Conflict-Based Belief Revision Operators in Possibilistic Logic

Guilin Qi[1,2]
[1]Key Laboratory of Computer Network and Information Integration of State Education Ministry
School of Computer Science and Engineering, Southeast University, China
[2] Key Laboratory of Symbolic Computation and Knowledge Engineering of Ministry of Education
Jilin University, Changchun, 130012, P.R. China

Kewen Wang
School of Computing and Information Technology
Griffith University
Australia

Abstract

In this paper, we investigate belief revision in possibilistic logic, which is a weighted logic proposed to deal with incomplete and uncertain information. Existing revision operators in possibilistic logic are restricted in the sense that the input information can only be a formula instead of a possibilistic knowledge base which is a set of weighted formulas. To break this restriction, we consider *weighted prime implicants* of a possibilistic knowledge base and use them to define novel revision operators in possibilistic logic. Intuitively, a weighted prime implicant of a possibilistic knowledge base is a logically weakest *possibilistic term* (i.e., a set of weighted literals) that can entail the knowledge base. We first show that the existing definition of a weighted prime implicant is problematic and need a modification. To define a revision operator using weighted prime implicants, we face two problems. The first problem is that we need to define the notion of a *conflict set* between two weighted prime implicants of two possibilistic knowledge bases to achieve minimal change. The second problem is that we need to define the *disjunction* of possibilistic terms. We solve these problems and define two conflict-based revision operators in possibilistic logic. We then adapt the well-known postulates for revision proposed by Katsuno and Mendelzon and show that our revision operators satisfy four of the basic adapted postulates and satisfy two others in some special cases.

Introduction

Belief revision plays an important role in knowledge maintenance. When a rational agent receives a new piece of information about the domain of interest that contradicts her original beliefs, she may have to drop some old beliefs to accommodate the new information. In their pioneer work, Alchourrón, Gärdenfors, and Makinson (AGM for short) propose a set of postulates for characterizing a rational belief revision operator (see (Gardenfors 1988)). Since that, belief revision receives a lot of attention from several interrelated fields, such as Artificial Intelligence, Databases and the Semantic Web. AGM's work is discussed in an abstract level. In (Katsuno and Mendelzon 1992), Katsuno and Mendelzon reformulate AGM postulates for revision in propositional logic and use them to evaluate existing revision operators.

It is well-known that possibilistic logic, which is a weighted logic proposed to deal with incomplete and uncertain information, is closely related to belief revision (Dubois and Prade 1991). A *possibilistic knowledge base* is a finite set of weighted formulas, where the weight of a formula is interpreted as the *necessity degree* of the formula. Semantically, possibilistic logic is defined by a *possibility distribution*, which is a mapping from the set of all interpretations to the interval [0,1]. Belief revision operators in possibilistic logic are often semantically defined by possibility distributions associated with the possibilistic knowledge bases. Given a possibility distribution and a newly received formula ϕ (or (ϕ, a)), the semantic revision operators result in a new possibility distribution that is transformed from the original possibility distribution and the formula. Some syntactic revision operators have also been proposed (Benferhat et al. 1993; Delgrande, Dubois, and Lang 2006; Qi 2008). However, the input information is restricted to a propositional formula. As far as we know, there is no revision operator in possibilistic logic that allows the newly received information to be a general possibilistic knowledge base.

In this paper, we take a different semantic characterization of possibilistic logic, called *weighted prime implicants* of a possibilistic knowledge base and use it to define new revision operators. Intuitively, a weighted prime implicant of a possibilistic knowledge base is a most compact *possibilistic term* (i.e., a set of weighted literals) that can entail the knowledge base. We show that the existing definition of a weighted prime implicant is not suitable to define revision operators in possibilistic logic and propose a new definition. Our revision operators are generalizations of the well-known Dalal revision operator, which has been defined by either a distance between two interpretations (Dalal 1988) or a distance between two prime implicants (Marchi, Bittencourt, and Perrussel 2010). To define a revision operator using weighted prime implicants, we face two problems. First, we need to define the notion of a *conflict set* between two weighted prime implicants of two possibilistic knowledge bases. Second, we need to define the *disjunction* of possibilistic terms. We solve these problems and define two conflict-based revision operators in possibilistic logic. We then adapt the well-known postulates for revision proposed by Katsuno and Mendelzon and show that our revision operators satisfy four of the basic adapted postulates and satisfy two others in some special cases.

The rest of this paper is organized as follows. We first briefly introduce possibilistic logic. We then define the notion of disjunction of possibilistic terms and present a new definition of weighted prime implicants. After that, we define our revision operators and discuss their logical properties. Before we conclude this paper, we give a discussion of related work.

Preliminaries

Propositional logic

We consider a propositional language $\mathcal{L}_{PS}$ defined from a finite set of propositional variables (also called atoms) PS and the usual connectives. Formulas are denoted by Greek letters ϕ, ψ, ... The classical consequence relation is denoted as $\vdash$. An interpretation is a total function from PS to $\{0, 1\}$. Ω denotes the set of all interpretations. The definition of an interpretation can be extended to formulas in a standard way. An interpretation is a model of a formula if it assigns truth value 1 to the formula. A *knowledge base K* is a finite set of propositional formulas. It is sometimes identified as the conjunction of its elements. An interpretation is a model of a knowledge base if it satisfies all the formulas in it. K is consistent if and only if it has a model. Two knowledge bases K_1 and K_2 are equivalent, denoted $K_1{\equiv}K_2$, if and only if they have the same set of models.

