Water Conservation Through Facilitation on Residential Landscapes

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Abstract

Plants can have positive effects on each other in numerous ways, including protection from harsh environmental conditions. This phenomenon, known as facilitation, occurs in water-stressed environments when shade from larger shrubs protects smaller annuals from harsh sun, enabling them to exist on scarce water. The topic of this paper is a model of this phenomenon that allows search algorithms to find residential landscape designs that incorporate facilitation to conserve water. This model is based in botany; it captures the growth requirements of real plant species in a fitness function, but also includes a penalty term in that function that encourages facilitative interactions with other plants on the landscape. To evaluate the effectiveness of this approach, two search strategies–simulated annealing and agent-based search–were applied to models of different collections of simulated plant types and landscapes with different light distributions. These two search strategies produced landscape designs with different spatial distributions of the larger plants. All designs exhibited facilitation and lower water use than designs where facilitation was not included.

Introduction

Irrigation of residential landscapes in arid and semi-arid regions accounts for a significant portion of household water use—up to 40 or even 70% (Hilaire et al. 2008). At the same time, landscapes can reduce energy and water use by shading structures (Bernatzky 1982; Shashua-Bar, Pearlmutter, and Erell 2009). Choosing effective, sustainable landscape designs is a matter of balancing this tradeoff. One approach to this task is to formulate it as a combinatorial optimization problem with a discrete set of locations for each plant on a grid, a fixed number of plants, and a fitness function that defines the performance of a plant at a location (Hoenigman, Bradley, and Barger 2010). The objective is to select the best k locations from n possibilities, where k is the number of plants and n is the number of cells on the landscape.

One important factor is missing from that formulation: the natural phenomenon known as facilitation (Callaway 1995). In water-scarce environments, larger shrubs can serve as benefactors to smaller annuals by generating conditions that protect them from harsh afternoon sun (Holzapfel et al. 2006). These shrubs, known as nurse plants, enable the annuals to survive on less water than they would need in the full sun (Whiting, Roll, and Vickerman 2007). The main result of this paper is a plant model that allows optimization strategies to find landscape designs that exploit facilitation to reduce water use.

In the work described in this paper, each plant is an agent with growth requirements, described in Section 2, that match those of real plants. A plant agent's fitness at a given location is defined by a fitness function, which is the topic of Section 3, that includes both those growth requirements and a penalty term designed to force facilitation. To evaluate the success of this approach, two optimization strategies were used, together with this model, to optimize placements of different collections of plants on three simulated landscapes. The algorithms involved—an agent-based search outlined in (Hoenigman, Bradley, and Barger 2010) and simulated annealing (SA) (Kirkpatrick, Gelatt, and Vecchi 1983)—are fundamentally different in how they operate. In the former, each agent acts locally and independently to improve its own fitness, in a manner that is designed to mimic how real plant communities evolve over time in response to environmental conditions. In SA, the fitness of the plant agent population is evaluated and controlled *globally*, rather than at the individual agent level. Both algorithms are described in more detail in Section 4. Sections 5 and 6 outline the numerical experiments and present and evaluate the results from both algorithms: the fitness scores and water use on each landscape, the presence of facilitation, and the spatial characteristics of the designs produced.

Modeling Plant Growth

In plant ecology, both negative (competition) and positive (facilitation) interactions between plants have been modeled in various ways. Agent-based models, also known as individual-based models (IBM) (Huston, DeAngelis, and Post 1988) in this field, have been used extensively to simulate local competition for resources, such as sunlight and rainfall, where the degree of competition is a function of the distance between plants, e.g. (Grimm and Railsback 2005). In some studies, this distance is calculated using a discrete grid, e.g. (Rademacher et al. 2004). Other spatial competition models include distance as well as resource use, e.g. (Wu et al. 1985). Models of plant facili-

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tation are less common, but have been used to show interactions in many types of ecosystems, including alpine environments (Callaway et al. 2002), and between shrubs and annuals along an aridity gradient (Holzapfel et al. 2006; Tielborger and Kadmon 2000).

