

Hybrid Semantics Accounting for Argument Types

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Abstract

Assessing the strength of arguments is essential for determining the outcomes of any argument-based system. A wide range of semantics has been proposed in the literature. These take as input a set of arguments—each assigned a basic weight and potentially subject to attacks from others—and compute a single strength value for each argument. Despite the diversity of argument types (or schemes), existing semantics apply uniform evaluation criteria across all arguments.

In this paper, we advocate for type-dependent evaluations, acknowledging that the impact of attacks can vary across types. Given that many argument-based systems involve heterogeneous types of arguments, we propose a broad family of hybrid semantics that combine distinct evaluation strategies, each tailored to specific argument types. We investigate their theoretical properties, present concrete instances within this family, and examine their computational complexity.

Introduction

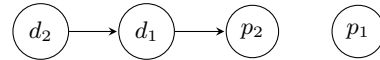
Argumentation is a powerful approach for addressing various reasoning tasks, such as non-monotonic reasoning (e.g., (Simari and Loui 1992; García and Simari 2004; Dung 1995; Prakken and Sartor 1997)) and decision making (e.g., (Bonet and Geffner 1996; Amgoud, Dimopoulos, and Moraitis 2008; Amgoud and Prade 2009; Baroni et al. 2015)) (see also (Baroni et al. 2018; Simari et al. 2021) for more applications). At its core, it involves justifying claims using arguments—reasons supporting a conclusion—which may carry an initial weight and can be attacked or supported by other arguments. Evaluating the *strength* of arguments—the level of support an argument provides to its claim—is key to determining the outcomes of argument-based systems. For instance, a decision-making system might favor options supported by the strongest arguments. In the literature, considerable attention has been devoted to formalizing this evaluation process through what are known as *semantics*.

Existing semantics generally treat arguments as abstract entities and evaluate them uniformly, regardless of their nature. However, as illustrated in (Walton, Reed, and Macagno 2008), arguments can take various *types* or *schemes*, such as causal, analogical, or practical. These types are often

classified based on the nature of the underlying reasoning—deductive, abductive, or inductive. When premises hold, deductive arguments are generally considered stronger, as their conclusions follow necessarily from the premises. In contrast, abductive and inductive arguments yield conclusions that are plausible but not guaranteed. Thus, types significantly influence the perceived strength of an argument. Besides, the impact of attacks can depend on the argument’s type. For example, a single strong attack may be enough to invalidate a deductive argument (Dung 1995), while analogical arguments tend to be more robust, requiring multiple attacks to be effectively disqualified (Amgoud 2020). Similarly, practical arguments are often sensitive to the quantity rather than the severity of attacks. Overlooking these distinctions can result in inaccurate evaluations, leading to flawed or suboptimal outcomes in systems that incorporate multiple argument types. The following example, based on a decision-making problem, illustrates this issue.

Example 1. Consider the following exchange between two friends deciding where to take their guest, Carla.

- p_1 : Since Carla likes art (ar), let’s go to the museum (mu);
 $p_1 = \langle \{ar, mu \rightarrow ar\}, mu \rangle$.
 p_2 : Let’s instead embark on a picnic (pi), as she likes fun (fu) when the weather is not rainy (ra);
 $p_2 = \langle \{\neg ra, fu, \neg ra \wedge pi \rightarrow fu\}, pi \rangle$.
 d_1 : It’s going to rain because the weather is cloudy (cl);
 $d_1 = \langle \{cl, cl \rightarrow ra\}, ra \rangle$.
 d_2 : The weather is not cloudy, the sun is shining (su);
 $d_2 = \langle \{su, su \rightarrow \neg cl\}, \neg cl \rangle$.



The two arguments d_1, d_2 are deductive, while p_1, p_2 are practical and so follow the following general schema:

- (Premise 1.) g is a goal,
 (Premise 2.) Choosing option A leads to g ($A \rightarrow g$),
 (Conclusion.) Therefore, choose A .

Fun and art are two goals of Carla. Assume that Carla prefers fun to art. Assume also that $w(d_1) = 0.98$, $w(d_2) = 1.0$, $w(p_1) = 0.7$, $w(p_2) = 1$, where $w(x) \in [0, 1]$ is the initial weight of x , $w(d_i)$ is the certainty degree of the premises of d_i while $w(p_i)$ represents the importance degree of the goal promoted by p_i , with $i \in \{1, 2\}$.

Any existing extension semantics would yield a single extension— $\{d_2, p_1, p_2\}$ —for this weighted argument graph (e.g., (Prakken and Sartor 1997; Amgoud and Vesic 2011)). Such semantics use the initial weights of arguments solely to block attacks from weaker sources against stronger targets.

The above evaluation of arguments implies that the two options, μ and π , are equally acceptable. However, the goal pursued in p_2 is more important than that in p_1 . Then, one would expect the system to recommend π over μ .

Let us now consider a gradual semantics, such as Mbs from (Amgoud et al. 2017), which better accounts for initial weights. This semantics yields the following strength values: $\delta(d_1) = 0.49$, $\delta(d_2) = 1$, $\delta(p_1) = 0.7$, and $\delta(p_2) = 0.68$. Based on these values, the decision system would recommend μ , since $\delta(p_1) > \delta(p_2)$. This outcome remains unsatisfactory, primarily due to an **overestimation** of the deductive attacker d_1 targeting p_2 . Recall that a deductive argument is valid only iff **all** its premises are true. Consequently, the falsification of even a single premise is sufficient to entirely disqualify the argument. In this example, the premise cl of d_1 is undermined by d_2 , rendering it false. Therefore, $\delta(d_1)$ should be infinitesimally small, or even zero. In that case, p_2 would retain its initial weight, making π the optimal decision. However, the gradual semantics Mbs satisfies a resilience property that prevents any argument x from receiving $\delta(x) = 0$ unless $\mathbf{w}(x) = 0$. As a result, Mbs is ill-suited for handling deductive arguments.