A literal is either an atom or the negation of an atom. A *clause* C is a *disjunction* of literals: $C = l_1 \vee ... \vee l_n$ and its dual clause, or *term* D, is a *conjunction* of literals: $D = l_1 \wedge ... \wedge l_n$. Sometimes, we represent a term as a set of literals. A term D is an implicant of formula ϕ iff $D \vdash \phi$ and D does not contain two complementary literals. A *prime implicant* of knowledge base K is an implicant D of K such that for every other implicant D' of K, $D \not\vdash D'$ ((Quine 1959)).

Possibilistic logic

We introduce some basic notions of possibilistic logic (more details can be found in (Dubois, Lang, and Prade 1994)). The semantics of possibilistic logic is based on the notion of a *possibility distribution* $\pi : \Omega \to [0, 1]$. The possibility degree $\pi(\omega)$ represents the degree of compatibility (resp. satisfaction) of the interpretation ω with the available beliefs about the real world. A possibility distribution is said to be *normal* if $\exists \omega_0 {\in} \Omega$, such that $\pi(\omega_0) = 1$. From a *possibility distribution* π, two measures can be determined: the possibility degree of formula ϕ, $\Pi_\pi(\phi) = max\{\pi(\omega) : \omega \in \Omega, \omega \models \phi\}$ and the necessity degree of formula ϕ, $N_\pi(\phi) = 1 - \Pi_\pi(\neg\phi)$.

At the syntactic level, a *possibilistic formula*, is represented by a pair $(\phi,\ a)$, where ϕ is a propositional formula and a is an element of the real interval $[0, 1]$, which means that the necessity degree of ϕ is at least equal to a, i.e. $N(\phi) \geq a$. A possibilistic literal is a pair (l, a), where l is a literal and a a weight in [0,1] and a possibilistic term is a set of possibilistic literals. In this paper, we assume that there do not exist two pairs (l, a) and (l, b) such that $a \neq b$ ($a > 0$ and $b > 0$) in a possibilistic term. In possibilistic logic, uncertain or prioritized pieces of information can be represented by a *possibilistic knowledge base* which is a finite set of possibilistic formulas of the form $B = \{(\phi_i,\ a_i) : i = 1, ..., n\}$. We use $\mathcal{B}$ to denote all the possibilistic knowledge bases built over $\mathcal{L}_{PS}$. The classical base associated with B, denoted B^*, is defined as $B^* = \{\phi_i | (\phi_i,\ a_i) \in B\}$. A possibilistic knowledge base B is consistent if and only if its classical base B^* is consistent. Given a possibilistic knowledge base B, a unique *possibility distribution*, denoted π_B, can be obtained by the principle of minimum specificity (Dubois, Lang, and Prade 1994). For all $\omega \in \Omega$,

$$\pi_B(\omega) = \begin{cases} 1 & \text{if } \forall(\phi_i,\ a_i) \in B, \omega \models \phi_i, \\ 1 - max\{a_i | \omega \not\models \phi_i, (\phi_i, a_i) \in B\} & \text{otherwise.} \end{cases} \tag{1}$$

The a-cut (resp. strict a-cut) of a possibilistic knowledge base B is $B_{\geq a} = \{\phi_i {\in} B^* | (\phi_i, b_i) {\in} B \ and \ b_i {\geq} a\}$ (resp. $B_{>a} = \{\phi_i {\in} B^* | (\phi_i, b_i) {\in} B \ \ and \ b_i {>} a\}$). The *inconsistency degree* of B is: $Inc(B) = max\{a_i : B_{\geq a_i} \ is \ inconsistent\}$ with $Max(\varnothing) = 0$. That is, the inconsistency degree of B is the largest weight a_i such that the a_i-cut of B is inconsistent. Two possibilistic knowledge bases B and B' are said to be equivalent, denoted $B \equiv_s B'$, iff $\forall\ a \in (0, 1],\ B_{\geq a} {\equiv} B'_{\geq a}$.

There are two entailment relations in possibilistic logic.

Definition 1 Let B be a possibilistic knowledge base. A possibilistic formula (ϕ, a) is a weak possibilistic consequence of B, denoted by $B \vdash (\phi, a)$, if $a > Inc(B)$ and $B_{\geq a} \vdash \phi$. A possibilistic formula (ϕ, a) is a possibilistic consequence of B, denoted $B \vdash_\pi (\phi,\ a)$, if (i) $B_{\geq a}$ is consistent; (ii) $B_{\geq a} \vdash \phi$; (iii) $\forall b {>} a$, $B_{\geq b} \not\vdash \phi$. The entailment relation $\vdash$ (resp. $\vdash_\pi$) can be extended to two possibilistic knowledge bases as follows: $B \vdash B'$ (resp. $B \vdash_\pi B'$) if $B \vdash (\phi, a)$ (resp. $B \vdash_\pi (\phi, a)$) for all $(\phi, a) \in B'$.

It is clear that if $B \vdash_\pi (\phi,\ a)$, then $B {\vdash} (\phi, a)$ but the converse does not hold in general.

Disjunction of Possibilistic Terms and Weighted Prime Implicants

In this section, we first define the notion of disjunction of possibilistic terms. We show that our definition of a disjunction satisfies some desirable properties, thus is suitable for defining revision operators in possibilistic logic. We then reinvestigate the definition of weighted prime implicants in possibilistic logic. We show that the existing definition is not reasonable and propose a new definition.

