The Inter-League Extension of the Traveling Tournament Problem and its Application to Sports Scheduling

Richard Hoshino and Ken-ichi Kawarabayashi

National Institute of Informatics

2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan

Abstract

With the recent inclusion of inter-league games to professional sports leagues, a natural question is to determine the "best possible" inter-league schedule that retains all of the league's scheduling constraints to ensure competitive balance and fairness, while minimizing the total travel distance for both economic and environmental efficiency. To answer that question, this paper introduces the *Bipartite Traveling Tournament Problem (BTTP)*, the inter-league extension of the well-studied Traveling Tournament Problem.

We prove that the $2n$-team *BTTP* is NP-complete, but for small values of n, a distance-optimal inter-league schedule can be generated from an algorithm based on minimum-weight 4-cycle-covers. We apply our algorithm to the 12-team Nippon Professional Baseball (NPB) league in Japan, creating an inter-league tournament that reduces total team travel by 16% compared to the actual schedule played by these teams during the 2010 NPB season. We also analyze the problem of inter-league scheduling for the 30-team National Basketball Association (NBA), and develop a tournament schedule whose total inter-league travel distance is just 3.8% higher than the trivial theoretical lower bound.

Introduction

In many professional sports leagues, the teams are divided into "conferences" based on historical rivalry or geographic proximity. During the season, each team plays *intra-league* games against teams from their own conference, as well as *inter-league* games against teams from the other conference. Many professional sports leagues adopt a two-conference structure, including the "Big Four" leagues of North America: the National Basketball Association (NBA), the National Football League (NFL), the National Hockey League (NHL), and Major League Baseball (MLB).

As teams must travel long distances to play their games during the course of a season, finding a schedule that reduces travel distance is important, for both economic and environmental reasons. Since the majority of regular season games occur within one's conference, much of the research in sports scheduling has focused on intra-league play, with

the goal of minimizing the sum total of distances traveled by all teams. The challenge of creating a distance-optimal intra-league schedule motivated the *Traveling Tournament Problem* (TTP), in which every pair of teams plays twice, with one game at each team's home stadium. The output is an optimal schedule that minimizes the sum total of distances traveled by the teams as they move from city to city, subject to several constraints that ensure competitive balance.

Since its introduction (Easton, Nemhauser, and Trick 2001), the TTP has emerged as a popular area of study (Kendall et al. 2010) within the operations research community due to its complexity and depth. Research on the TTP has led to the development of powerful techniques in integer programming, constraint programming, as well as heuristics such as simulated annealing and hill-climbing (Lim, Rodrigues, and Zhang 2006). The TTP has direct applications to scheduling optimization, and can aid professional sports leagues as they make their regular season schedules more efficient, saving time and money, as well as reducing greenhouse gas emissions.

In this paper, we extend the TTP to inter-league play, connecting the techniques and methods of the Traveling Tournament Problem to the theory of bipartite tournaments, thus producing new directions for research in scheduling optimization. Determining distance-optimal inter-league schedules is a natural next step in the field of sports scheduling, especially given the recent introduction of inter-league play to pro leagues. For example, in Major League Baseball, inter-league play began only in 1997, despite having first been proposed in the 1930s.

We introduce the *Bipartite Traveling Tournament Problem (BTTP)*, the inter-league analogue of the TTP. We prove that *BTTP* on $2n$ teams is NP-complete by obtaining a reduction from 3-SAT, the well-known NP-complete problem on boolean satisfiability (Garey and Johnson 1979). Despite its computational intractability for general n, we present a powerful heuristic based on minimum-weight 4-cycle-covers and apply it to the 12-team Nippon Professional Baseball (NPB) league in Japan, as well as the 30-team National Basketball Association (NBA). We solve *BTTP* for the NPB, producing a distance-optimal schedule that represents a 16% reduction compared to the actual distance traveled by the teams during the 2010 NPB season. We conclude the paper by finding a nearly-optimal solution to *BTTP* for the NBA.

Definitions of *BTTP* and *BTTP**

Let there be $2n$ teams, with n teams in each league. Let X and Y be the two leagues, with $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$. Let D be the $2n \times 2n$ *distance matrix*, where entry $D_{p,q}$ is the distance between the home stadiums of teams p and q. By definition, $D_{p,q} = D_{q,p}$ for all $p, q \in X \cup Y$, and all diagonal entries $D_{p,p}$ are zero.

Similar to the original TTP, we require the following conditions: that each team play one game per day; that no team has a home stand or road trip lasting more than three games; that no team play against the same opponent in two consecutive games; and that for all $1 \leq i, j \leq n$, teams x_i and y_j play twice, once in each other's home venue.

To illustrate, Table 1 provides two examples of a feasible tournament satisfying all of the above conditions for the case $n = 3$. In this table, as in all other schedules that will be subsequently presented, home games are marked in bold.

Team	1	2	3	4	5	6
x_1	y_1	y_2	y_3	$\mathbf{y_1}$	$\mathbf{y_2}$	$\mathbf{y_3}$
x_2	y_2	y_3	y_1	$\mathbf{y_2}$	$\mathbf{y_3}$	$\mathbf{y_1}$
x_3	y_3	y_1	y_2	$\mathbf{y_3}$	$\mathbf{y_1}$	$\mathbf{y_2}$
y_1	$\mathbf{x_1}$	$\mathbf{x_3}$	$\mathbf{x_2}$	x_1	x_3	x_2
y_2	$\mathbf{x_2}$	$\mathbf{x_1}$	$\mathbf{x_3}$	x_2	x_1	x_3
y_3	$\mathbf{x_3}$	$\mathbf{x_2}$	$\mathbf{x_1}$	x_3	x_2	x_1

Team	1	2	3	4	5	6
x_1	$\mathbf{y_1}$	$\mathbf{y_2}$	y_3	y_2	$\mathbf{y_3}$	y_1
x_2	y_3	$\mathbf{y_1}$	$\mathbf{y_2}$	y_1	y_2	$\mathbf{y_3}$
x_3	y_2	$\mathbf{y_3}$	y_1	y_3	$\mathbf{y_1}$	$\mathbf{y_2}$
y_1	x_1	x_2	$\mathbf{x_3}$	$\mathbf{x_2}$	x_3	$\mathbf{x_1}$
y_2	$\mathbf{x_3}$	x_1	x_2	$\mathbf{x_1}$	$\mathbf{x_2}$	x_3
y_3	$\mathbf{x_2}$	x_3	$\mathbf{x_1}$	$\mathbf{x_3}$	x_1	x_2

Table 1: Two feasible inter-league tournaments for $n = 3$.

Following the convention of the TTP, whenever a team is scheduled for a road trip consisting of multiple away games, the team doesn't return to their home city but rather proceeds directly to their next away venue. Furthermore, we assume that every team begins the tournament at home, and returns home after its last away game. For example, in Table 1, team x_1 would travel a distance of $D_{x_1,y_1} + D_{y_1,y_2} + D_{y_2,y_3} + D_{y_3,x_1}$ when playing the top schedule and a distance of $D_{x_1,y_3} + D_{y_3,y_2} + D_{y_2,x_1} + D_{x_1,y_1} + D_{y_1,x_1}$ when playing the bottom schedule. The desired solution to *BTTP* is the tournament schedule that minimizes the total distance traveled by all $2n$ teams subject to the given conditions.