This example shows a key insight: extension semantics tend to produce more appropriate results for deductive arguments, whereas gradual semantics are better suited for evaluating practical ones. Moreover, as shown in (Amgoud 2020), analogical arguments are best assessed using gradual semantics that account for all attackers, but with a nuanced, graded impact. More broadly, *different argument types require different evaluation criteria*. To our knowledge, no existing semantics evaluates arguments differently based on their type within the same graph. This paper addresses this gap by introducing the first semantics that explicitly account for argument types. Focusing on weighted argumentation graphs—where arguments and attacks are assigned initial weights— it defines the concept of *hybrid semantics* that integrates distinct evaluation strategies, each tailored to specific argument types. The paper’s contributions are fourfold.

1. It introduces a novel principle that provides a foundation for comparing arguments across different types. Additionally, it adapts the set of principles introduced in (Amgoud and Doder 2019) to the context of hybrid semantics. Most of these principles are type-dependent—meaning that a given semantics may satisfy a principle for certain types of arguments while violating it for others.
2. It proposes a broad family of hybrid semantics and analyzes their theoretical properties.
3. It examines some of its representative instances.
4. It establishes the computational complexity of some existing gradual semantics.

The paper is structured as follows. It begins with the formal setting, followed by the presentation of the principles. Next, it introduces the family of hybrid semantics,

with its instances discussed in a subsequent section. The final section covers related work and concluding remarks. Detailed proofs are in (Amgoud, Hanocq, and Lagasquie-Schiex 2025).

Background

In this paper, we consider *typed argumentation graphs*, where nodes represent arguments—each associated with a type (or scheme) and an initial weight (e.g., reflecting user preferences). Edges denote attacks (i.e., conflicts) between arguments, and each edge also carries an initial weight. Let args denote the set of all possible arguments, and let $\text{types} = \{t_1, \dots, t_k\}$ be the non-empty set of possible types (e.g., the schemes identified in (Walton, Reed, and Macagno 2008)). The set types is equipped with a **fixed** binary relation \succeq , which is a preorder—that is, a reflexive and transitive relation. For any $t_i, t_j \in \text{types}$, the notation $t_i \succeq t_j$ expresses that type t_i is at least as robust to attacks as type t_j . We assume that \succeq is total, that is, for all $t_i, t_j \in \text{types}$, $t_i \succeq t_j$ or $t_j \succeq t_i$. The notation $t_i \succ t_j$ is a shortcut for $t_i \succeq t_j$ and $t_j \not\succeq t_i$. For example, if t_i is deductive and t_j is practical, then we may have $t_j \succ t_i$. We abstract from the precise specification of \succeq , as this is beyond our scope. The set types is partitioned into disjoint equivalence classes, $\text{types} = \mathbf{T}_1 \cup \dots \cup \mathbf{T}_m$, where all types in each \mathbf{T}_i are equally robust, and are more robust than those in \mathbf{T}_j , with $j < i$. So, \mathbf{T}_m contains the **most robust** types.

Definition 1 (TAF). A typed argumentation framework (TAF) is a tuple $\mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle$ such that:

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| <ul style="list-style-type: none"> • $\mathcal{A} \subseteq \text{args}$, \mathcal{A} being finite, • $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$, | <ul style="list-style-type: none"> • $\mathbf{t} : \mathcal{A} \rightarrow \text{types}$, • $\mathbf{w} : \mathcal{A} \cup \mathcal{R} \rightarrow [0, 1]$. |
|--|---|

TAF is the set of all typed argumentation frameworks.

Note that an argument may be hybrid, in the sense that it combines elements from two or more types. This richer form of argument is left for future work. In this paper, we do not impose any constraints on the attack relation—for instance, we do not prohibit attacks from arguments of type t to arguments of type t' . Some works (e.g., (Hadidi, Dimopoulos, and Moraitis 2010)) disallow practical arguments from attacking deductive ones. We assume that such constraints, if required, are incorporated into the definition of the attack relation \mathcal{R} , as it is commonly done in the literature.

Notations: For $\mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}$ and $a \in \mathcal{A}$, $\text{att}_{\mathbf{H}}(a) = \{b \in \mathcal{A} \mid (b, a) \in \mathcal{R}\}$, i.e., the *direct attackers* of a in \mathbf{H} . Let $\mathbf{H}' = \langle \mathcal{A}', \mathbf{t}', \mathcal{R}', \mathbf{w}' \rangle \in \text{TAF}$ be such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$. We define $\mathbf{H} \oplus \mathbf{H}' = \langle \mathcal{A} \cup \mathcal{A}', \mathbf{t}'' , \mathcal{R} \cup \mathcal{R}' , \mathbf{w}'' \rangle \in \text{TAF}$ such that $\forall a \in \mathcal{A}$ (resp. $a \in \mathcal{A}'$), $\mathbf{t}''(a) = \mathbf{t}(a)$ (resp. $\mathbf{t}''(a) = \mathbf{t}'(a)$), and $\forall x \in \mathcal{A} \cup \mathcal{R}$ (resp. $x \in \mathcal{A}' \cup \mathcal{R}'$), $\mathbf{w}''(x) = \mathbf{w}(x)$ (resp. $\mathbf{w}''(x) = \mathbf{w}'(x)$).

We start by recalling the notion of *path* in a graph, and then define an *isomorphism* between two TAFs.

Definition 2 (Path - Isomorphism). Let $\mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle$, $\mathbf{H}' = \langle \mathcal{A}', \mathbf{t}', \mathcal{R}', \mathbf{w}' \rangle \in \text{TAF}$, and $a, b \in \mathcal{A}$.

- A path from b to a is a finite non-empty sequence $\langle x_1, \dots, x_n \rangle$ of arguments in \mathcal{A} such that $x_1 = b$, $x_n = a$, and $\forall 1 \leq i < n$, $(x_i, x_{i+1}) \in \mathcal{R}$.
- An isomorphism from \mathbf{H} to \mathbf{H}' is a bijective function $F : \mathcal{A} \rightarrow \mathcal{A}'$ such that:

- $\forall a \in \mathcal{A}, \mathbf{t}(a) = \mathbf{t}'(F(a))$ and $\mathbf{w}(a) = \mathbf{w}'(F(a))$,
- $\forall a, b \in \mathcal{A}, (a, b) \in \mathcal{R}$ iff $(F(a), F(b)) \in \mathcal{R}'$,
- $\forall (a, b) \in \mathcal{R}, \mathbf{w}((a, b)) = \mathbf{w}'((F(a), F(b)))$.