Disjunction of possibilistic terms

We first define the disjunction of two possibilistic terms. Given two possibilistic terms $D_1 = \{(l_1, a_1), ..., (l_n, a_n)\}$ and $D_2 = \{(l'_1, b_1), ..., (l'_m, b_m)\}$, following the definition of disjunction of two terms in propositional logic, we should take the disjunction of l_i and l'_j, where $l_i \in D_1$ and $l'_j \in D_2$. However, the question is, which aggregation function should we use to aggregate the weights of l_i and l'_j? One may argue that we should use the maximum since $l_i \vee l'_j$ can be inferred by both D_1 and D_2. However, this definition is not desirable. For example, if we use the maximum to aggregate a_i and b_j which are weights of l_i and l'_j respectively, then $(l_i \vee l'_j, max(a_i, b_j))$ (assume $a_i, b_j > 0$) will be included in the disjunction of D_1 and D_2. But it cannot be inferred by both (l_i, a_i) and (l'_j, b_j) unless $a_i = b_j$. Thus, it seems to be reasonable to use the minimum to aggregate the weights of weighted literals in different possibilistic terms. So the definition of disjunction, denoted as $\vee$, is given as

$D_1 \vee D_2 = \{(l_i \vee l'_j, min(a_i, b_j)) | (l_i, a_i) \in D_1, (l'_j, b_j) \in D_2\}$.

It is easy to check that $\vee$ is associative and commutative. Thus the disjunction of more than two possibilistic terms can be easily defined.

We show an important property of disjunction $\vee$. It says that a possibilistic formula is inferred from the disjunction of a set of possibilistic terms if and only if it is inferred from each of them w.r.t. the weak possibilistic entailment.

Lemma 1 Given n possibilistic terms D_1, D_2, ..., D_n, we have $(D_1 \vee ... \vee D_n)_{\geq a} = (D_1)_{\geq a} \vee ... \vee (D_n)_{\geq a}$ for all $a \in [0, 1]$.

Proposition 1 Given n possibilistic terms D_1, D_2, ..., D_n that do not contain any conflicting literals, for any possibilistic formula (ϕ, a), we have

$$D_1 \vee ... \vee D_n \vdash (\phi, a) \text{ iff } D_i \vdash (\phi, a) \text{ for all } i.$$

By Lemma 1 and Proposition 1, we can show the following proposition.

Proposition 2 Given n possibilistic terms D_1, D_2, ..., D_n that do not contain any conflicting literals, for any possibilistic formula (ϕ, a), we have

$D_1 \vee ... \vee D_n \vdash_\pi (\phi, a)$ iff $D_i \vdash (\phi, a)$ for all i and there exists j such that $D_j \vdash_\pi (\phi, a)$.

Propositions 1 and 2 ensure that the disjunction of possibilistic terms has the same inferential power as the set of possibilistic terms (w.r.t two entailment relations in possibilistic logic). Thus, it is desirable to use $\vee$ to construct a possibilistic knowledge base from a set of possibilistic terms.

A new definition of weighted prime implicants

We first introduce the notion of weighted prime implicants of a possibilistic knowledge base given in (Qi, Liu, and Bell 2010). Let $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$ be a possibilistic knowledge base where ϕ_i is a clause[1]. A weighted implicant of B is a possibilistic term $D = \{(\psi_1, b_1), ..., (\psi_k, b_k)\}$, such that $D \vdash_\pi B$, where ψ_i are literals such that no two complementary literals exist. Let D and D' be two weighted implicants of B, D is said to be *subsumed* by D', denoted as $D \prec_s D'$, iff $D \neq D'$, $D'^* \subseteq D^*$ and for all $(\psi_i, a_i) \in D$, there exists $(\psi_i, b_i) \in D'$ with $b_i \leq a_i$ (b_i is 0 if $\psi_i \in D^*$ but $\psi_i \notin D'^*$). In other words, D is subsumed by D' iff $D \neq D'$, and every literal appearing in D' must appear in D with higher or same necessity degree.

Definition 2 (Qi, Liu, and Bell 2010) Let $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$ be a possibilistic knowledge base where ϕ_i are clauses. A weighted prime implicant (WPI) of B is a weighted implicant of B such that there does exist another weighted implicant D' of B such that D is subsumed by D'.

This definition looks reasonable and it reduces to the definition of prime implicants in the propositional case (when all weights of formulas are 1). However, it has some drawbacks as it does not satisfy the following two desirable properties:

Property 1: for any possibilistic knowledge base, there is at least one WPI of it.

Property 2: for any possibilistic knowledge base B and any formula ϕ, $B^* \vdash \phi$ iff $\bigvee_{D_i \in \mathsf{WPI}(B)} D_i^* \vdash \phi$.

To see why Property 1 is violated, let us consider $B = \{(p \vee q, 0.8), (p, 0.7), (q, 0.7)\}$. To infer $(p \vee q, 0.8)$, any WPI D under Definition 2 should contain either $(p, 0.8)$ or $(q, 0.8)$. If it contains $(p, 0.8)$, then we do not have $D \vdash_\pi (p, 0.7)$. If it contains $(q, 0.8)$, we do not have $D \vdash_\pi (q, 0.7)$. Thus, there is no WPI for B. This example also shows that Property 2 is violated.

The following example shows another problem of Definition 2.

Example 1 Let $B = \{(q \vee r, 0.9), (q, 0.8)\}$. We can check that $D = \{(r, 0.9), (q, 0.8)\}$ is a WPI of B under Definition 2. However, $D' = \{(q, 0.9)\}$ is not a weighted implicant of B under Definition 2 because $D' \nvdash_\pi (q, 0.8)$. Thus, the only WPI of B is D, which is logically stronger than B. Intuitively, D' should also be a WPI of B and we can show that $B \equiv_s D \vee D'$.

In Example 1, D' should be a WPI of B, but it is excluded by the definition of weighted implicants as the possibilistic inference is used.