Let *BTTP** be the restriction of *BTTP* to the set of tournament schedules where on any given day, the teams in each league either all play at home, or all play on the road. For example, the top schedule in Table 1 is a feasible solution of both *BTTP** and *BTTP*. We say that such schedules are *uniform*. While this uniformity constraint significantly reduces the number of potential tournaments, it allows us to quickly generate an approximate solution to *BTTP* from an algorithm based on minimum-weight 4-cycle-covers. We now prove that both *BTTP* and *BTTP** are NP-complete, by obtaining a reduction from 3-SAT.

NP-completeness of *BTTP* and *BTTP**

Let $S = C_1 \wedge C_2 \wedge \ldots \wedge C_m$ be the conjunction of m clauses with three literals on the variables $u_1, u_2, \ldots, u_l$. From S, we will define the sets X_S and Y_S representing the teams in leagues X and Y. From this set of $|X_S| + |Y_S|$ vertices, we will construct a complete graph and assign edge weights to produce the distance matrix D_S. We then prove the existence of an integer $T = T(m)$ for which the solutions to *BTTP* and *BTTP** have total travel distance $\leq T$ iff S is satisfiable. This will establish the desired results.

We can assume that literals u_i and $\overline{u_i}$ occur equally often in S for each $1 \leq i \leq l$. To see why, assume without loss that u_i occurs less frequently than $\overline{u_i}$. By repeated addition of the tautological clause $(u_i \vee u_{i+1} \vee \overline{u_{i+1}})$, which does not affect the satisfiability of S, we can ensure that the number of occurrences of u_i in S matches that of $\overline{u_i}$.

Let $r(i)$ denote the number of occurrences of u_i in S. In Figure 1, we present a "gadget" for each variable u_i, where the vertices $u_{i,r}$ and $\overline{u}_{i,r}$ correspond respectively to the rth occurrence of u_i and $\overline{u_i}$ in S, vertex $a_{i,r}$ is adjacent to $\overline{u}_{i,r-1}$ and $u_{i,r}$, and vertex $b_{i,r}$ is adjacent to $u_{i,r}$ and $\overline{u}_{i,r}$. (Note: we set $\overline{u}_{i,0} := \overline{u}_{i,r(i)}$ for all i.)
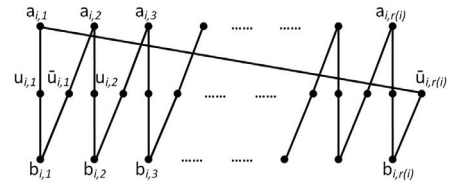

Figure 1: Gadget for 3-SAT reduction.

This gadget was used to establish the NP-completeness of deciding whether an undirected graph contains a given number of vertex-disjoint s-t paths of a specified length (Itai, Perl, and Shiloach 1982) and to prove that the original TTP is NP-complete (Thielen and Westphal 2010).

There are l gadgets, one for each u_i, $i = 1, 2, \ldots, l$. Now we define the *gadget graph* G_S. We create vertices c_j and d_j for $1 \leq j \leq m$, one pair for each clause in S. Join c_j to d_j. Now connect c_j to vertex $u_{i,r}$ iff clause C_j contains the rth occurrence of u_i in S. Similarly, connect c_j to vertex $\overline{u}_{i,r}$ iff clause C_j contains the rth occurrence of $\overline{u_i}$ in S.

To illustrate, let $S = C_1 \wedge C_2 \wedge C_3 \wedge C_4 \wedge C_5 \wedge C_6 \wedge C_7 \wedge C_8$, where $C_1 = (u_1 \vee u_2 \vee u_3)$, $C_2 = (\overline{u_1} \vee \overline{u_2} \vee \overline{u_3})$, $C_3 = (u_1 \vee \overline{u_2} \vee u_4)$, $C_4 = (u_2 \vee \overline{u_3} \vee u_4)$, $C_5 = (\overline{u_1} \vee u_3 \vee u_4)$, $C_6 = (u_1 \vee \overline{u_2} \vee \overline{u_4})$, $C_7 = (u_2 \vee \overline{u_3} \vee \overline{u_4})$, and $C_8 = (\overline{u_1} \vee u_3 \vee \overline{u_4})$. By definition, S is an instance of 3-SAT. The gadget graph G_S is given in Figure 2.

Since each literal occurs as often as its negation, and each clause has three literals, the number of clauses in S must be even. Hence, $m = 2k$ for some integer $k \geq 1$. From the instance S, we will define a set X_S with $18k$ vertices corresponding to the teams in league X. We will then define another set Y_S, with just 3 vertices (labeled p, q, and r), and place $6k$ teams at each of these three vertices. This will create a $36k$-team league, with $18k$ teams in both X and Y. The
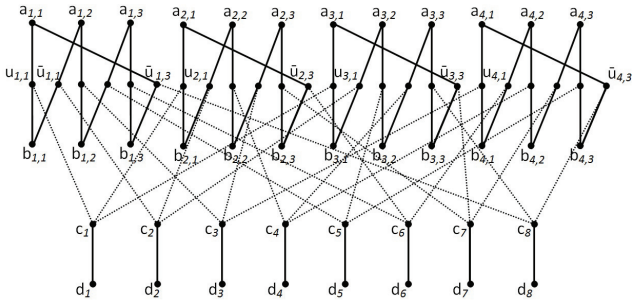
Figure 2: The gadget graph G_S for the instance S.

	$p \in Y_S$	$q \in Y_S$	$r \in Y_S$
$a \in A$	z^2	$z^2 + z$	$2z^2 - 1$
$b \in B$	z^2	$2z^2 - 1$	$z^2 + z$
$c \in C$	$2z^2 - 1$	z^2	$z^2 + z$
$d \in D$	z^2	$2z^2 - 1$	z^2
$e \in E$	$2z^2 - 1$	$z^2 + z$	z^2
$f \in F$	z^2	z^2	$2z^2 - 1$
$u \in U$	$z^2 + z$	$z^2 + 2z$	$z^2 + 2z$

Table 2: Weights of all edges connecting X_S to Y_S.

weight of each edge will just correspond to the distance between the teams located at those vertices. Using the gadget graph G_S, we will define the edge weights in such a way that S is satisfiable iff the solutions to *BTTP* and *BTTP** have total distance at most $T = T(k) = 96k^2(2900k^2 + 375k + 11)$. This will establish the desired reduction from 3-SAT.

We first define X_S. Let $C = \{c_1, c_2, \ldots, c_{2k}\}$ and $D = \{d_1, d_2, \ldots, d_{2k}\}$, which are the same set of vertices from the corresponding gadget graph G_S. Let U be the set of $6k$ vertices of the form $u_{i,r}$ or $\overline{u}_{i,r}$ that appear in G_S, and let A and B be respectively the set of vertices of the form $a_{i,r}$ and $b_{i,r}$ that appear in G_S. Finally, we present two additional sets, $E = \{e_1, e_2, \ldots, e_k\}$ and $F = \{f_1, f_2, \ldots, f_k\}$, which will be matched up to the vertices of U in our cycle cover.