A semantics evaluates the strength of arguments in typed argumentation frameworks. Formally, it is a function that assigns to each argument a value from the ordered scale $[0, 1]$, with higher values indicating stronger arguments.

Definition 3. A semantics is a function \mathbf{S} mapping any $\mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}$ to $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}} : \mathcal{A} \rightarrow [0, 1]$. For $a \in \mathcal{A}$, $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a)$ is the strength degree of a in \mathbf{H} under \mathbf{S} .

Principles

Properties, often referred to as *principles*, have been proposed in the literature (e.g., (Cayrol and Lagasque 2005; Baroni and Giacomin 2007; Amgoud and Ben-Naim 2016; Bonzon et al. 2016, 2017; Amgoud et al. 2017; Baroni, Rago, and Toni 2018; Amgoud and Doder 2019)). They are desirable features that semantics may exhibit. They help clarify design choices behind semantics, enable formal comparisons between semantics, and serve as a guide for selecting suitable semantics in practical applications.

We introduce thirteen principles for semantics that account for argument types. The first twelve are adaptations of the principles defined in (Amgoud and Doder 2019), while the thirteenth is a new contribution for which there is no analogue in the literature. We organize the adapted principles into two groups. The first comprises four *type-independent* principles that must hold regardless of the specific type of each argument. The second group comprises *type-dependent* principles—properties that must hold when attention is restricted to arguments of a fixed type. These principles may be satisfied for one type and violated for another.

Throughout the paper, given any principle \mathbf{P} , we say that a semantics \mathbf{S} satisfies $t\text{-}\mathbf{P}$ if the conditions of \mathbf{P} hold when one considers only arguments of type $t \in \text{types}$; \mathbf{S} satisfies \mathbf{P} if it satisfies $t\text{-}\mathbf{P}$ for every $t \in \text{types}$.

Let us now introduce the first group. *Independence* ensures that the strength of an argument does not depend on unrelated arguments; *Directionality* further requires that influence between arguments occurs only along directed paths; *Maximality* states that every non-attacked argument should retain its initial weight; *Weakening* holds that an argument should lose weight if it receives at least one serious attack.

Independence: $\forall \mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle, \mathbf{H}' = \langle \mathcal{A}', \mathbf{t}', \mathcal{R}', \mathbf{w}' \rangle \in \text{TAF}$ s.t. $\mathcal{A} \cap \mathcal{A}' = \emptyset$, it holds: $\forall a \in \mathcal{A}, \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) = \mathbf{V}_{\mathbf{H} \oplus \mathbf{H}'}^{\mathbf{S}}(a)$.

Directionality: $\forall \mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}, \forall a, b \in \mathcal{A}, \forall \mathbf{H}' = \langle \mathcal{A}', \mathbf{t}', \mathcal{R}', \mathbf{w}' \rangle \in \text{TAF}$ s.t. $\mathcal{A}' = \mathcal{A}, \mathbf{t}' = \mathbf{t}, \mathcal{R}' = \mathcal{R} \cup \{(a, b)\}, \forall x \in \mathcal{A} \cup \mathcal{R}, \mathbf{w}'(x) = \mathbf{w}(x)$, it holds: $\forall x \in \mathcal{A}$, if there is no path from b to x , then $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(x) = \mathbf{V}_{\mathbf{H}'}^{\mathbf{S}}(x)$.

Maximality: $\forall \mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}, \forall a \in \mathcal{A}$, if $\text{att}_{\mathbf{H}}(a) = \emptyset$, then $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) = \mathbf{w}(a)$.

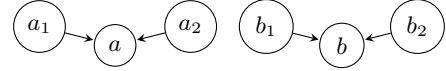
Weakening: $\forall \mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}, \forall a \in \mathcal{A}$, if $\mathbf{w}(a) > 0$ and $\exists b \in \text{att}_{\mathbf{H}}(a)$ such that $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(b) > 0$ and $\mathbf{w}((b, a)) > 0$, then $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) < \mathbf{w}(a)$.

The first principle in the second group is *Anonymity*, which ensures that the strength assigned to an argument is

independent of its identity. *Equivalence* states that an argument's strength depends only on its *type*, *initial weight*, and the *strengths of its direct attackers*. *Counting* ensures sensitivity to the number of attackers, and *Resilience* guarantees that an argument does not lose all of its initial weight, provided that the latter is positive. In what follows, we assume a **fixed** but arbitrary type $t \in \text{types}$.

Anonymity: $\forall \mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle, \mathbf{H}' = \langle \mathcal{A}', \mathbf{t}', \mathcal{R}', \mathbf{w}' \rangle \in \text{TAF}$, for any isomorphism F from \mathbf{H} to \mathbf{H}' , it holds: $\forall a \in \mathcal{A}, \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) = \mathbf{V}_{\mathbf{H}'}^{\mathbf{S}}(F(a))$.

Example 2. Consider the typed argumentation graph depicted below, where $\mathbf{w} \equiv 1$, i.e., all arguments and attacks are assigned an initial weight of 1. Assume also that $\mathbf{t}(a) = t_1, \mathbf{t}(b) = t_2, \mathbf{t}(a_1) = \mathbf{t}(a_2) = \mathbf{t}(b_1) = \mathbf{t}(b_2) = t_3$.



Observe that the subgraph on the right is not isomorphic to the left one since a and b have different types. Thus, these two arguments may get different strength values.

Equivalence: $\forall \mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}, \forall a, b \in \mathcal{A}$, if:

- $\mathbf{t}(a) = \mathbf{t}(b) = t$,
- $\mathbf{w}(a) = \mathbf{w}(b)$,
- there exists a bijective function $F : \text{att}_{\mathbf{H}}(a) \rightarrow \text{att}_{\mathbf{H}}(b)$ such that $\forall x \in \text{att}_{\mathbf{H}}(a)$,
 - $\mathbf{w}((x, a)) = \mathbf{w}(F(x), b)$ and
 - $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(x) = \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(F(x))$,

then $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) = \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(b)$.

