Super Level Sets and Exponential Decay: A Synergistic Approach to Stable Neural Network Training (Abstract Reprint)

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Abstract

This paper presents a theoretically grounded optimization framework for neural network training that integrates an Exponentially Decaying Learning Rate with Lyapunov-based stability analysis. We develop a dynamic learning rate algorithm and prove that it induces connected and stable descent paths through the loss landscape by maintaining the connectivity of super-level sets S = { n : () }. Under the condition that the Lyapunov function V() = () satisfies V() () 0, we establish that these super-level sets are not only connected but also equiconnected across epochs, providing uniform topological stability. We further derive convergence guarantees using a second-order Taylor expansion and demonstrate that our exponentially scheduled learning rate with gradient-based modulation leads to a monotonic decrease in loss. The proposed algorithm incorporates this schedule into a stability-aware update mechanism that adapts step sizes based on both curvature and energy-level geometry. This work formalizes the role of topological structure in convergence dynamics and introduces a provably stable optimization algorithm for high-dimensional, non-convex neural networks.

References

Chaudhary, J.; Nidhi, D.; Heikkonen, J.; Merisaari, H.; and Kanth, R. K. 2025. Super Level Sets and Exponential Decay: A Synergistic Approach to Stable Neural Network Training. *Journal of Artificial Intelligence Research*, 83.