On residential landscapes, both competition and facilitation play important roles due to heterogeneity in the types of plants and in the landscape conditions. There are, however, few examples in the currently available ecological models that are able to capture both types of interactions, e.g. (Holmgren, Scheffer, and Huston 1997). Effective analysis and optimization of residential landscapes involves modeling both individual plant performance and the effects of both competition and facilitation. To accomplish this, the approach used here represents resources and conditions explicitly in each cell on the simulated landscape—an approach that is simpler and more natural than the currently available ecological models. Using this approach, the growth rates of a heterogeneous set of individuals emerge directly from the resources available to each individual at its location.

Plants need light, water, and nutrients to grow. As they do so, they influence their surroundings primarily by removing water from the landscape and generating shade. The plant-growth model used here includes these basic features, with one exception: nutrients are not considered as these can be modified on built landscapes. This growth model, which is described in more detail in (Hoenigman, Bradley, and Barger 2010), is based on empirical data for how real plants respond to different levels of light and water availability (Harvey 1979). In it, each plant is represented as an agent whose growth is determined by the amount of light and water available at its location on the landscape. Plant growth is nonlinear: it increases with increasing light, up to a certain point, known as the light saturation point (Crawley 1997). Above this level, additional light does not result in additional growth and can actually decrease growth due to other processes necessary for plant survival. The amount of water that a plant requires—the other key factor in the model proposed here—is based on its growth rate, size, water-use category, and light level. Light and water effects are not independent; high growth rates and larger sizes require more water, for instance, and plants need more water in full sun than in partial sun because they use it to stay cool (Gardingen and Grace 1991).

Plant agents in this work are classified into discrete categories, similar to those used to categorize real nursery plants: three levels of light requirements—low, medium, and high—and two levels of water requirements (low and high). This results in six types of plants. The landscape conditions in which these plant agents grow are represented using a discrete grid, where each cell in the grid is one square foot. This size is the average space needed by summer annuals, the smallest plants considered here. Only one plant can occupy a cell at each time step. Each landscape cell has three parameters: morning light, afternoon light, and water, representing the amount of that resource present in the cell at a given time. Considering morning and afternoon light separately is critical to modeling facilitation, since the effects of nurse plants come into play in the strong afternoon sun.

Plant agents interact with the landscape—and, indirectly, with each other—in two ways: by removing water from their own and surrounding cells and by generating shade. Both of these effects occur in proportion to their size. Plant agents one foot or shorter can only use water from their own cell; those taller than one foot can access water in surrounding cells proportional to their size over one foot. The water that a plant agent needs to grow is withdrawn evenly from all cells that it can access. This is an approximation of water use in real plants—larger plants have larger root structures and an increased reach for water. Light effects are similar and also include directionality. Plant agents generate shade in surrounding cells, again in proportion to their size. Each foot of height over one foot shades one cell in the horizontal direction; for each two feet of height over one foot, the plant also shades one cell in the vertical direction. Cells to the right of the plant agent (east) are shaded in the afternoon and cells to the left (west) are shaded in the morning. If a cell is shaded, the light in that cell is reduced by 30%. If two plants shade the same cell, the shade effects are multiplicative, the 30% reduction is applied twice, once to the original value, and then again to the reduced value. This profile does not entirely reflect real-world shading, as cells south of the plant should never be shaded (in the northern hemisphere); these effects will be addressed in future work. This model only includes the mid-summer sun angle because the primary concern here is reducing water use, which is highest in the summer.