We give another definition of WPIs based on the weak possibilistic inference.

Definition 3 A weak weighted implicant of a possibilistic knowledge base B is a possibilistic term $D = \{(\psi_1, b_1), ..., (\psi_k, b_k)\}$, such that $D \vdash B$, where ψ_i are literals such that no two complementary literals exist.

To define WPIs based on weak weighted implicants, we can still use Definition 2, and we will use the name WPI for the new definition if no confusion is caused. This new definition works well for Example 1 because D' is now a WPI of B under the new definition. In the following, we use $\mathsf{WPI}(B)$ and $\mathsf{WI}(B)$ to denote the set of all WPIs of B and the set of all weak weighted implicants of B respectively. Since there are only finite weights in a possibilistic knowledge base, we can check that there are only finite many WPIs of a possibilistic knowledge base. For any possibilistic knowledge base B, we can easily find a WPI D of it by the following steps. First, D is initialized as an empty set. Second, for each (ϕ, a), suppose $\phi = l_1 \vee ... \vee l_k$, then

[1] A possibilistic formula of the form $(\phi_1 \wedge ... \wedge \phi_n, a)$ can be equivalently decomposed into a set of formulas (ϕ_1, a),...,(ϕ_n, a) due to the min-decomposability of necessity measures.

we randomly select one l_i and add (l_i, a) to D. Third, we remove redundant weighted literals from D to make it a WPI. Thus, our new definition of WPIs satisfies Property 1.

We need to point out that there are knowledge bases that cannot be captured by WPIs.

Example 2 Let $B = \{(q \vee r, 0.9), (\neg r, 0.8)\}$. Any WPI D of B should include $(\neg r, 0.8)$, thus q must appear in it. Since D can infer $(q \vee r, 0.9)$, it must contain $(q, 0.9)$. Thus $D \vdash_\pi (q, 0.9)$ and D is the only WPI of B. However, we can check that $B \vdash_\pi (q, 0.8)$.

As we can see from this example, WPIs may overestimate the weight of a formula when they are used to make inference. However, we are able to show that our new definition of WPIs satisfies Property 2, thus they will not infer new formulas.

Proposition 3 $B^* \vdash \phi$ iff $\bigvee_{D_i \in \mathsf{WPI}(B)} D_i^* \vdash \phi$.

Definition 4 Given a possibilistic knowledge base B, we say B is WPI-definable if $B \equiv_s \bigvee_{D_i \in \mathsf{WPI}(B)} D_i$.

That is, a possibilistic knowledge base B is WPI-definable if it is equivalent to the disjunction of its WPIs. The following is a corollary of Proposition 2. It gives a necessary and sufficient condition for WPI-definability.

Corollary 1 A possibilistic knowledge base B is WPI-definable iff the following statement holds: for any possibilistic formula (ϕ, a), $B \vdash_\pi (\phi, a)$ iff $D \vdash (\phi, a)$ for all $D \in \mathsf{WPI}(B)$ and there exists $D \in \mathsf{WPI}(B)$, $D \vdash_\pi (\phi, a)$.

There are many possibilistic knowledge bases that are WPI-definable. We give two simple examples: (1) any possibilistic knowledge base consisting of a single possibilistic formula is WPI-definable; (2) any possibilistic knowledge base which does not contain complementary literals is WPI-definable. Of course, there are more complicated WPI-definable possibilistic knowledge bases. For those possibilistic knowledge bases B that are not WPI-definable, we say that another possibilistic knowledge base B' is a lower WPI-definable approximation of B if B' is WPI-definable and $B' \vdash B$. Then $\mathsf{WPI}(B)$ can be viewed as the greatest lower WPI-definable approximation of B according to the following proposition.

Proposition 4 Given a possibilistic knowledge base B, we have (1) $\bigvee_{D_i \in \mathsf{WPI}(B)} D_i \vdash B$ and (2) for any lower WPI-definable approximation B' of B, $B' \vdash \bigvee_{D_i \in \mathsf{WPI}(B)} D_i$.

Conflict-based Revision Operators in Possibilistic Logic

In belief revision, a basic requirement is that the newly received information (which can be either a formula or a knowledge base) should be inferred by the revised knowledge base. In this section, we consider the weak possibilistic entailment when defining our revision operators. That is, given two possibilistic knowledge bases B_1 and B_2, the revised possibilistic knowledge base $B_1 \circ B_2$ by the revision operator $\circ$ should infer every possibilistic formula in B_2 w.r.t. the weak possibilistic entailment relation. We first give two revision operators in possibilistic logic. We then adapt KM postulates for revision to possibilistic logic and show that our revision operators satisfy all of them. In this paper, we assume that all the possibilistic knowledge bases in $\mathcal{B}$ share a common scale.

Revision operators

We first adapt a notion used in (Marchi, Bittencourt, and Perrussel 2010).

Definition 5 Let $\Gamma : \mathcal{B} \times \mathcal{B} \mapsto \mathcal{B}$ be a function defined as follows: let B and B' be two possibilistic knowledge bases, we have

$$\begin{aligned}\Gamma(B, B') \quad = \quad & \{f(D_1, D_2) | D_1 \in \mathsf{WPI}(B) \text{ and} \\ & D_2 \in \mathsf{WPI}(B')\},\end{aligned}$$

where $f(D_1, D_2) = r(D_2 \cup \{(l, a) \in D_1 | l^c \notin D_2^*\})$ with r a function that removes redundant possibilistic literals from a possibilistic term, i.e., $r(D) = \{(l, a) | (l, a) \in D, a = max\{a_i | (l, a_i) \in D\}\}$. Informally, for any two weighted literals (l, a) and (l, b), if $a > b$, then r removes (l, b) as it is redundant. l^c is the complement of literal l.