We define $X_S = A \cup B \cup C \cup D \cup E \cup F \cup U$. Hence, $|X_S| = |A| + |B| + |C| + |D| + |E| + |F| + |U| = 3k + 3k + 2k + 2k + k + k + 6k = 18k$.

Having defined X_S, we now define the edge weights connecting each pair of vertices in X_S, thus producing a complete graph on $18k$ vertices. The weight of each edge will be a function of k. For readability, we will express each weight as a function of z, where $z := 20k + 1$. To each edge in this complete graph, we assign a weight from the set $\{z^2, z^2 + z, 2z^2 - 1\}$ as follows:

(1) A weight of z^2 is given to every edge that appears in the gadget graph G_S, the $6k^2$ edges from U to E, and the k edges connecting e_i to f_i (for each $1 \le i \le k$).

(2) A weight of $z^2 + z$ is given to the $6k^2$ edges from U to F, the $6k$ edges connecting A to B through a common neighbour in U, and the $6k$ edges connecting D to U through a common neighbour in C.

(3) A weight of $2z^2 - 1$ is given to every other edge.

We now create an inter-league tournament with $36k$ total teams. First, we assign the $18k$ teams in league X to be the $18k$ vertices of graph X_S, where the distance between the home venues of two teams is just the edge weight between the corresponding two vertices in the complete graph.

Now let $Y_S = \{p, q, r\}$. Now define the $18k$ teams in league Y as follows: place $6k$ teams at point p, $6k$ teams at point q, and $6k$ teams at point r.

Therefore, $|X_S \cup Y_S| = 18k + 3$. We now extend our complete graph on $18k$ vertices to include these three additional vertices. To assign an edge weight connecting each pair of "inter-league" vertices, we apply Table 2.

For example, the edge from c_i to p is given a weight of $2z^2 - 1$, for all $i = 1, 2, \ldots, 2k$. We repeat the same process for each of the $7 \times 3 = 21$ pairs connecting a vertex in $X_S = A \cup B \cup C \cup D \cup E \cup F \cup U$ to a vertex in $Y_S = \{p, q, r\}$.

Finally, let the weights of edges pq, pr, and qr all be $2z^2 - 1$. As a result, we have now created a complete graph on the vertex set $X_S \cup Y_S$, and assigned a weight to each edge. Moreover, the weight of each edge appears in the set $\{z^2, z^2 + z, z^2 + 2z, 2z^2 - 1\}$, where $z = 20k + 1$. As most versions of the TTP require the teams to be located at points satisfying the Triangle Inequality, we have chosen the weights in our inter-league variant to ensure that the Triangle Inequality holds for any triplet of points in $X_S \cup Y_S$.

We now partition the $18k$ vertices of X_S into groups of cardinality at most three and attach them to each $y \in \{p, q, r\} = Y_S$ to produce a union of cycles of length at most 4. More formally, we define the following:

Definition 1 *For each $y \in Y_S$, a y-rooted 4-cycle-cover is a union of cycles of length at most 4, where every cycle contains y, no cycle contains a vertex from $Y_S \backslash \{y\}$, and every vertex of X_S appears in exactly one cycle.*

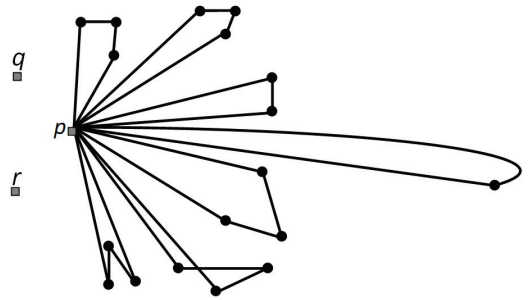Figure 3 gives a p-rooted 4-cycle-cover with $|X_S| = 18$.

Figure 3: A p-rooted 4-cycle-cover with 18 vertices in set X_S.

This definition is motivated by our tournament construction, where we will show that the total travel distance is minimized by creating a uniform schedule where each team takes the maximum number of three-game road trips to play their $18k$ away games. In the case of the $6k$ teams of Y_S located at vertex p, their $6k$ three-game road trips will correspond to the $6k$ 4-cycles in the minimum weight p-rooted 4-cycle-cover. For example, if p-$u_{1,1}$-c_5-d_5-p appears as one of the $6k$ cycles, then each team in Y_S located at vertex p

will play three consecutive road games during the tournament against the teams of X_S located at $u_{1,1}$, c_5, and d_5.

So the total distance traveled by each team at $y \in Y_S$ is bounded below by the sum of the edge weights of the minimum weight y-rooted 4-cycle-cover.

Definition 2 *We define three special types of cycles that may appear in a p-rooted 4-cycle-cover.*

(1) A (p, a, u, b, p)-cycle is a 4-cycle with vertices p, a, u, b in that order, where $p \in Y_S$, $a \in A$, $u \in U$, $b \in B$, where au and ub are both edges in the gadget graph G_S.

(2) A (p, u, c, d, p)-cycle is a 4-cycle with vertices p, u, c, d in that order, where $p \in Y_S$, $u \in U$, $c \in C$, $d \in D$, where uc and cd are both edges in the gadget graph G_S.

(3) A (p, u, e, f, p)-cycle is a 4-cycle with vertices p, u, e, f in that order, where $p \in Y_S$, $u \in U$, $e \in E$, $f \in F$, where e and f have the same index (i.e., e_i and f_i for some $1 \le i \le k$.)

For example, for our instance S whose gadget graph was illustrated in Figure 2, p-$a_{1,2}$-$\overline{u}_{1,1}$-$b_{1,1}$-p is a (p, a, u, b, p)-cycle, but p-$a_{1,2}$-$\overline{u}_{1,1}$-$b_{4,2}$-p is not. Similarly, p-$\overline{u}_{4,3}$-c_8-d_8-p is a (p, u, c, d, p)-cycle, but p-$\overline{u}_{4,3}$-c_7-d_7-p is not.

Following the convention of the TTP (Easton, Nemhauser, and Trick 2001), we define ILB_t to be the *individual lower bound* for team t. This value represents the minimum possible distance that can be traveled by team t in order to complete all of their games under the constraints of *BTTP*, independent of the other teams' schedules. By definition, for each team t located at $y \in Y_S$, the value of ILB_t is the minimum weight of a y-rooted 4-cycle-cover.

Similarly, we define the *league lower bound LLB_T* to be the minimum possible distance traveled by all of the teams t in league T, and the *tournament lower bound TLB* to be the minimum possible distance traveled by all the teams in both leagues. We note the following trivial inequalities:

$$TLB \ge LLB_X + LLB_Y$$
$$LLB_X \ge \sum_{t \in X} ILB_t , \ \ LLB_Y \ge \sum_{t \in Y} ILB_t .$$

By definition, the solution to *BTTP* is a tournament schedule whose total travel distance is TLB.