Plant Agent Fitness

The fitness score measures plant growth and water use in different light and water conditions. It includes a growth portion that is based on the plant agent's biological properties and an additional penalty for high-water-use conditions. The plant-growth model and the landscape conditions determine the biological portion of the score. The penalty term is engineered to encourage facilitation; it is a function of the available light and the agent's size and light saturation point. The fitness for an arrangement of plant agents is the sum of the fitness for each individual agent. The dynamics of irrigation require that the fitness be calculated over a multi-day period: a plant with high water requirements or exposure to harsh sun may not have enough water to survive between irrigation cycles in an arid or semi-arid region. The fitness calculation proceeds as follows:

- **Step 1**—Calculate morning fitness using the agent's light response curve, the water needed to support this growth, and the available light and water using the equation

$$mGrowth = \frac{mBiomass * \frac{wAvailable}{wNeeded}}{maxMBiomass} \quad (1)$$

where $mBiomass$ is the expected biomass given the plant agent's morning light, $wNeeded$ is the amount of water needed to support growth at this light level, $wAvailable$ is the amount of water the agent can extract from the soil to support growth, and $maxMBiomass$ is the expected biomass under optimal light conditions.

- **Step 2**—Calculate the amount of water used by each plant agent during the morning period and update the landscape conditions accordingly. See (Hoenigman, Bradley, and Barger 2010) for more detail.
- **Step 3**—Calculate afternoon fitness using same variables as in the morning fitness calculation and the equation

$$aGrowth = \frac{aBiomass * \frac{wAvailable}{wNeeded}}{maxABiomass} \quad (2)$$

- **Step 4**—Calculate the amount of water used by each plant agent during the afternoon period and update the landscape conditions accordingly.
- **Step 5**—Calculate the agent's daily fitness score from the morning and afternoon fitness scores.

$$dayGrowth = mGrowth + aGrowth \quad (3)$$

- **Step 6**—Calculate the penalty value for light conditions beyond the plant agent's light saturation point.

$$penalty = \alpha * (maxH - h) * (\frac{mL + aL - 2 * lS}{2 * lS}) \quad (4)$$

where mL is morning light, aL is afternoon light, α is a user-defined weight parameter, $maxH$ is a user-defined height that controls how the penalty affects different-sized plants, h is the size of the plant, and lS is the plant-specific light saturation point. The α parameter adjusts how much the penalty contributes to the total fitness score. The $maxH$ parameter controls how the penalty affects plant agents of different sizes, which is a key element of the fitness function. The penalty term is designed to encourage facilitation by allowing larger nurse plants to be located in full sun to provide shade for smaller plants. Without the $maxH$ parameters, the nurse plants would have low fitness scores in full-sun conditions, which would not achieve the objectives of the model or reflect observed behavior on real landscapes (Holzapfel et al. 2006). When the plant agent's height, h, is equal to $maxH$, the penalty is zero. The penalty increases with decreasing plant height. The penalty also increases with increasing light above lS.

- **Step 7**—Calculate final daily fitness score including growth and penalty components

$$dayFit = dayGrowth - penalty \quad (5)$$

- **Step 8**—Repeat Steps 1-7 for user-defined number of days to calculate agent's final multi-day fitness score

$$agentFitness = \frac{\sum_{d=1}^{days} dayFit_d}{days} \quad (6)$$

where d is the day number, and $days$ is the total number of days being simulated. The growth score has a maximum value of one, and the penalty term can only reduce the final fitness score.

- **Step 9**—Calculate fitness score for the arrangement

$$landscapeFitness = \sum_{i=1}^{n} agentFitness_i \quad (7)$$

where n is the number of plants.

Search Algorithms

Calculating the optimal solution in combinatorial problems is computationally prohibitive for anything but the simplest situations. As a result, metaheuristics, such as simulated annealing (SA) and genetic algorithms, are often used to search for good solutions (Alp and Erkut 2003; Otto and Kokai 2008). An objective function measures the global fitness for a solution; the search process identifies solutions that improve the global fitness. A very different type of metaheuristic relies on the notion of autonomous agents that act independently to improve their own individual fitness. In these schemes, the global fitness "emerges" from the individual actions and interactions (Arentz and Timmermans 2007; Moujahed, Simonin, and Koukam 2009).