The function f is used to combine two possibilistic terms D_1 and D_2 by giving preference to D_2, that is, any possibilistic literal in D_1 that is in conflict with a possibilistic literal in D_2 will be removed after combination. The intuition behind it is that any belief in B' should be accepted after revision, thus any possibilistic term of B' is preferred to any possibilistic term in B. Given a possibilistic knowledge base B, we define $\Gamma(B) = \bigcup_{B' \in \mathcal{B}} \Gamma(B, B')$.

Example 3 Suppose there are four atoms p, q, r and s, where

- p represents "red light is on"
- q represents "green light is off"
- r represents "press the button"
- s represents "yellow light is on"

Suppose we have a possibilistic knowledge base $B = \{(\neg q \to r, 0.8), (p \to \neg r, 0.7), (\neg s \to \neg r, 0.6)\}$ that consists of three uncertain rules. Later on, we get another possibilistic knowledge base which contains some uncertain facts, i.e., we have $B' = \{(\neg q, 0.9), (p, 0.8), (\neg s, 0.7)\}$, and we want to revise B by incorporating B'. We have $\mathsf{WPI}(B) = \{D_1, D_2, D_3, D_4\}$, where
$D_1 = \{(q, 0.8), (\neg p, 0.7), (s, 0.6)\}$,
$D_2 = \{(q, 0.8), (\neg p, 0.7), (\neg r, 0.6)\}$,
$D_3 = \{(q, 0.8), (\neg r, 0.7)\}$ and
$D_4 = \{(r, 0.8), (\neg p, 0.7), (s, 0.6)\}$.
It is easy to check that B is WPI-definable. We have $\mathsf{WPI}(B') = B'$ so it is also WPI-definable. Finally, we have $\Gamma(B, B') = \{f(D_1, B'), f(D_2, B'), f(D_3, B'), f(D_4, B')\}$, where
$f(D_1, B') = \{(\neg q, 0.9), (p, 0.8), (\neg s, 0.7)\}$,
$f(D_2, B') = \{(\neg q, 0.9), (p, 0.8), (\neg s, 0.7), (\neg r, 0.6)\}$,
$f(D_3, B') = \{(\neg q, 0.9), (p, 0.8), (\neg r, 0.7), (\neg s, 0.7)\}$,
$f(D_4, B') = \{(\neg q, 0.9), (r, 0.8), (p, 0.8), (\neg s, 0.7)\}$.

Our revision operators are inspired from the revision operator defined by prime implicants given in (Marchi, Bittencourt, and Perrussel 2010). The basic idea is that we first define a preference relation over $\Gamma(B)$, then define the result of revision by the set of minimal possibilistic terms in $\Gamma(B, B')$ w.r.t. the preference relation. In the following, we give two such preference relations: one is defined by the quantity of conflict between two WPIs of two possibilistic knowledge bases given in (Qi, Liu, and Bell 2010), and the other is defined by the lexicographic ordering.

Suppose B is a possibilistic knowledge base and $D = f(D_1, D_2)$ is in $\Gamma(B)$, where $D_1 \in \mathsf{WPI}(B)$ and $D_2 \in \mathsf{WPI}(B')$ for some possibilistic knowledge base B'. The quantity of conflict between D_1 and D_2 is defined as $q_{Con}(D_1, D_2) = \sum_{(l,a)\in D_1 \ and \ (l^c,b)\in D_2} min(a, b)$ (Qi, Liu, and Bell 2010). Our first preference relation $\preceq^B_{sum}$ over $\Gamma(B)$ (we omit the superscript when it is clear from the context) is defined as follows: given $D = f(D_1, D_2)$ and $D' = f(D'_1, D'_2)$ in $\Gamma(B)$,

$D \preceq_{sum} D'$ iff $q_{Con}(D_1, D_2) \leq q_{Con}(D'_1, D'_2)$.

That is, D is preferred to D' iff the quantity of conflict between D_1 and D_2 is less than or equal to the quantity of conflict between D'_1 and D'_2.

For any $D = f(D_1, D_2)$ in $\Gamma(B)$, we use S_D to denote the set of conflicting weighted literals, i.e.,

$S_D = \{(l, c) : (l, a) \in D_1, (l^c, b) \in D_2, c = min(a, b)\}$.

We define a preference relation $\preceq^B_{lex}$ over $\Gamma(B)$ (we omit the superscript when it is clear from the context) as follows: for any $D = f(D_1, D_2)$ and $D' = f(D'_1, D'_2)$ in $\Gamma(B)$, suppose $c_1 > c_2 > ... > c_n$ are all distinct weights appearing in $D \cup D'$, then $D_1 \prec_{lex} D_2$, iff $|\{l|(l, c_i) \in S_D\}| < |\{l'|(l', c_i) \in S_{D'}\}|$ for all i or there exists c_i such that $|\{l|(l, c_i) \in S_D\}| < |\{l'|(l', c_i) \in S_{D'}\}|$ and $|\{l|(l, c_j) \in S_D\}| = |\{l'|(l', c_j) \in S_{D'}\}|$ for any $j < i$. $D_1 \preceq_{lex} D_2$ iff $D_1 \prec_{lex} D_2$ or $|\{l|(l, c_i) \in S_D\}| = |\{l'|(l', c_i) \in S_{D'}\}|$ for all i. Let $\preceq$ be either $\preceq_{sum}$ or $\preceq_{lex}$, we define $D \prec D'$ iff $D \preceq D'$ but $D' \not\preceq D$.

We are now ready to define our revision operators.