Counting: $\forall \mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}, \forall a, b \in \mathcal{A}$, if

- $\mathbf{t}(a) = \mathbf{t}(b) = t$,
- $\mathbf{w}(a) = \mathbf{w}(b)$,
- $\text{att}_{\mathbf{H}}(b) = \text{att}_{\mathbf{H}}(a) \cup \{x\}$, with $x \in \mathcal{A} \setminus \text{att}_{\mathbf{H}}(a)$ and:
 - $\forall z \in \text{att}_{\mathbf{H}}(a), \mathbf{w}((z, a)) = \mathbf{w}((z, b))$,
 - $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(x) > 0$ and $\mathbf{w}((x, b)) > 0$,
- $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) > 0$,

then $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) > \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(b)$.

Resilience: $\forall \mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}, \forall a \in \mathcal{A}$ such that $\mathbf{t}(a) = t, \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) > 0$ whenever $\mathbf{w}(a) > 0$.

Four additional principles belong to the second group; their definitions are deferred to (Amgoud, Hanocq, and Lagasque-Schiex 2025) for reasons of space: *Neutrality* states that worthless attacks have no impact on their targets; *Proportionality* asserts that the strength of an argument is sensitive to its initial weight; *Reinforcement* ensures that an argument's strength reflects the quality of its attackers; *Attack-sensitivity* guarantees sensitivity to attacks quality.

Principles of the second group ensure equal treatment of arguments of the same type. But, they do not address how to compare arguments of different types. To highlight the importance of addressing this gap, consider the next example.

Example 2 (Cont) By *Maximality*, $\mathbf{V}(a_i) = \mathbf{V}(b_i) = 1, \forall i \in \{1, 2\}$. Note that the only distinction between a and b lies in their types. As a result, the equivalence principle

does not apply in this case. Assume now that a is deductive and b is practical, hence $t_2 \succ t_1$. Thus, one may expect $\mathbf{V}(a) < \mathbf{V}(b)$ since b would be more robust to attacks than a .

Thus we introduce a novel principle—absent from the existing literature—that explicitly accounts for argument types when comparing arguments of different kinds. This principle states that, all else being equal, an argument whose type is more robust to attacks should be assigned a higher strength value. Naturally, the principle is applicable only when there is at least one significant attack, as in the absence of attacks or in case all attacks are worthless, the Maximality and Neutrality principles would yield the same result—namely, the argument’s initial strength.

Type-Sensitivity: $\forall \mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}, \forall a, b \in \mathcal{A}$, if

- $\mathbf{t}(a) \succ \mathbf{t}(b)$,
- $\mathbf{w}(a) = \mathbf{w}(b)$,
- $\exists x \in \text{att}_{\mathbf{H}}(a)$ such that $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(x) > 0$ and $\mathbf{w}((x, a)) > 0$,
- there exists a bijective function $F : \text{att}_{\mathbf{H}}(a) \rightarrow \text{att}_{\mathbf{H}}(b)$ such that $\forall z \in \text{att}_{\mathbf{H}}(a)$:
 $\mathbf{w}((z, a)) = \mathbf{w}((F(z), b))$ and $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(z) = \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(F(z))$
- $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) > 0$,

then $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) > \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(b)$.

We show that the principles are compatible—that is, they can all be simultaneously satisfied by a single semantics.

Theorem 1. *There exists at least one semantics that satisfies all the principles.*

Hybrid Semantics

Certain types of arguments, such as analogical arguments, are inherently robust to attacks. In these cases, arguments do not collapse under attack but may lose strength in a nuanced, graded manner. Semantics thus should support such graduality. One effective approach is to define semantics using *aggregation functions*, as originally proposed in the seminal work by (Cayrol and Lagasque 2005) for flat graphs—that is, graphs without initial weights. In our paper, we consider three distinct functions, \mathbf{f} , \mathbf{g} , \mathbf{h} , whose roles are as follows: \mathbf{h} aggregates the strength of an attacker with the initial weight of the corresponding attack; \mathbf{g} merges the strengths of all incoming attacks; and \mathbf{f} combines an argument’s initial weight with the aggregated strength of its attacks. Tuples of such functions should satisfy some required constraints—a subset of those used in (Amgoud and Doder 2019).

Definition 4. A well-behaved evaluation method¹ is a tuple $\mathbf{M} = \langle \mathbf{f}, \mathbf{g}, \mathbf{h} \rangle$, where:

- $\mathbf{f} : [0, 1] \times [0, +\infty) \rightarrow [0, 1]$
- $\mathbf{g} : \bigcup_{n=0}^{+\infty} [0, 1]^n \rightarrow [0, +\infty)$
- $\mathbf{h} : [0, 1] \times [0, 1] \rightarrow [0, 1]$

and the following conditions hold.

¹The term well-behaved is used in (Leite and Martins 2011; Amgoud and Doder 2019) for evaluation methods that satisfy other sets of constraints.

1. a) $\mathbf{f}(x_1, y) > \mathbf{f}(x_2, y)$ whenever $x_1 > x_2$,
b) $\mathbf{f}(y, x_1) > \mathbf{f}(y, x_2)$ whenever $x_1 < x_2$ and $y \neq 0$,
c) $\mathbf{f}(x, 0) = x$,
d) $\mathbf{f}(0, x) = 0$,
e) \mathbf{f} is continuous in the second variable,
f) $\lambda \mathbf{f}(x_1, \lambda x_2) < \mathbf{f}(x_1, x_2), \forall \lambda < 1, x_1 \neq 0$.
2. a) $\mathbf{g}() = 0$,
b) $\mathbf{g}(x_1, \dots, x_n) = \mathbf{g}(x_1, \dots, x_n, 0)$,
c) $\mathbf{g}(x_1, \dots, x_n, y) \leq \mathbf{g}(x_1, \dots, x_n, z)$ whenever $y \leq z$,
d) \mathbf{g} is commutative, i.e., $\mathbf{g}(x_1, \dots, x_n) = \mathbf{g}(x_{\rho(1)}, \dots, x_{\rho(n)})$, for any permutation ρ of the set $\{1, \dots, n\}$;
e) \mathbf{g} is continuous.
3. a) $\mathbf{h}(0, x) = \mathbf{h}(x, 0) = 0$,
b) $\mathbf{h}(x, y) > 0$ whenever $xy > 0$,
c) \mathbf{h} is non-decreasing in both variables,
d) \mathbf{h} is continuous in the second variable.
4. $\mathbf{g}(\mathbf{h}(y_1, \lambda x_1), \dots, \mathbf{h}(y_n, \lambda x_n)) \geq \lambda \mathbf{g}(\mathbf{h}(y_1, x_1), \dots, \mathbf{h}(y_n, x_n)), \forall \lambda \in [0, 1]$.