In the landscaping problem presented in this paper, the objective is to select the best *k* out of *n* total cells on a landscape, where *k* is the number of plants. There are no repetitions of the *k* elements on the landscape, and assigning a plant to a cell effectively generates an ordering that is meaningful to the solution. This process is a *k-permutation of n* with $\frac{n!}{(n-k)!}$ possible permutations. The two search algorithms evaluated in this paper—simulated annealing, a global control search routine, and an agent-based search routine—were selected to represent different search methodologies used for combinatorial problems. Both algorithms use the plant-growth model and fitness function described above to define the behavior of the individual plant agents. The main difference between the two algorithms is in how individual moves are evaluated. In SA, a random location is selected for each plant agent. If that new location improves the global fitness score, the agent moves to that location. If the new location does not improve the global fitness, the agent still moves—with a probability that decreases as the search progresses. These non-improving moves are designed to keep the algorithm from getting stuck in local optima. The search stops when the probability of non-improving moves reaches zero and there are no moves that improve fitness. An outline of the SA search routine used here is shown in Algorithm 1. In the agent-based search algorithm, plant agents employ a combination of local search and random jumps to improve their own individual fitness scores—without concern for global fitness. Agents have a fixed number of times that they can move during the search process; which guarantees that the algorithm will converge to a solution and models the physical reality of the effects of re-planting. Agents first search locally within a pre-defined search radius that designed to allow agents to move out of the influence zone of other agents. An increasing threshold score controls the search process—agents move if their score is below this threshold. This value is initially set low and increased as agents find better conditions on the landscape, which is reflected in higher fitness scores. The search stops when fitness scores can no longer be improved above this threshold and agents have used up their allowable moves. An outline of the algorithm is shown in Algorithm 2; additional details about the algorithm can be found in (Hoenigman, Bradley, and Barger 2010).

Algorithm 1 Simulated annealing

```
Set starting temperature to 0.9
Generate land //random locations for all plant agents
Calculate f_land //fitness score for landscape
repeat
    for all agents do
        Select new random location for plant agent
        Calculate f_land' //new fitness score
        if f_land' > f_land then
            land = land'
        else
            Move with uniform probability, pr(move) < heat
        end if
    end for
    heat = heat - 0.01
until heat = 0
```

Numerical Experiments

In order to evaluate the utility of the model proposed here, a series of experiments was performed using both search strategies to place different collections of plants on three simulated landscapes:

- **Scenario 1**—The landscape size is 25x25 cells, all of which receive full morning and afternoon sun. A collection of 46 plants is to be placed on this landscape, including six, six-foot nurse plants that require full sun and low water. All of the shade created on this landscape will be due to the placement of these nurse plants. The other 40 plants, which represent smaller annuals, have similar requirements and a height of one foot.
- **Scenario 2**—The plants are the same as in Scenario 1, but the landscape is 25x30 and 50% of its area is modeled with existing afternoon shade (e.g., from buildings on two sides). The purpose of this scenario is to examine how the presence of existing shade affects both the smaller plants and the nurse plants.
- **Scenario 3**–This scenario uses the same landscape conditions as in Scenario 2, but includes a more-diverse plant collection: 10 full-sun, low-water nurse plants; 10 1-foot full-sun, low-water plants; 10 2-foot full-sun, high-water plants; 10 1-foot shade-loving, high-water plants; and 10 1.5-foot partial-sun, high-water plants. This scenario is designed to test the effect of facilitation on a realistically diverse group of plants with higher water requirements.

These experiments represent conditions observed on real landscapes in terms of the types of plants, variability in lighting, and plant density. The search algorithms were evaluated by calculating the fitness scores, water use, role of facilitation, and spatial characteristics of the final solutions in each case. The water use was also compared to that of a random placement of each plant collection on each landscape. The water on the landscapes was initialized to represent recently irrigated soils in many areas in the U.S. Southwest. Plant agents grew for five days.