Definition 6 A sum-based revision operator $\circ_{sum} : \mathcal{B}\times\mathcal{B} \mapsto \mathcal{B}$ is defined as follows:

$$B \circ_{sum} B' = \vee(min(\Gamma(B, B'), \preceq_{sum})).$$

Similarly, we can define a lex-based revision operator $\circ_{lex}$ as follows:

$$B \circ_{lex} B' = \vee(min(\Gamma(B, B'), \preceq_{lex})).$$

Example 4 (Continue Example 3) We compute the quantity of conflict between D_i and B' as follows: $q_{Con}(D_1, B') = 2.1$, $q_{Con}(D_2, B') = 1.5$, $q_{Con}(D_3, B') = 0.8$ and $q_{Con}(D_4, B') = 1.3$. So $B \circ_{sum} B' = f(D_3, B') = \{(\neg q, 0.9), (p, 0.8), (\neg r, 0.7), (\neg s, 0.7)\}$. that is, $(q \vee r, 0.8)$ are kept but other formulas are dropped. In this case, we believe that we should not press the button. We then have $S_{f(D_1,B')} = \{(q, 0.8), (p, 0.7), (s, 0.6)\}$, $S_{f(D_2,B')} = \{(q, 0.8), (p, 0.7)\}$, $S_{f(D_3,B')} = \{(q, 0.8)\}$ and $S_{f(D_4,B'')} = \{(p, 0.7), (s, 0.6)\}$. Thus, $B \circ_{lex} B' = f(D_4, B') = \{(\neg q, 0.9), (r, 0.8), (p, 0.8), (\neg s, 0.7)\}$, that is, $(\neg p \vee \neg r, 0.7)$ and $(s \vee \neg r, 0.6)$ are kept but $(q \vee r, 0.8)$ is dropped. In this case, we believe that we should press the button.

In Example 4, different possibilistic knowledge bases are obtained by different revision operators. We cannot say that one revision operator is better than the other but rather say that they are applicable for different settings. The former is perhaps more meaningful when weights attached with formulas are given quantitative meaning, whilst the latter is more meaningful when weights attached with formulas are given qualitative meaning.

Logic properties

Suppose B_1, B_2, B_3, B'_1 and B'_2 are possibilistic knowledge bases, we adapt the KM postulates for revision as follows.

(RP1) $B_1 \circ B_2 \vdash B_2$
(RP2) If $B_1 \cup B_2$ is consistent then $(B_1 \circ B_2)^* \equiv (B_1 \cup B_2)^*$
(RP3) If B_2 is consistent then $B_1 \circ B_2$ is also consistent
(RP4) If $B_1 \equiv_s B_2$ and $B'_1 \equiv_s B'_2$ then $B_1 \circ B_2 \equiv_s B'_1 \circ B'_2$
(RP5) If $(B_1 \circ B_2) \cup B_3$ is WPI-definable then $(B_1 \circ B_2) \cup B_3 \vdash B_1 \circ (B_2 \cup B_3)$
(RP6) If $(B_1 \circ B_2) \cup B_3$ is consistent then $B_1 \circ (B_2 \cup B_3) \vdash (B_1 \circ B_2) \cup B_3$

Before we verify the logical properties of our revision operators, we show a lemma.

Lemma 2 Suppose $B \cup B'$ is consistent. For any $D \in \mathsf{WPI}(B \cup B')$, there exist $D_1 \in \mathsf{WPI}(B)$ and $D_2 \in \mathsf{WPI}(B')$ such that $D = r(D_1 \cup D_2)$.

Proposition 5 Our revision operators $\circ_{sum}$ and $\circ_{lex}$ satisfy (RP1)-(RP4). But they may not satisfy (RP5) and (RP6).

Proof. We only show that $\circ_{sum}$ satisfies (RP1)-(RP6) as properties of $\circ_{lex}$ can be checked similarly.

(RP1), (RP3) and (RP4) can be easily shown. We consider (RP2). We only need to show that $\mathsf{WPI}(B_1 \cup B_2) = min(\Gamma(B, B'), \preceq_{sum})$ when $B_1 \cup B_2$ is consistent. Suppose $D \in min(\Gamma(B, B'), \preceq_{sum})$. So $D = f(D_1, D_2)$ where $D_1 \in \mathsf{WPI}(B)$ and $D_2 \in \mathsf{WPI}(B')$. We prove $D \in \mathsf{WI}(B)$ by absurdity. Suppose $D \notin \mathsf{WI}(B)$. Since $B \cup B'$ is consistent, $\mathsf{WPI}(B \cup B')$ is not empty. Suppose $D' \in \mathsf{WPI}(B \cup B')$. According to Lemma 2, $D' \in \Gamma(B, B')$. Since $D' \in \mathsf{WI}(B)$ and $D \notin \mathsf{WI}(B)$, we have $D' \prec_{sum} D$, contradiction. Therefore, $D \in \mathsf{WI}(B_1 \cup B_2)$. We show $D \in \mathsf{WPI}(B_1 \cup B_2)$ by absurdity. Suppose $D \notin \mathsf{WPI}(B_1 \cup B_2)$, then there exists $D' \in \mathsf{WPI}(B_1 \cup B_2)$ such that $D' \prec_{sum} D$' However, according to Lemma 2, $D' \in \Gamma(B, B')$, a contradiction. Next, suppose $D \in \mathsf{WPI}(B_1 \cup B_2)$, according to Lemma 2, $D \in \Gamma(B, B')$. Since $D \in \mathsf{WI}(B)$, $D \in min(\Gamma(B, B'), \preceq_{sum})$. So $(B \circ_{sum} B')^* \equiv \bigvee_{D_i \in \mathsf{WPI}(B \cup B')} D^*_i$. By Proposition 3, we have $(B_1 \circ B_2)^* \equiv (B_1 \cup B_2)^*$.