\mathbb{M} is the set of all possible well-behaved evaluation methods, \mathbb{F} (respectively \mathbb{G} and \mathbb{H}) is the set of all functions \mathbf{f} (resp. \mathbf{g} , \mathbf{h}) that satisfy the above corresponding conditions.

Table 1 presents examples of functions that satisfy the above constraints. They form well-behaved tuples.

Proposition 1. Let $\mathbf{M} = \langle \mathbf{f}, \mathbf{g}, \mathbf{h} \rangle$ be a tuple such that $\mathbf{h} \in \{\mathbf{h}_{\text{prod}}, \mathbf{h}_{\text{prod}, \alpha}, \mathbf{h}_{\text{min}}\}$.

- If $\mathbf{f} \in \{\mathbf{f}_{\text{frac}}, \mathbf{f}_{\text{frac}, \alpha}\}$ and $\mathbf{g} \in \{\mathbf{g}_{\text{sum}}, \mathbf{g}_{\text{sum}, \alpha}, \mathbf{g}_{\text{max}}\}$, then $\mathbf{M} \in \mathbb{M}$.
- If $\mathbf{f} = \mathbf{f}_{\text{exp}}$ and $\mathbf{g} = \mathbf{g}_{\text{max}}$, then $\mathbf{M} \in \mathbb{M}$.

Evaluation methods form the foundation of semantics, as they determine the properties that a semantics can exhibit. Thus, certain methods may be well-suited to specific types of arguments, while others may not. For instance, in analogical arguments, each individual attacker affects its target (Amgoud 2020), rendering the function \mathbf{g}_{max} unsuitable. We now introduce a broad family of semantics, which combine multiple evaluation methods tailored to argument types. Since a semantics should be capable of evaluating arguments in any typed graph—including those that involve all \mathbf{k} types—we assume that one evaluation method is associated with each argument type. To ensure generality, types may nonetheless be evaluated using distinct or the same methods.

Definition 5. Let $\text{types} = \{t_1, \dots, t_k\}$, $\mathbf{M}_1, \dots, \mathbf{M}_k \in \mathbb{M}$, with $\mathbf{M}_{i=1, \dots, k} = \langle \mathbf{f}_i, \mathbf{g}_i, \mathbf{h}_i \rangle$. A semantics based on $(\mathbf{M}_1, \dots, \mathbf{M}_k)$ is a function \mathbf{S} mapping any $\mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}$ into $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}} : \mathcal{A} \rightarrow [0, 1]$ such that $\forall a \in \mathcal{A}$,

$$\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) = \mathbf{f}_i \left(\mathbf{w}(a), \mathbf{g}_i \left(\mathbf{h}_i \left(\mathbf{w}(a_1, a), \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a_1) \right), \dots, \mathbf{h}_i \left(\mathbf{w}(a_n, a), \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a_n) \right) \right) \right)$$

where $\mathbf{t}(a) = t_i$ and $\{a_1, \dots, a_n\} = \text{att}_{\mathbf{H}}(a)$. \mathbf{S} is called base semantics if $\mathbf{M}_1 = \dots = \mathbf{M}_k$; otherwise, it is hybrid.

The idea is to associate each type t_i with an evaluation method \mathbf{M}_i , and to assess all arguments of type t_i using \mathbf{M}_i . Let us illustrate the approach with an example.

f	g	h
$\mathbf{f}_{\text{frac}}(x_1, x_2) = \frac{x_1}{1+x_2}$	$\mathbf{g}_{\text{sum}}(x_1, \dots, x_n) = \sum_{i=1}^n x_i$	$\mathbf{h}_{\text{prod}}(x_1, x_2) = x_1 x_2$
$\mathbf{f}_{\text{frac}, \alpha}(x_1, x_2) = \frac{x_1}{1+\alpha x_2}, \alpha > 0$	$\mathbf{g}_{\text{sum}, \alpha}(x_1, \dots, x_n) = \left(\sum_{i=1}^n (x_i)^\alpha\right)^{\frac{1}{\alpha}}, \alpha > 0$	$\mathbf{h}_{\text{prod}, \alpha}(x_1, x_2) = x_1^\alpha x_2, \alpha > 0$
$\mathbf{f}_{\text{exp}}(x_1, x_2) = x_1 e^{-x_2}$	$\mathbf{g}_{\text{max}}(x_1, \dots, x_n) = \max\{x_1, \dots, x_n\}$	$\mathbf{h}_{\text{min}}(x_1, x_2) = \min(x_1, x_2)$

Table 1: Examples of functions **f**, **g**, **h**.

Example 1 (Cont) Assume that $\text{types} = \{t_1, t_2, t_3\}$,

$$\mathbf{t}(d_1) = \mathbf{t}(d_2) = t_1 \quad \mathbf{t}(p_1) = \mathbf{t}(p_2) = t_2.$$

A semantics **S** should be based on three evaluation methods: $(\mathbf{M}_1, \mathbf{M}_2, \mathbf{M}_3)$, with $\mathbf{M}_{i \in \{1,2,3\}} = \langle \mathbf{f}_i, \mathbf{g}_i, \mathbf{h}_i \rangle$ being associated to the type t_i . **S** evaluates the arguments d_1, d_2 by \mathbf{M}_1 and p_1, p_2 by \mathbf{M}_2 . We get the following equations.