Algorithm 2 Agent-based search

```
repeat
    for all agents do
        Calculate fitness score for plant agent.
        if score > threshold then
            Leave the plant alone
        else
            if plant moves < moves allowed then
                Search locally for better location
                if local search successful then
                    if location unoccupied then
                        Plant agent relocates to new location.
                    else
                        Plant agents compete. Loser gets new random location.
                    end if
                else
                    Plant agent moves to random unoccupied location.
                end if
                Increment plant moves
            end if
        end if
    end for
until scores > threshold for all plants or no more moves allowed
If a solution is found for all plants above the threshold, increase threshold and repeat the algorithm.
```

The parameters for each algorithm, which include the number of moves allowed and the local search neighborhood for the agent search and the cooling speed for SA, were selected by running the algorithms for a range of these parameters and selecting the parameters that generated the highest fitness scores. For the agent search, agents were allowed to move 10 times and search five cells locally. For SA, a cooling parameter of 0.01 was used. In each scenario, the agent-based search routine was applied from 20 initial conditions and SA was applied from 30 initial conditions, where an initial condition is a random location for each plant agent. This number of initial conditions for each algorithm generated comparable computational efforts and, therefore, provides a good comparison. Each search routine was repeated 100 times, resulting in 100 solutions to evaluate for each scenario. The results reported in this paper are the mean results from these 100 runs for each scenario. The water use for 100 randomly placed configurations for each scenario was used for comparison to the agent-based and SA solutions. In these random configurations, no optimization strategy was used to encourage facilitation on the landscape.

Results

The results from both search algorithms show facilitation in all three scenarios. The average fitness scores for all 100 solutions in each scenario were within 15% percent of each other, with the SA scores consistently higher than the agent search scores. Figure 1 shows a visual example on one lay-

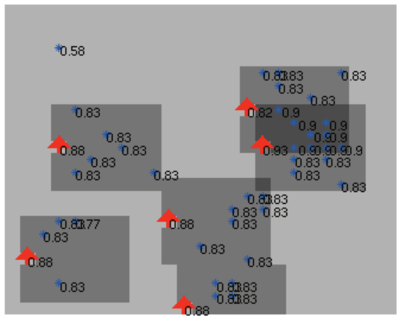

Figure 1: Example SA solution for Scenario 1. In this example, all but one of the smaller annuals (shown in blue) ended up in protected conditions generated by the nurse plants.

out produced by SA for Scenario 1. The nurse plants are the red triangles and the small annuals are the blue stars. The number next to each plant is the plant's fitness score at that location. The shaded regions show the afternoon shade generated by the nurse plants; the lighter areas represent full sun. In this solution, only one plant ended up in unprotected conditions—the one with the lower fitness score at the top left of the image. The higher scores for the shaded annuals reflect the improved growing conditions created by facilitation, in the form of afternoon shade. This pattern is generic across all three scenarios for both algorithms. In Scenario 1, for example, 93% of the small annuals in the agent solutions and 97% of small annuals in the SA solutions benefit from facilitation.

The spatial configuration effects created by facilitation greatly reduced water use on these landscapes. In Scenario 1, for instance, the average small annual in the agent-based design uses 0.98 gallons over the five-day simulation, as compared to 0.87 gallons in the design produced by SA. In all three scenarios, water use in the random placements is higher than in the SA or agent solutions—about 5-10% higher in Scenario 1, for example. In Scenario 3, this difference is even more dramatic, underscoring the potential for facilitation-based water savings. Here, the randomly placed plants use an average of 3 gallons over the five-day simulation, while the agent and SA-placed plants use 1.9 gallons and 1.74 gallons respectively—a savings of 37 and 42%.

There are some interesting differences between the SA and agent-based solutions, primarily in the spatial distribution of the plants and the nurse plants' effect on the landscape. Figure 2 shows an example of these spatial distributions for both algorithms for Scenario 2. In both solutions, all small annuals are in protected conditions and most nurse plants are in full sun. The nurse plants' contributions to facilitation are different in the solutions produced by the two algorithms, however. In the SA solution, the nurse plants have very few neighbors—i.e., most of the smaller annuals are placed in the *existing* shade. In contrast, many of the smaller annuals in the agent solution are placed in the nurse plants' shade. While facilitation is at work in both solutions, its characteristics are different. This pattern is particularly pronounced in Scenario 2, where 61% (standard deviation