We give a counterexample to show that (RP5) may not hold. Suppose $B_1 = \{(q \vee r, 0.8), (p \vee \neg r, 0.7)\}$, $B_2 = \{((\neg p \wedge r) \vee (\neg q \wedge \neg p), 0.6)\}$ and $B_3 = \{(\neg q, 0.8)\}$, we have $\mathsf{WPI}(B_1) = \{D_1, D_2, D_3\}$, where $D_1 = \{(q, 0.8), (p, 0.7)\}$, $D_2 = \{(q, 0.8), (\neg r, 0.7)\}$ and $D_3 = \{(r, 0.8), (p, 0.7)\}$, and

$\mathsf{WPI}(B_2) = \{\{(\neg p, 0.6), (r, 0.6)\}, \{(\neg q, 0.6), (\neg p, 0.6)\}\}$. Thus $(B_1 \circ_{sum} B_2) \cup B_3 = \{(\neg q, 0.8), (\neg p, 0.6)\}$ (it is WPI-definable). On the other hand, $B_1 \circ_{sum} (B_2 \cup B_3) = \{(\neg p, 0.6), (\neg q, 0.8), (r, 0.8)\}$. So $(B_1 \circ_{sum} B_2) \cup B_3 \not\vdash B_1 \circ_{sum} (B_2 \cup B_3)$, (RP5) does not hold.

For (RP6), suppose $B_1 = \{(p, 0.8), (q, 0.6)\}$, $B_2 = \{(\neg p \vee r, 0.6), (\neg p, 0.4)\}$ and $B_3 = \{(\neg p \vee \neg q, 0.6)\}$, then $(B_1 \circ_{sum} B_2) \cup B_3 = \{(r, 0.6), (q, 0.6), (\neg p, 0.6)\}$. On the other hand, $B_1 \circ_{sum} (B_2 \cup B_3) = \{(\neg p, 0.6), (q, 0.6)\}$. Since $B_1 \circ_{sum} (B_2 \cup B_3) \not\vdash (B_1 \circ_{sum} B_2) \cup B_3$, (RP6) does not hold. □

The reason why (RP5) and (RP6) are falsified is that when we combine a WPI D of $B_1 \circ B_2$ (assume $D = f(D_1, D_2)$, where $D_1 \in \mathsf{WPI}(B_1)$ and $D_2 \in \mathsf{WPI}(B_2)$) and a WPI D_3 of B_3, then $q_{Con}(D_1, r(D_2 \cup D_3))$ may be greater than $q_{Con}(D_1, D_2)$ even if $D \cup D_3$ is consistent. Thus, $f(D_1, r(D_2 \cup D_3))$ may be not in $min_{\preceq_{B_1}}(\Gamma(B_1, B_2 \cup B_3))$.

There are still some cases where (RP5) and (RP6) can be satisfied by our revision operators.

Proposition 6 Our revision operators $\circ_{sum}$ and $\circ_{lex}$ satisfy (RP5) and (RP6) when any weight appearing in B_2 is greater than or equal to all weights appearing in B_3.

Proof. We only consider (RP5) as (RP6) can be shown similarly. Given $D \in \mathsf{WPI}((B_1 \circ_{sum} B_2) \cup B_3)$, by Lemma 2, there exist $D' \in \mathsf{WPI}(B_1 \circ_{sum} B_2)$ and $D_3 \in \mathsf{WPI}(B_3)$ such that $D = r(D' \cup D_3)$. By Proposition 2, we can assume $D' = f(D_1, D_2)$ is in $min(\Gamma(B, B'), \preceq_{sum})$, where $D_1 \in \mathsf{WPI}(B_1)$ and $D_2 \in \mathsf{WPI}(B_2)$. We can check that $D = f(D_1, r(D_2 \cup D_3))$. Thus, $D \in \Gamma(B_1, B_2 \cup B_3)$. Suppose $D \notin min_{\preceq_{B_1}}(\Gamma(B_1, B_2 \cup B_3))$, then there exists $D'' \in min_{\preceq_{B_1}}(\Gamma(B_1, B_2 \cup B_3))$ such that $D'' \prec_{sum} D$. Assume $D'' = f(D'_1, r(D'_2 \cup D'_3))$. Since $D' \in min(\Gamma(B, B'), \preceq_{sum})$, $q_{Con}(D_1, D_2) \leq q_{Con}(D'_1, D'_2)$. Since $D' \cup D_3$ is consistent and any weight of literals in D_2 is great than or equal to all weights in D_3, $q_{Con}(D_1, D_2) = q_{Con}(D_1, D_2 \cup D_3)$ thus $q_{Con}(D_1, D_2 \cup D_3) \leq q_{Con}(D'_1, D'_2 \cup D'_3)$. So $D \preceq_{sum} D''$, this is a contradiction. Thus, $D \in min_{\preceq_{B_1}}(\Gamma(B_1, B_2 \cup B_3))$. It follows that (RP5) holds. □

We compare our revision operators and the two revision operators given in (Benferhat et al. 2002) when the new information is a formula. Let B be a possibilistic knowledge base and ϕ be a formula. Suppose $a = Inc(B \cup \{(\phi)\})$. The first revision operator in (Benferhat et al. 2002) is defined as $B \circ_1 (\phi, 1) = \{(\psi, b) : (\psi, b) \in B \; and \; b > a\} \cup \{(\phi, 1)\}$, and the second revision operator in (Benferhat et al. 2002) is defined as $B \circ_2 (\phi, 1) = \{(\psi, f(b)) : (\psi, b) \in B \; and \; b > a\} \cup \{(\phi, 1)\}$, where $f(b) = \frac{b-a}{1-a}$.