We now have all of the definitions we need to complete the proof of the NP-completeness of *BTTP* and *BTTP**. We will create an inter-league tournament between the $18k$ teams of X_S and the $18k$ teams of Y_S (with one-third of the teams at each vertex of Y_S), and show that there exists a distance-optimal (uniform) tournament with total distance $T(k) = 96k^2(2900k^2 + 375k + 11)$ iff S is satisfiable. The desired result will follow from the next four lemmas.

In each lemma, we let K_S be the complete graph on the $18k + 3$ vertices of $X_S \cup Y_S$, with edge weights as described in our construction.

Lemma 1 *The following statements are equivalent:*

(i) $S = C_1 \wedge C_2 \wedge \ldots \wedge C_{2k}$ is satisfiable.

(ii) There exists a p-rooted 4-cycle-cover of K_S with exactly $3k$ (p, a, u, b, p)-cycles, $2k$ (p, u, c, d, p)-cycles, and k (p, u, e, f, p)-cycles.

Proof First, we prove $(i) \rightarrow (ii)$. If S is satisfiable, then there exists a valid truth assignment, i.e., a function ϕ for which $\phi(u_i) \in \{\text{TRUE}, \text{FALSE}\}$ for every $1 \le i \le l$ that ensures that each clause C_j evaluates to TRUE for all $1 \le j \le 2k$. From ϕ, we build a p-rooted 4-cycle-cover of K_S with exactly $3k$ (p, a, u, b, p)-cycles, $2k$ (p, u, c, d, p)-cycles, and k (p, u, e, f, p)-cycles.

We first identify the $3k$ (p, a, u, b, p)-cycles. For each $1 \le i \le l$, if $\phi(u_i)$ is FALSE, then select all 4-cycles of the form p-$a_{i,r}$-$u_{i,r}$-$b_{i,r}$-p for $r = 1, 2, \ldots, r(i)$. And if $\phi(u_i)$ is TRUE, then select all 4-cycles of the form p-$a_{i,r+1}$-$\overline{u}_{i,r}$-$b_{i,r}$-p, where $a_{i,r(i)+1} := a_{i,1}$. Repeating this construction for each i, we produce $3k$ (p, a, u, b, p)-cycles, covering the $6k$ vertices of $A \cup B$, as well as $3k$ vertices of U.

Now consider any clause C_j. Since ϕ is a valid truth assignment, at least one of the three literals in C_j evaluates to TRUE. Thus, there exists some index i for which $u_i \in C_j$ and $\phi(u_i)$ is TRUE, or $\overline{u_i} \in C_j$ and $\phi(u_i)$ is FALSE.

In the former case, where $u_i \in C_j$ and $\phi(u_i)$ is TRUE, there exists some index r for which $u_{i,r}$-c_j is an edge of the gadget graph G_S. Then p-$u_{i,r}$-c_j-d_j-p is a (p, u, c, d, p)-cycle. Note that $u_{i,r}$ has not been previously selected in a (p, a, u, b, p)-cycle since $\phi(u_i)$ is TRUE (and so only the vertices $\overline{u}_{i,1}, \overline{u}_{i,2}, \ldots, \overline{u}_{i,r(i)}$ were covered earlier.)

In the latter case, where $\overline{u_i} \in C_j$ and $\phi(u_i)$ is FALSE, there exists some index r for which $\overline{u}_{i,r}$-c_j is an edge of the gadget graph G_S. Then p-$\overline{u}_{i,r}$-c_j-d_j-p is a (p, u, c, d, p)-cycle. Note that $\overline{u}_{i,r}$ has not been previously selected in a (p, a, u, b, p)-cycle since $\phi(u_i)$ is FALSE (and so only the vertices $u_{i,1}, u_{i,2}, \ldots, u_{i,r(i)}$ were covered earlier.)

Repeating this construction for each j, we produce $2k$ (p, u, c, d, p)-cycles, covering the $4k$ vertices of $C \cup D$. Note that no $u \in U$ can be chosen twice since each vertex in U is adjacent to only one vertex in C. Thus, these $2k$ cycles cover a set of $6k$ vertices in X_S, completely disjoint from the $9k$ vertices covered by the previously-constructed $3k$ (p, a, u, b, p)-cycles. As a result, we are left with $3k$ vertices in X_S still to be covered, specifically k vertices in each of U, E, and F. These vertices can be trivially partitioned into k (p, u, e, f, p)-cycles by just ensuring that e_i and f_i belong to the same cycle for each $1 \le i \le k$. When this process is complete, our p-rooted 4-cycle-cover of K_S will contain exactly $3k$ (p, a, u, b, p)-cycles, $2k$ (p, u, c, d, p)-cycles, and k (p, u, e, f, p)-cycles.

We now prove $(ii) \rightarrow (i)$. Consider a p-rooted 4-cycle-cover of K_S containing exactly $3k$ (p, a, u, b, p)-cycles, $2k$ (p, u, c, d, p)-cycles, and k (p, u, e, f, p)-cycles. We prove there exists a function ϕ that is a satisfying truth assignment for S, where $\phi(u_i) \in \{\text{TRUE}, \text{FALSE}\}$ for each $1 \le i \le l$.

Define an *a-b path* to be any path on three vertices whose endpoints are $a_{i,j}$ and $b_{i,k}$, for some indices i, j, k. Consider the problem of maximizing the number of vertex-disjoint a-b paths in the ith gadget. A maximum packing of a-b paths occurs iff the $r(i)$ paths are chosen by taking all paths of the form $a_{i,r}, u_{i,r}, b_{i,r}$ for each $1 \le r \le r(i)$, or all paths of the form $a_{i,r+1}, \overline{u}_{i,r}, b_{i,r}$ for each $1 \le r \le r(i)$.

The former case corresponds to selecting our a-b paths *vertically*; the latter, *diagonally*. These are the only two ways to achieve a maximum packing. Thus, in our p-rooted

4-cycle-cover containing $3k$ (p,a,u,b,p)-cycles, one of the following scenarios must hold true in the ith gadget:

(1) For each $r = 1, 2, \ldots, r(i)$, vertex $u_{i,r}$ appears in some (p,a,u,b,p)-cycle, while no vertex $\overline{u}_{i,r}$ appears in any (p,a,u,b,p)-cycle.

(2) For each $r = 1, 2, \ldots, r(i)$, vertex $\overline{u}_{i,r}$ appears in some (p,a,u,b,p)-cycle, while no vertex $u_{i,r}$ appears in any (p,a,u,b,p)-cycle.

In our given p-rooted 4-cycle-cover of K_S, for each i define $\phi(u_i) =$ FALSE in scenario (1) and define $\phi(u_i) =$ TRUE in scenario (2). We claim that this is our desired function ϕ. To prove this, consider the $2k$ (p,u,c,d,p)-cycles in our 4-cycle-cover. For each $1 \le j \le 2k$, the (p,u,c,d,p)-cycle containing c_j also contains some other vertex in U. This vertex is either $u_{i,r}$ or $\overline{u}_{i,r}$, for some indices i and r.