- $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(d_2) = \mathbf{f}_1(\mathbf{w}(d_2), \mathbf{g}_1()) = \mathbf{w}(d_2) = 1$
- $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(p_1) = \mathbf{f}_2(\mathbf{w}(p_1), \mathbf{g}_2()) = \mathbf{w}(p_1) = 0.7$
- $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(d_1) = \mathbf{f}_1(\mathbf{w}(d_1), \mathbf{g}_1((\mathbf{h}_1(\mathbf{w}((d_2, d_1))), \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(d_2))))$
- $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(p_2) = \mathbf{f}_2(\mathbf{w}(p_2), \mathbf{g}_2((\mathbf{h}_2(\mathbf{w}((d_1, p_2))), \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(d_1))))$

Note that $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(d_2) = \mathbf{w}(d_2)$ and $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(p_1) = \mathbf{w}(p_1)$, since from the condition 2.a) in Definition 4, $\mathbf{g}_1() = \mathbf{g}_2() = 0$ and from the condition 1.c), $\mathbf{f}_i(x, 0) = x$, for any f_i . Moreover, in this example, \mathbf{M}_3 will not be invoked since there is no argument of type t_3 in the typed argumentation framework.

Notations: Let $\text{Sem} = \text{HSem} \cup \text{BSem}$ denote the set of all semantics formed from \mathbb{M} , where HSem is the set of hybrid semantics and BSem the set of base semantics, i.e., $\text{BSem} = \{\mathbf{S} \text{ is a semantics based on } \underbrace{(\mathbf{M}, \dots, \mathbf{M})}_{\mathbf{k}} \mid \mathbf{M} \in \mathbb{M}\}$.

Remarks: Base semantics ignore argument types and evaluate arguments in a uniform manner. They therefore apply to weighted argumentation graphs of the form $\langle \mathcal{A}, \mathcal{R}, \mathbf{w} \rangle$, which have been discussed in (Egilmmez, Martins, and Leite 2013; Amgoud and Doder 2019). Indeed, for any semantics **S** based on $\underbrace{(\mathbf{M}, \dots, \mathbf{M})}_{\mathbf{k}}$ with $\mathbf{M} = (\mathbf{f}, \mathbf{g}, \mathbf{h})$, for any $\mathbf{H} =$

$$\langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}, a \in \mathcal{A}, \{a_1, \dots, a_n\} = \text{att}_{\mathbf{H}}(a),$$

$$\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) = \mathbf{f} \left(\mathbf{w}(a), \mathbf{g} \left(\mathbf{h} \left(\mathbf{w}(a_1, a), \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a_1) \right), \dots, \mathbf{h} \left(\mathbf{w}(a_n, a), \mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a_n) \right) \right) \right).$$

Table 2 presents examples of base semantics, among which eHbs, eMbs, and Ebs were introduced in (Amgoud and Doder 2019). Our setting generalizes that work by subsuming its base semantics and introduces new ones, such as the parameterized semantics pHbs and many others that rely on functions **h** which violate the constraint $\mathbf{h}(1, x) = x$ imposed in that paper. An example of such function is \mathbf{h}_{max} , where $\mathbf{h}_{\text{max}}(x, y) = \max(x, y)$.

As illustrated by the example, evaluating arguments amounts to solving a system of equations—one per argument. This raises two natural questions: *does a solution exist, and is it unique?* The following theorem answers both

in the affirmative. Uniqueness is equivalent to the following property: if two semantics are based on the same well-behaved evaluation methods, then they coincide. This result guarantees that all semantics in the set Sem are well-defined.

Theorem 2. Let $\text{types} = \{t_1, \dots, t_k\}$, $\mathbf{S} \in \text{Sem}$ with **S** being based on $(\mathbf{M}_1, \dots, \mathbf{M}_k)$, and $\mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}$. For every $a \in \mathcal{A}$ with $\mathbf{t}(a) = t_i$, $\mathbf{M}_i = \langle \mathbf{f}_i, \mathbf{g}_i, \mathbf{h}_i \rangle$ and $\text{att}_{\mathbf{H}}(a) = \{a_1, \dots, a_n\}$, we define the sequence $(\mathbf{u}(a)^{(m)})_{m=1}^{+\infty}$ in the following way :

- $\mathbf{u}(a)^{(1)} = \mathbf{w}(a)$
- $\mathbf{u}(a)^{(m+1)} = \mathbf{f}_i \left(\mathbf{w}(a), \mathbf{g}_i \left(\mathbf{h}_i \left(\mathbf{w}(a_1, a), \mathbf{u}(a_1)^{(m)} \right), \dots, \mathbf{h}_i \left(\mathbf{w}(a_n, a), \mathbf{u}(a_n)^{(m)} \right) \right) \right)$

For every $a \in \mathcal{A}$, $(\mathbf{u}(a)^{(m)})_{m=1}^{+\infty}$ converges and $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) = \lim_{m \rightarrow +\infty} \mathbf{u}(a)^{(m)}$.

The set Sem is non-empty since it includes semantics constructed from the well-behaved function combinations identified in Proposition 1 from Table 1.

Proposition 2. Let **S** be a semantics based on a tuple $(\mathbf{M}_1, \dots, \mathbf{M}_k)$ such that every \mathbf{M}_i is a well-behaved tuple in Proposition 1. It holds that $\mathbf{S} \in \text{Sem}$.

The following result shows that the strength of any argument is bounded by its initial weight, irrespective of its type.

Theorem 3. Let $\mathbf{S} \in \text{Sem}$ and $\mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}$. For every $a \in \mathcal{A}$, $\mathbf{V}_{\mathbf{H}}^{\mathbf{S}}(a) \in [0, \mathbf{w}(a)]$.

Before analyzing the new semantics, we introduce three relations that order the functions in \mathbb{F} , \mathbb{G} , and \mathbb{H} according to their *level of generosity*, from most to least generous.

