is FastForward (Helmer 2006). For each planning task, we compute the heuristics value  $h(I)$  for its initial state  $I$ . The heuristics used for comparison are  $h_{max}$ ,  $LM-cut$ , multi-valued landmark heuristics (denoted as  $mval-LM$ ) and the optimal heuristic  $h^*$ . The optimal heuristic is computed by running a heuristic search planner with  $LM-cut$  until it finds a solution. The result is shown in figure 2. We can see that the use of multi-valued landmarks significantly improve the heuristic estimate in three domains: blocksworld, SAT and visitall. Note that in SAT and visitall, multi-valued landmark heuristic is very close to  $h^*$ . In the satellite and logistics domains, the difference between multi-valued landmarks heuristic and  $LM-cut$  is less noticeable.

## References

- Bonet, B., and Castillo, J. 2011. A complete algorithm for generating landmarks. In *ICAPS*.
- Helmer, M., and Domshlak, C. 2009. Landmarks, Critical Paths and Abstractions: What's the Difference Anyway? In *International Conference on Automated Planning and Scheduling/Artificial Intelligence Planning Systems*.
- Helmer, M. 2006. The Fast Downward Planning System. *Journal of Artificial Intelligence Research* 26:191–246.