In the former case, $u_{i,r}$ and c_j appear in the same (p,u,c,d,p)-cycle, implying that $u_{i,r}$-c_j is an edge of the gadget graph G_S, and that u_i is a literal in clause C_j. Since $u_{i,r}$ appears in this (p,u,c,d,p)-cycle and therefore not in any (p,a,u,b,p)-cycle, this implies scenario (2). Since $\phi(u_i) =$ TRUE and $u_i \in C_j$, clause C_j evaluates to TRUE.

In the latter case, $\overline{u}_{i,r}$ and c_j appear in the same (p,u,c,d,p)-cycle, implying that $\overline{u}_{i,r}$-c_j is an edge of the gadget graph G_S, and that $\overline{u_i}$ is a literal in clause C_j. Since $\overline{u}_{i,r}$ appears in this (p,u,c,d,p)-cycle and therefore not in any (p,a,u,b,p)-cycle, this implies scenario (1). Since $\phi(u_i) =$ FALSE and $\overline{u_i} \in C_j$, clause C_j evaluates to TRUE.

Since C_j evaluates to TRUE for all $1 \le j \le 2k$, this implies that ϕ is a valid truth assignment. We conclude that $S = C_1 \wedge C_2 \wedge \ldots \wedge C_{2k}$ is satisfiable. ∎

We illustrate Lemma 1 with an example. We can show that the instance S, whose gadget graph was illustrated in Figure 2, is not satisfiable. Therefore, there is no p-rooted 4-cycle-cover of K_S with 12 (p,a,u,b,p)-cycles, 8 (p,u,c,d,p)-cycles, and 4 (p,u,e,f,p)-cycles.

Lemma 2 *The following statements are equivalent:*

(i) A p-rooted 4-cycle-cover of K_S has exactly $3k$ (p,a,u,b,p)-cycles, $2k$ (p,u,c,d,p)-cycles, and k (p,u,e,f,p)-cycles.

(ii) A p-rooted 4-cycle-cover of K_S has total edge weight $k(24z^2 + 3z)$.

Proof First, we prove $(i) \rightarrow (ii)$. In a (p,a,u,b,p)-cycle, the edges au and ub appear in the gadget graph G_S. Therefore, the edge weights of au and ub are both z^2. From Table 2, a (p,a,u,b,p)-cycle has edge weight $z^2 + z^2 + z^2 + z^2 = 4z^2$. Similarly, a (p,u,c,d,p)-cycle has edge weight $(z^2 + z) + z^2 + z^2 + z^2 = 4z^2 + z$, and a (p,u,e,f,p)-cycle has edge weight $(z^2 + z) + z^2 + z^2 + z^2 = 4z^2 + z$. So if a p-rooted 4-cycle-cover of K_S has exactly $3k$ (p,a,u,b,p)-cycles, $2k$ (p,u,c,d,p)-cycles, and k (p,u,e,f,p)-cycles, then its total edge weight is exactly $3k(4z^2) + 2k(4z^2 + z) + k(4z^2 + z) = k(24z^2 + 3z)$.

We now prove $(ii) \rightarrow (i)$. Let R be a p-rooted 4-cycle-cover of K_S which is the union of r cycles, with total edge weight $k(24z^2 + 3z)$. Since each of the 18k vertices of X_S is covered by exactly one cycle of R, the number of edges in

R is $|X_S| + r = 18k + r$. Since no cycle has length greater than 4, we have $r \ge \frac{18k}{3} = 6k$. Now suppose $r \ge 6k + 1$. Then there are at least $24k + 1$ edges in R, all of which have weight at least z^2 given the construction of our complete graph K_S. Hence, the total edge weight of R is at least $(24k + 1)z^2 = 24kz^2 + z^2 = 24kz^2 + z(20k + 1) > 24kz^2 + 3zk = k(24z^2 + 3z)$, a contradiction.

It follows that $r = 6k$, and that R must be the union of $6k$ cycles of length 4. Recall that the weight of each edge appears in the set $\{z^2, z^2 + z, z^2 + 2z, 2z^2 - 1\}$. Suppose that one of these $24k$ edges has weight $2z^2 - 1$. Then the total edge weight of R is at least $(24k - 1)z^2 + (2z^2 - 1) > k(24z^2 + 3z)$, a contradiction. Hence, all edges of R must have weight z^2, $z^2 + z$, or $z^2 + 2z$.

From Table 2, we see that no edges p-c and p-e can appear in our 4-cycle-cover R, since all edges from p to $C \cup E$ have weight $2z^2 - 1$. Thus, there must exist $2k$ 4-cycles of the form p-?-c_i-?-p and k 4-cycles of the form p-?-e_i-?-p, with each of these $2k + k = 3k$ 4-cycles containing a unique element from $C \cup E$. Each blank space (denoted by a question mark) can only be filled with a vertex from D, F, or U, as the weights of edges ca, cb, ea, eb are all $2z^2 - 1$ for all $a \in A$, $b \in B$, $c \in C$, and $e \in E$.

Since edge p-u has weight $z^2 + z$, if some vertex $u \in U$ is chosen to appear in one of these $3k$ 4-cycles, then this adds edge weight $z^2 + z$, producing a 4-cycle of weight at least $4z^2 + z$. But if no vertices $u \in U$ are chosen to replace these blank spaces, then the cycles must be of the form p-d_j-c_i-d_k-p or p-f_j-e_i-f_k-p, both of which lead to the addition of at least one edge of weight $2z^2 - 1$ (since we cannot simultaneously have $i = j$, $i = k$, and $j \ne k$). It follows that these $2k + k = 3k$ 4-cycles containing the vertices of $C \cup E$ must each have weight at least $4z^2 + z$, thus contributing at least $k(12z^2 + 3z)$ to the total distance of R.

Since the given 4-cycle-cover R has weight $k(24z^2 + 3z)$, this implies that the rest of the $3k$ 4-cycles must each have weight exactly $4z^2$, and that in each of the $2k$ cycles of the form p-?-c_i-?-p and k cycles of the form p-?-e_i-?-p, the total edge weight must be *exactly* $4z^2 + z$ to ensure that the total edge weight of R does not exceed $k(24z^2 + 3z)$. This implies that in these two scenarios, we cannot replace the two blank spaces with two distinct vertices from U, as that would create a cycle of weight $4z^2 + 2z$. It follows that R *must* have $2k$ (p,u,c,d,p)-cycles and k (p,u,e,f,p)-cycles.