Definition 6. We define \succ_f (resp. \succ_g, \succ_h) as a preorder on \mathbb{F} (resp. \mathbb{G}, \mathbb{H}), s.t. $\forall \mathbf{f}_1, \mathbf{f}_2 \in \mathbb{F}, \forall \mathbf{g}_1, \mathbf{g}_2 \in \mathbb{G}, \forall \mathbf{h}_1, \mathbf{h}_2 \in \mathbb{H}$,

- $\mathbf{f}_1 \succ_f \mathbf{f}_2$ iff $\forall x, y$ such that $xy > 0$, $\mathbf{f}_1(x, y) > \mathbf{f}_2(x, y)$.
- $\mathbf{g}_1 \succ_g \mathbf{g}_2$ iff $\forall x_1, \dots, x_n \in [0, 1]$ such that $\exists i, j \in [1, n], x_i \neq x_j$, $\mathbf{g}_1(x_1, \dots, x_n) > \mathbf{g}_2(x_1, \dots, x_n)$.
- $\mathbf{h}_1 \succ_h \mathbf{h}_2$ iff $\forall x, y \in [0, 1]$ such that $xy > 0$, $\mathbf{h}_1(x, y) > \mathbf{h}_2(x, y)$.

Let us now introduce some additional properties for functions **g** and **h**.

Definition 7. Let $\mathbf{g} : \bigcup_{n=0}^{+\infty} [0, 1]^n \rightarrow [0, +\infty)$ and $\mathbf{h} : [0, 1] \times [0, 1] \rightarrow [0, 1]$.

$\mathbf{V}_H^{\text{eHbs}}(a) = \frac{\mathbf{w}(a)}{1 + \sum_{(b_i, a) \in \mathcal{R}} \min(\mathbf{w}((b_i, a)), \mathbf{V}_H^{\text{eHbs}}(b_i))}$	$\mathbf{V}_H^{\text{eMbs}}(a) = \frac{\mathbf{w}(a)}{1 + \max_{(b_i, a) \in \mathcal{R}} \min(\mathbf{w}((b_i, a)), \mathbf{V}_H^{\text{eMbs}}(b_i))}$
$\mathbf{V}_H^{\text{pHbs}}(a) = \frac{\mathbf{w}(a)}{1 + \alpha \sum_{(b_i, a) \in \mathcal{R}} \min(\mathbf{w}((b_i, a)), \mathbf{V}_H^{\text{pHbs}}(b_i))}, \alpha \in [1, +\infty[$	$\mathbf{V}_H^{\text{Ebs}}(a) = \mathbf{w}(a) e^{-\max_{(b_i, a) \in \mathcal{R}} \mathbf{w}((b_i, a)) \times \mathbf{V}_H^{\text{Ebs}}(b_i)}$

Table 2: Examples of base semantics (for eHbs, pHbs and eMbs, $\mathbf{h} = \mathbf{h}_{\min}$, and for Ebs, $\mathbf{h} = \mathbf{h}_{\text{prod}}$).

- \mathbf{g} is strictly increasing iff $\mathbf{g}(x_1, \dots, x_n, y) > \mathbf{g}(x_1, \dots, x_n, z)$ whenever $y > z$.
- \mathbf{h} is strictly increasing in the first variable iff $\mathbf{h}(x_1, y) > \mathbf{h}(x_2, y)$ whenever $x_1 > x_2$ and $y \neq 0$.
- \mathbf{h} is strictly increasing in the second variable iff $\mathbf{h}(y, x_1) > \mathbf{h}(y, x_2)$ whenever $x_1 > x_2$ and $y \neq 0$.

The next result evaluates the newly introduced semantics against the thirteen principles presented in the previous section. It shows that all semantics satisfy nine of these principles unconditionally for all types. The remaining four principles are satisfied under certain constraints on the evaluation methods employed by the semantics. For instance, type-sensitivity is ensured when the semantics uses identical aggregation functions \mathbf{g} and \mathbf{h} , along with more generous functions \mathbf{f} for argument types that are considered more robust.

Theorem 4. Let $\text{types} = \{t_1, \dots, t_k\}$, $\mathbf{S} \in \text{Sem}$ such that \mathbf{S} is based on $(\mathbf{M}_1, \dots, \mathbf{M}_k)$.

- \mathbf{S} satisfies independence, directionality, maximality and weakening, anonymity, equivalence, neutrality, proportionality, and resilience.
- If $\mathbf{S} \in \text{BSem}$, then \mathbf{S} violates type-sensitivity.
- If $\mathbf{S} \in \text{HSem}$, then \mathbf{S} satisfies type-sensitivity if $\forall t_i, t_j \in \text{types}$ s.t. $t_i \succ t_j$, one of the following conditions holds:
 - $\mathbf{f}_i \succ_f \mathbf{f}_j$, $\mathbf{g}_i = \mathbf{g}_j$, and $\mathbf{h}_i = \mathbf{h}_j$.
 - $\mathbf{f}_i = \mathbf{f}_j$, $\mathbf{g}_j \succ_g \mathbf{g}_i$, and $\mathbf{h}_i = \mathbf{h}_j$.
 - $\mathbf{f}_i = \mathbf{f}_j$, $\mathbf{g}_i = \mathbf{g}_j$ with \mathbf{g}_i being strictly increasing, and $\mathbf{h}_j \succ_h \mathbf{h}_i$.
- If for any \mathbf{M}_i \mathbf{g}_i is strictly increasing, then:
 - \mathbf{S} satisfies t_i -counting.
 - If for any \mathbf{M}_i \mathbf{h}_i is strictly increasing in the first variable, then \mathbf{S} satisfies t_i -attack-sensitivity.
 - If for any \mathbf{M}_i \mathbf{h}_i is strictly increasing in the second variable, then \mathbf{S} satisfies t_i -reinforcement.

The above results are general, as they abstract from aggregation functions. In what follows, we investigate the properties of semantics that are grounded on the functions of Table 1. Recall that the set of types is partitioned into \mathbf{m} equivalence classes, $\mathbf{T}_1, \dots, \mathbf{T}_m$, where $\mathbf{T}_m \succ \dots \succ \mathbf{T}_1$.