Proposition 7 Given a possibilistic knowledge base B and a formula ϕ, we have (1) $B \circ_{lex} \{(\phi, 1)\} \vdash B \circ_1 (\phi, 1)$ and (2) $(B \circ_{lex} \{(\phi, 1)\})^* \vdash (B \circ_2 (\phi, 1))^*$.

Proposition 7 does not hold if we replace $\circ_{lex}$ by $\circ_{sum}$. Consider Example 3 again, suppose $\phi = \neg q \wedge p \wedge \neg s$. Since $Inc(B \cup \{(\phi, 1)\}) = 0.7$, we have $B \circ_1 (\phi, 1) = \{(\phi, 1), (q \vee r, 0.8)\}$. However, $B \circ_{sum} \{(\phi, 1)\} = \{(\neg q, 1), (p, 1), (\neg s, 1), (\neg r, 0.7)\}$. Thus, we do not have $B \circ_{lex} \{(\phi, 1)\} \vdash B \circ_1 (\phi, 1)$.

Related Work

There have been some works on belief revision in possibilistic logic. In (Dubois and Prade 1991), the authors propose a syntactic revision operator in possibilistic logic that is restricted to revise possibilistic knowledge bases that are linearly ordered, i.e., different formulas are attached with different weights, and the new information is a sure formula of the form $(\phi, 1)$. There are other more general syntactic revision operators that select some maximal consistent subsets and take their disjunction as the result of revision (ref. (Benferhat et al. 1993; Delgrande, Dubois, and Lang 2006; Qi 2008)). These revision operators are syntax-dependent and they are not applicable to revision with uncertain input.

Semantic revision operators in possibilistic logic are often defined by conditioning in possibility theory (ref. (Dubois and Prade 1992; 1997; Benferhat et al. 2002; 2010; Benferhat, Tabia, and Sedki 2011)). That is, given a possibilistic knowledge base, the result of semantic revision is often a new possibility distribution that is transformed from the possibility distribution associated with the knowledge base and the newly received formula (either a sure formula or an arbitrary possibilistic formula) by a conditioning. These works are closely related to belief revision operators in Ordinal Conditional Function framework, such as Adjustment in (Williams 1994) and Maxi-Adjustment in (Williams 1996). As we have shown, when the newly received information is a sure formula, our revision operator $\circ_{lex}$ can preserve more information than the semantic revision operators defined in (Benferhat et al. 2002). In (Benferhat et al. 2002), uncertain input information of the form (ϕ, a) is considered. Unlike the syntactic approach, an uncertain input (ϕ, a) is interpreted as $N(\phi) = a$, i.e., the necessity degree of ϕ is exactly equal to a. The revision operators are then defined by some kind of conditioning, i.e., product-based conditioning or sum-based conditioning. The syntactic counterpart of the semantic revision is also proposed. Even if the incoming formula ϕ is consistent with the original possibilistic knowledge base B, we still need to drop some information from B if B is inconsistent with $\neg\phi$. In contrast, in our work, (ϕ, a) is considered as a possibilistic formula, i.e., (ϕ, a) is interpreted as $N(\phi) \geq a$, and our revision operators are reduced to classical revision operators in the flat case. Our revision operators are also different from the revision operators given in (Benferhat et al. 2010; Benferhat, Tabia, and Sedki 2011) where the input information is a possibility distribution over the elements of a partition of Ω.

This work is different from belief revision or update with probabilities (such as the work given in (Grünwald and Halpern 2003) (van Benthem, Gerbrandy, and Kooi 2009)) in several aspects. First, our work is based on possibilistic logic, which is different from probabilistic logic according to (Dubois and Prade 2001). Second, our revision operators are generalizations of existing revision operators in propositional logic, whilst belief revision in the face of probabilities takes a different mechanism from belief revision in propositional logic.

Our revision operators may be considered as a kind of "prioritized merging" operators, i.e., two knowledge bases with different reliability are merged. In this sense, it is related to the work on prioritized merging given in (Delgrande, Dubois, and Lang 2006). However, in (Delgrande, Dubois, and Lang 2006), the knowledge bases to be merged are propositional knowledge bases. The work presented in (Benferhat et al. 1999) proposes a prioritized merging operator in possibilistic logic, but it drops the commensurability assumption.

In summary, our revision operators are the first ones that allow the input information to be a general possibilistic knowledge base and are at the same time syntax-independent (in the sense it satisfies (RP4)).

Conclusion and Future Work

In this paper, we considered the problem of revising a possibilistic knowledge base by another one. We first defined the disjunction of possibilistic terms and presented a new definition of weighted prime implicants of a possibilistic knowledge base. We then defined two conflict-based revision operators based on these two notions. After that, we adapted the six KM postulates and show that our revision operators satisfy four of the basic ones but may not satisfy other two. We also showed that in some special cases, our revision operators satisfy all of the adapted postulates. We showed that one of our revision operators, i.e., $\circ_{lex}$, is more powerful than the revision operators in (Benferhat et al. 2002) in terms of minimal change when the input information is a sure formula.

As a future work, we will develop an algorithm to compute all weighted prime implicants of a possibilistic knowledge base. Based on this algorithm, we may be able to provide a procedure to check if a possibilistic knowledge base is WPI-definable. As another future work, we will apply weighted prime implicants to define update operators and merging operators in possibilistic logic.

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