We are now left with $3k$ vertices from each of A, B, and U to form our remaining $3k$ 4-cycles. For the total edge weight of R to not exceed $k(24z^2 + 3z) = 3k(4z^2 + z) + 12kz^2$, each of the remaining $12k$ edges must have weight z^2. Since edge p-u has weight $z^2 + z$ for all $u \in U$, the $3k$ remaining vertices in U must each appear in a unique 4-cycle, none adjacent to the root vertex p. Thus, the remaining $3k$ 4-cycles of R must all be (p,a,u,b,p)-cycles. ∎

Lemma 3 *Let ILB_y be the minimum total edge weight of a y-rooted 4-cycle-cover of K_S. Then*

$$ILB_y = \begin{cases} k(24z^2 + 3z) & \text{if } y = p \\ k(24z^2 + 20z) & \text{if } y = q \\ k(24z^2 + 19z) & \text{if } y = r \end{cases}$$

Proof From the proof of Lemma 2, we see that $ILB_p = k(24z^2 + 3z)$. The other two cases ($y = q$ and $y = r$) follow similarly, and so we omit the details. ∎

Just as we defined special 4-cycles rooted at p (e.g. (p, a, u, b, p)-cycles), we can similarly define 4-cycles rooted at q and r. In the above lemma, the lower bound ILB_q occurs when the q-rooted 4-cycle-cover consists of $3k$ (q, u, b, a, q)-cycles, $2k$ (q, c, d, u, q)-cycles, and k (q, f, u, e, q)-cycles, with total edge weight $3k(4z^2 + 4z) + 2k(4z^2 + 3z) + k(4z^2 + 2z) = k(24z^2 + 20z)$. The lower bound ILB_r occurs when the r-rooted 4-cycle-cover consists of $3k$ (r, b, a, u, r)-cycles, $2k$ (r, d, u, c, r)-cycles, and k (r, e, f, u, r)-cycles, with total edge weight $3k(4z^2 + 4z) + 2k(4z^2 + 2z) + k(4z^2 + 3z) = k(24z^2 + 19z)$.

Recall that LLB_{Y_S} is the league lower bound for the $18k$ teams in $Y_S = \{p, q, r\}$. By Lemma 3, it follows that $LLB_{Y_S} \geq 6k(ILB_p + ILB_q + ILB_r) = 6k^2(24z^2 + 3z) + 6k^2(24z^2 + 20z) + 6k^2(24z^2 + 19z) = 6k^2(72z^2 + 42z)$.

We now construct a uniform double round-robin bipartite tournament so that this value of LLB_{Y_S} is achieved whenever S is satisfiable. In this schedule, each team in Y_S plays three consecutive road games followed by three consecutive home games and repeats that pattern until the end of the tournament. Thus, each team has $6k$ three-game road trips.

Let ϕ be a satisfying truth assignment for S. By the proof of Lemma 1, ϕ generates $3k$ (p, a, u, b, p)-cycles, $2k$ (p, u, c, d, p)-cycles and k (p, u, e, f, p)-cycles. These $6k$ cycles form a p-rooted 4-cycle-cover of K_S with total edge weight $ILB_p = k(24z^2 + 3z)$.

We first schedule the $6k$ three-game road trips for each team at $p \in Y_S$ according to this minimum weight p-rooted 4-cycle-cover, ensuring that every team p plays a unique opponent in each of the $18k$ time slots. For each $1 \leq i \leq 6k$, we use Table 3 below to generate the schedules for teams q_i and r_i, based on the schedule of team p_i.

Game	1	2	3	1	2	3	1	2	3
p_i	a^*	u^*	b^*	u^*	c^*	d^*	u^*	e^*	f^*
q_i	u^*	b^*	a^*	c^*	d^*	u^*	f^*	u^*	e^*
r_i	b^*	a^*	u^*	d^*	u^*	c^*	e^*	f^*	u^*

Table 3: Construction of road games for teams in Y_S.

For example, suppose that some team p_i plays against teams $a^* \in A$, $u^* \in U$, and $b^* \in B$ in slots 1, 2, and 3. Then team q_i will play against u^* in slot 1, b^* in slot 2, and a^* in slot 3. It is easy to see that this construction generates one-half of a uniform schedule where each team $y \in Y_S = \{p, q, r\}$ has total travel distance ILB_y, corresponding to the minimum-weight y-rooted 4-cycle-cover.

We now determine the value of ILB_t for each $t \in X_S = A \cup B \cup C \cup D \cup E \cup F \cup U$. Every team $t \in X_S$ plays a road game against each of the $18k$ teams in Y_S, with $6k$ teams located at points p, q, and r. Team t must make at least $\frac{6k}{3} = 2k$ trips to each of p, q, and r, since the maximum length of a road trip is three games. Therefore, $ILB_t \geq 2k(D_{t,p} + D_{t,q} + D_{t,r})$, where $D_{t,y}$ is the distance from $t \in X_S$ to $y \in Y_S$ for all choices of t and y. Note that

equality can occur, specifically when the road trips of team t are scheduled in the most efficient way, with each trip consisting of three consecutive games against three teams located at the same point.

From Table 2, $ILB_t = 2k(D_{t,p} + D_{t,q} + D_{t,r}) = 4k(4z^2 + z - 1)$ for all $t \in A \cup B \cup C \cup E$. Similarly, $ILB_t = 4k(4z^2 - 1)$ for all $t \in D \cup F$, and $ILB_t = 4k(3z^2 + 5z)$ for all $t \in U$. Thus, $LLB_{X_S} \geq 4k(4z^2 + z - 1)(|A| + |B| + |C| + |E|) + 4k(4z^2 - 1)(|D| + |F|) + 4k(3z^2 + 5z)(|U|) = k^2(264z^2 + 156z - 48)$.

Therefore, $TLB \geq LLB_{X_S} + LLB_{Y_S} \geq \sum ILB_t = k^2(264z^2 + 156z - 48) + 6k^2(72z^2 + 42z) = k^2(696z^2 + 408z - 48) = 96k^2(2900k^2 + 375k + 11)$.

We can quickly construct the other half of our uniform schedule, where the teams in X_S play on the road, ensuring that each team $t \in X_S$ has total travel distance ILB_t. All that is required when putting the schedules together is to ensure the *no-repeat* rule, which is a simple matter given all of the flexibility we have in constructing this half of the tournament schedule. Therefore, we have proven the following lemma:

Lemma 4 *If S is satisfiable, then there exists a uniform schedule (i.e., a solution to BTTP and BTTP*) whose total travel distance is $\sum ILB_t = 96k^2(2900k^2 + 375k + 11)$.*

Having provided all of the lemmas, we can now prove the main theorem of this paper.

Theorem 1 *BTTP and BTTP* are NP-complete.*

Proof Let S be an instance of 3-SAT with $2k$ clauses, and create sets X_S and Y_S, with edge weights as described in our construction. Consider an inter-league tournament between the $18k$ teams at X_S and the $18k$ teams at Y_S (with one-third of the teams at each vertex of Y_S).

By Lemma 4, if S is satisfiable, then there exists a tournament with total distance at most $96k^2(2900k^2 + 375k + 11)$. Since this tournament is uniform, it is a feasible solution to *BTTP* and *BTTP**. We now prove the converse statement.

Let $T(k) = 96k^2(2900k^2 + 375k + 11)$. Consider an inter-league tournament between these $36k$ teams with total travel distance at most $T(k)$. By Lemma 4, $T(k) = \sum ILB_t$. Hence, *every* team $t \in X_S \cup Y_S$ must travel the shortest possible distance of ILB_t to play all of their games. By Lemma 3, this implies that every team located at $p \in Y_S$ must travel a distance of $ILB_p = k(24z^2 + 3z)$.