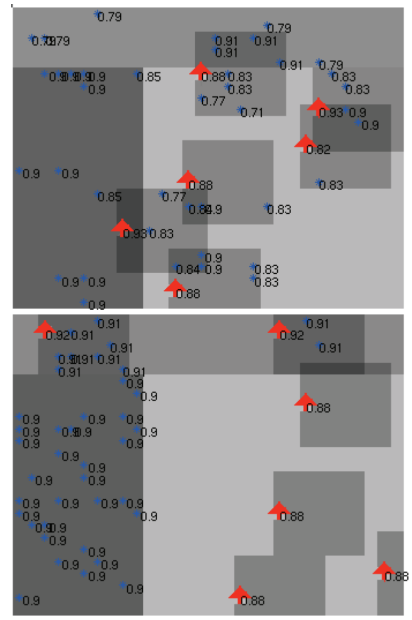

Figure 2: Example optimized plant placements produced by agent search (top) and SA (bottom) for Scenario 2. Smaller annuals were more likely to cluster around the nurse plants in the former than in the latter.

11%) of small annuals in the agent solutions are in protected conditions generated by the nurse plants, as compared to only 35% (standard deviation 10%) in the SA solutions.

Another difference in spatial distribution manifests in the placement of the nurse plants in relation to the shade on the landscape. This difference can be seen in Figure 3, which shows example solutions for Scenario 3. In these solutions, many more nurse plants are located in the existing shade in the SA solution than in the agent solution, where nurse plants tend to cluster in the full sun. This pattern is generic; in Scenario 3, for example, 31% (standard deviation 14%) of nurse plants are in the full-sun region in the agent solutions as compared to only 15% (standard deviation 12%) in the SA solutions.

While these spatial patterns enhance water savings, they have some limitations. The existing shade on the landscape produced higher fitness scores than the shade from the nurse plants, as it generated light conditions closer to the agents' light saturation points. An effective search algorithm could find these higher fitness scores. The smaller plants clustered near the nurse plants could indicate, for example, that they found acceptable conditions during the local search portion of the algorithm but did not have enough optimization pressure to continue searching for better conditions. On the other hand, this is the nature of an agent-based search algorithm, which is a natural match to the problem at hand. Recall that the objective here is to design landscapes that use facilitation to improve growing conditions while also reducing water use. By presenting design patterns with plants consistently placed in the same region on the landscape, SA may

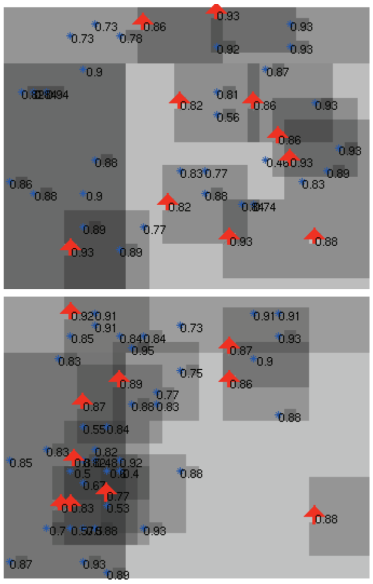

Figure 3: Example optimized plant placements produced by agent search (top) and SA (bottom) for Scenario 3. The nurse plants tended to cluster in the full sun in the agent-based solution and in the shade in the SA solution.

not be an appropriate algorithm for this problem. This is particularly true if aesthetics and subjective human choice are involved in the process. While SA produced higher average fitness scores, the agent-based search algorithm generated more space on the landscape with good growing conditions.

Conclusion

This computational model for optimizing landscapes represents a new approach for water conservation in residential systems. The results show that optimization strategies that take facilitation into account can generate significant water savings, particularly on heterogeneous landscapes containing high-water plants, which are common residential landscape design elements—even in water-scarce regions. The results presented here are based on a growth model generated from empirical data from real plants. The next stage in this work will involve experimenting with real plants in order to tune the plant-growth model (i.e., the relationship between light, water, and growth in landscaping plants, and the effects of plant interactions) and to validate the role of facilitation in water conservation on residential landscapes.

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