By Lemma 2, if each team $p \in Y_S$ travels a distance of $k(24z^2 + 3z)$, then the graph K_S contains a p-rooted 4-cycle-cover with exactly $3k$ (p, a, u, b, p)-cycles, $2k$ (p, u, c, d, p)-cycles, and k (p, u, e, f, p)-cycles. And by Lemma 1, this occurs iff S is satisfiable.

Therefore, we have constructed a uniform inter-league tournament K_S on $36k$ teams with distance matrix D_S for which the solutions to *BTTP* and *BTTP** have total distance $\leq T(k)$ iff the instance S with $2k$ clauses is satisfiable. This establishes the desired reduction from 3-SAT, proving the NP-hardness of *BTTP* and *BTTP**. Finally, we note that both problems are clearly in NP, since the distance traveled by the teams can be calculated in polynomial time. Therefore, we conclude that *BTTP* and *BTTP** are NP-complete. ∎

Application 1: Japanese Baseball ($n = 6$)

Nippon Professional Baseball (NPB) is Japan's largest professional sports league. In the NPB, the teams are split into two leagues (i.e., conferences) of six teams, with each team playing 120 intra-league and 24 inter-league games in the regular season. The intra-league problem was analyzed recently (Hoshino and Kawarabayashi 2011b), where the authors developed a multi-round generalization of the TTP based on Dijkstra's shortest path algorithm and applied it to produce a distance-optimal schedule reducing the total travel distance by over 60000 kilometres (a 25% reduction) as compared to the 2010 NPB intra-league schedule.

We consider the inter-league problem, where the six teams in the NPB Pacific League ($\{p_i : 1 \leq i \leq 6\}$) each play four games against all six teams in the NPB Central League ($\{c_i : 1 \leq i \leq 6\}$), with one two-game *set* played at the home of the Pacific League team, and the other two-game set played at the home of the Central League team. All inter-league games take place during a five-week stretch between mid-May and mid-June, with no intra-league games occurring during that period. Thus, the NPB inter-league scheduling problem is precisely *BTTP*, for the case $n = 6$.

We determine the 12×12 distance matrix, representing the distances between each pair of teams from $\{p_1, \ldots, p_6, c_1, \ldots, c_6\}$. The locations of each team's home stadium is marked in Figure 4. For the actual distance matrix, we refer the reader to our journal paper (Hoshino and Kawarabayashi 2011c). In the 2010 NPB inter-league schedule, the teams traveled a total of 51134 kilometres.
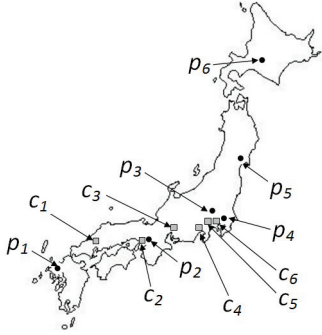

Figure 4: Location of the 12 teams in the NPB.

First, we determine ILB_{p_i} and ILB_{c_i} for each team in the two leagues. By the Triangle Inequality, if some team t plays a single road set sandwiched between two home sets (e.g. HH-RR-H**H**-**R**-**H**H-RRR), then the time slots can be re-ordered to reduce the total travel distance. Hence, the value of ILB_t occurs when team t plays its six road sets in two blocks of three (e.g. HHH-RRR-HHH-RRR) or three blocks of two (e.g. HH-RR-HH-RR-H-RR-H).

For each of these two scenarios, we consider all $6! = 720$ ways of permuting the six road games, from which we can determine all possible distances that can be traveled by that team for the given home-road pattern. We check all 2×720 cases and the minimum distance corresponds to ILB_t. We find that in all twelve cases, the value of ILB_t occurs

when team t plays its six road sets in two blocks of three, i.e., the minimum-weight t-rooted 4-cycle-cover consists of just two cycles. We determine that $\sum ILB_t = \sum ILB_{p_i} + \sum ILB_{c_i} = 16686 + 26077 = 42763$.

Therefore, the optimal solution to *BTTP* for the NPB distance matrix is a tournament requiring at least 42763 kilometres of total team travel. In Table 4 below, we present a feasible solution with total distance 42950 kilometres.

	1	2	3	4	5	6	7	8	9	10	11	12
p_1	c_3	c_5	c_1	c_3	c_2	c_1	c_6	c_2	c_4	c_5	c_6	c_4
p_2	c_5	c_3	c_2	c_1	c_3	c_6	c_1	c_4	c_5	c_6	c_4	c_2
p_3	c_4	c_2	c_6	c_5	c_4	c_3	c_5	c_1	c_3	c_2	c_1	c_6
p_4	c_2	c_4	c_5	c_4	c_6	c_5	c_3	c_6	c_1	c_3	c_2	c_1
p_5	c_1	c_6	c_4	c_6	c_5	c_2	c_4	c_3	c_2	c_1	c_5	c_3
p_6	c_6	c_1	c_3	c_2	c_1	c_4	c_2	c_5	c_6	c_4	c_3	c_5
c_1	p_5	p_6	p_1	p_2	p_6	p_1	p_2	p_3	p_4	p_5	p_3	p_4
c_2	p_4	p_3	p_2	p_6	p_1	p_5	p_6	p_1	p_5	p_3	p_4	p_2
c_3	p_1	p_5	p_6	p_1	p_2	p_3	p_4	p_5	p_3	p_4	p_6	p_2
c_4	p_3	p_4	p_5	p_4	p_3	p_6	p_5	p_2	p_1	p_6	p_2	p_1
c_5	p_2	p_1	p_4	p_3	p_5	p_4	p_3	p_6	p_2	p_1	p_5	p_6
c_6	p_6	p_5	p_3	p_5	p_4	p_2	p_1	p_4	p_6	p_2	p_1	p_3

Table 4: Solution to *BTTP* with total distance 42950 km.

We claim that this solution is optimal. Let $M = 42950 - 42763 = 187$, and let Φ be a distance-optimal solution to *BTTP*. For each team t, let S_t^* be the set of team schedules whose total distance is at most $ILB_t + M$. Note that team t's schedule must appear in S_t^* as otherwise the total distance traveled would exceed $\sum ILB_t + M = 42950$.

As teams p_5 and p_6 are located quite far away from the other ten teams (see Figure 4), we find that $S_{p_5}^*$ and $S_{p_6}^*$ only consist of schedules where the six road sets are played in two blocks of three. Furthermore, each Central League team c_j must play their road sets against p_5 and p_6 in two consecutive time slots, as otherwise that team would travel a distance exceeding $ILB_{c_j} + M$, contradicting the optimality of Φ. Based on these two observations, a simple lemma (Hoshino and Kawarabayashi 2011a) shows that the team schedules for p_5 and p_6 must have the pattern HH-RRR-HH-RRR-HH, with the six home sets having the following structure for some permutation (a, b, c, d, e, f) of $\{1, 2, 3, 4, 5, 6\}$:

	1	2	3	4	5	6	7	8	9	10	11	12
p_5	c_a	c_b	c	c	c	c_c	c_d	c	c	c	c_e	c_f
p_6	c_b	c_a	c	c	c	c_d	c_c	c	c	c	c_f	c_e

The forced structure of p_5 and p_6 significantly reduces the search space. We include this constraint in a simple Maplesoft program that generates all possible bipartite tournaments satisfying the conditions of *BTTP*, where the schedule of each team t appears in S_t^* (Hoshino and Kawarabayashi 2011a). Using a Toshiba laptop under Windows with a single 2.10 GHz processor and 2.75 GB RAM, we find 28 optimal solutions in 34716 seconds (just under 10 hours). Each optimal solution (e.g. Table 4) has feasible value 42950, representing a reduction of 8184 kilometres, or 16%, compared to the actual distance traveled by the teams in the 2010 NPB season. Thus, we have solved *BTTP* for the NPB.

Our algorithm requires 10 hours of processing time. We conjecture that a much faster solution could be generated using a IP, CP, or some hybrid combination of the two.

Application 2: American Basketball ($n = 15$)

The National Basketball Association (NBA) is one of the world's most lucrative sports leagues, with over four billion dollars in annual revenue, and an average franchise value of $400 million dollars. There are 15 teams in the Western Conference and 15 teams in the Eastern Conference. Every NBA team plays 82 regular-season games, of which 30 are inter-league (with one home game and one away game against each of the 15 teams from the other conference.)

Given that NBA teams play inter-league games, we consider *BTTP* for this league, where we attempt to find a distance-optimal inter-league tournament. In this theoretical problem, we will assume that all inter-league games take place during a consecutive stretch in the regular season, as is done currently in the Japanese NPB. We will also enforce all the constraints of *BTTP*, including no team having a home stand or road trip lasting more than 3 games. We note that these strict conditions are not part of the NBA scheduling requirement, as evidenced by the San Antonio Spurs playing 6 consecutive home games followed immediately by 8 consecutive road games during the 2009-10 regular season.

	1 2 3	4 5 6	7 8 9	10 11 12	13 14 15
PT	MB IP CU	**MB CB CC**	AH OM MH	**CU AH NK**	WW CB PS
GW	TR CC DP	**CC MB CB**	MB IP CU	**NK CU AH**	MH OM AH
SK	NK NN BC	**CB CC MB**	TR CC DP	**AH NK CU**	MB IP CU
LC	IP CU MB	**NN MH TR**	OM MH AH	**CC CB MB**	CB PS WW
LL	WW CB PS	**TR NN MH**	NK NN BC	**MB CC CB**	TR CC DP
PS	BC NK NN	**MH TR NN**	CC DP TR	**CB MB CC**	IP CU MB
UJ	OM MH AH	**DP PS OM**	PS WW CB	**TR NN MH**	BC NK NN
DN	CC DP TR	**OM DP PS**	IP CU MB	**MH TR NN**	OM AH MH
OT	AH OM MH	**IP WW BC**	WW CB PS	**DP OM PS**	NK NN BC
SS	CU MB IP	**BC IP WW**	MH AH OM	**PS DP OM**	PS WW CB
DM	DP TR CC	**WW BC IP**	CU MB IP	**OM PS DP**	AH MH OM
HR	PS WW CB	**AH NK CU**	BC NK NN	**WW BC IP**	DP TR CC
MT	CB PS WW	**PS OM DP**	NN BC NK	**NN MH TR**	CC DP TR
MG	NN BC NK	**CU AH NK**	DP TR CC	**IP WW BC**	CU MB IP
NH	MH AH OM	**NK CU AH**	CB PS WW	**BC IP WW**	NN BC NK

	16 17 18	19 20 21	22 23 24	25 26 27	28 29 30
PT	**BC IP WW**	NK NN BC	**DP OM PS**	TR CC DP	**TR MH NN**
GW	**WW BC IP**	PS WW CO	**PS DP OM**	NN NK BC	**NN TR MH**
SK	**IP WW BC**	AH OM MH	**OM PS DP**	WW CO PS	**MH NN TR**
LC	**NK AH CU**	NN BC NK	**IP WW BC**	DP TR CC	**DP OM PS**
LL	**CU NK AH**	MB CU IP	**BC IP WW**	OM MH AH	**PS DP OM**
PS	**AH CU NK**	MH AH OM	**WW BC IP**	PS WW CO	**OM PS DP**
UJ	**MB CC CO**	CC DP TR	**CU NK AH**	CU IP MB	**BC WW IP**
DN	**CO MB CC**	CO PS WW	**AH CU NK**	BC NN NK	**IP BC WW**
OT	**TR MH NN**	DP TR CC	**MB CO CC**	MB CU IP	**CU AH NK**
SS	**NN TR MH**	BC NK NN	**CC MB CO**	CC DP TR	**NK CU AH**
DM	**MH NN TR**	WW CO PS	**CO CC MB**	NK BC NN	**AH NK CU**
HR	**OM PS DP**	IP MB CU	**MH NN TR**	AH OM MH	**CO CC MB**
MT	**CC CO MB**	CU IP MB	**NK AH CU**	MH AH OM	**WW IP BC**
MG	**DP OM PS**	OM MH AH	**TR MH NN**	CO PS WW	**MB CO CC**
NH	**PS DP OM**	TR CC DP	**NN TR MH**	IP MB CU	**CC MB CO**

Table 5: A feasible solution for the NBA inter-league problem.

We determine the 30×30 NBA distance matrix from an online website[1] that lists the flight distance (in statute miles) between each pair of major cities in North America. From this, we calculate ILB_t for each team t, giving $LLB_W \geq \sum_{t \in W} ILB_t = 251795$, $LLB_E \geq \sum_{t \in E} ILB_t = 266137$, and $TLB \geq LLB_W + LLB_E \geq 517932$.

Unlike the 12-team NPB where we could solve *BTTP*, it appears highly unlikely that we can solve this problem for the 30-team NBA. Nonetheless, we can generate a *uniform* inter-league tournament (i.e., a solution to *BTTP**) whose total distance is close to the trivial lower bound of $\sum ILB_t$,

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[1] http://www.savvy-discounts.com/discount-travel/JavaAirportCalc.html

by determining for each team t the set of t-rooted 4-cycle-covers containing exactly 5 cycles whose weights are close to ILB_t. Table 5 presents a uniform tournament schedule found by grouping the fifteen teams in each conference into five groups of three, and matching triplets from opposing leagues. This tournament has total distance 537791, which is just 3.8% more than the trivial lower bound of $\sum ILB_t$.

Our journal paper (Hoshino and Kawarabayashi 2011c) explains all the details, as well as providing the 30×30 distance matrix and the labeling of all 30 teams (e.g. PT = Portland Trailblazers, MB = Milwaukee Bucks, etc.)

While we are certain that the trivial lower bound of $\sum ILB_t$ cannot be achieved for either the *BTTP* or *BTTP**, we conjecture that the 3.8% figure can be reduced using more sophisticated techniques. But how close can we get? We leave this as a challenge for the interested reader.

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