Theorem 5. Let $\text{types} = \{t_1, \dots, t_k\} = \mathbf{T}_1 \cup \dots \cup \mathbf{T}_m$, $F = \{\mathbf{f}_{\text{frac}}, \mathbf{f}_{\text{frac}, \alpha}, \mathbf{f}_{\text{exp}}\}$, $G = \{\mathbf{g}_{\text{sum}}, \mathbf{g}_{\text{sum}, \alpha}, \mathbf{g}_{\text{max}}\}$ and $H = \{\mathbf{h}_{\text{prod}}, \mathbf{h}_{\text{prod}, \alpha}, \mathbf{h}_{\min}\}$. Let \mathbf{S} be a semantics based on $(\mathbf{M}_1, \dots, \mathbf{M}_k)$ such that $\forall i \in [1, k]$, $\mathbf{M}_i = (\mathbf{f}_i, \mathbf{g}_i, \mathbf{h}_i)$, $\mathbf{f}_i \in F$, $\mathbf{g}_i \in G$, and $\mathbf{h}_i \in H$.

- \mathbf{S} satisfies independence, directionality, maximality, weakening, anonymity, equivalence, neutrality, proportionality, and resilience.

- If $\mathbf{S} \in \text{BSem}$, then \mathbf{S} violates type-sensitivity.
- If $\mathbf{g}_i \in \{\mathbf{g}_{\text{sum}}, \mathbf{g}_{\text{sum}, \alpha}\}$, then:
 - \mathbf{S} satisfies t_i -counting (t_i is the type associated to \mathbf{M}_i).
 - If $\mathbf{h}_i \in \{\mathbf{h}_{\text{prod}}, \mathbf{h}_{\text{prod}, \alpha}\}$, then \mathbf{S} satisfies t_i -reinforcement and t_i -attack-sensitivity.
- \mathbf{S} satisfies type-sensitivity if the following hold:
 - $\forall l \in [1, \mathbf{m}]$, $\forall t_i \in \mathbf{T}_l$, $\mathbf{f}_i = \mathbf{f}_{\text{frac}, \alpha_l}$, with $\alpha_l > 0$,
 - $\alpha_1 > \dots > \alpha_m$,
 - $\forall i, j \in [1, \mathbf{k}]$, $\mathbf{g}_i = \mathbf{g}_j$ and $\mathbf{h}_i = \mathbf{h}_j$.

The above result illustrates a case where type-sensitivity is satisfied—namely, when only the function \mathbf{f} varies across types. A natural question arises: would a semantics based on $\mathbf{M}_1 = \langle \mathbf{f}, \mathbf{g}_{\text{max}}, \mathbf{h} \rangle$ and $\mathbf{M}_2 = \langle \mathbf{f}, \mathbf{g}_{\text{sum}}, \mathbf{h} \rangle$ also satisfy the principle? The answer is negative due to the equality $\mathbf{g}_{\text{max}}(x) = \mathbf{g}_{\text{sum}}(x)$. As a result, arguments with a single attacker receive the same valuation regardless of their type, thereby violating type-sensitivity.

Instances of the family

Theorem 5 unveils hybrid semantics—instances of HSem —that satisfy all thirteen principles. Each such semantics is defined by evaluation methods that share a common aggregation function \mathbf{h} from the set $\{\mathbf{h}_{\text{prod}}, \mathbf{h}_{\text{prod}, \alpha}\}$ and a common function $\mathbf{g} \in \{\mathbf{g}_{\text{sum}}, \mathbf{g}_{\text{sum}, \alpha}\}$. Additionally, it employs \mathbf{m} distinct functions $\mathbf{f}_{\text{frac}, \alpha_1}, \dots, \mathbf{f}_{\text{frac}, \alpha_m}$, one for each equivalence class of argument types, where the parameters satisfy the inequalities $\alpha_1 > \dots > \alpha_m$, and \mathbf{m} is the number of equivalence classes ($\text{types} = \mathbf{T}_1 \cup \dots \cup \mathbf{T}_m$). In this setup, all types within the same equivalence class are evaluated using the same evaluation method, while more robust types are assigned less generous evaluation functions \mathbf{f} . Below a schematic summary of these semantics.

$$\begin{array}{ccc} \underbrace{\mathbf{T}_m}_{t_{m_1}, \dots, t_{m_l}} & \succ \dots \succ & \underbrace{\mathbf{T}_1}_{t_{1_1}, \dots, t_{1_n}} \\ \langle \mathbf{f}_{\text{frac}, \alpha_m}, \mathbf{g}, \mathbf{h} \rangle & \dots & \langle \mathbf{f}_{\text{frac}, \alpha_1}, \mathbf{g}, \mathbf{h} \rangle \\ \alpha_m & < \dots < & \alpha_1 \end{array}$$

To sum up, such semantics use \mathbf{m} distinct evaluation methods and every argument whose type belongs to the equivalence class \mathbf{T}_j is evaluated using $\mathbf{M}_j = \langle \mathbf{f}_{\text{frac}, \alpha_j}, \mathbf{g}, \mathbf{h} \rangle$. In what follows, we discuss an example of such semantics, called THbs, and that is grounded on $\mathbf{f}_{\text{frac}, \alpha}$, \mathbf{g}_{sum} , and \mathbf{h}_{prod} .

Definition 8 (THbs). Let $\text{types} = \mathbf{T}_1 \cup \dots \cup \mathbf{T}_m$, $\mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}$, and $a \in \mathcal{A}$.

$$\mathbf{V}_H^{\text{THbs}}(a) = \frac{\mathbf{w}(a)}{1 + \alpha_i \sum_{b \in \text{att}_H(a)} \mathbf{w}((b, a)) \mathbf{V}_H^{\text{THbs}}(b)},$$

Related Work and Summary

where $\mathbf{t}(a) \in \mathbf{T}_i$, $\alpha_i = \mathbf{m} - i + 1$, and $\sum_{b \in \text{att}_{\mathbf{H}}(a)} \mathbf{w}((b, a)) \mathbf{V}_{\mathbf{H}}^{\text{THbs}}(b) = 0$ if $\text{att}_{\mathbf{H}}(a) = \emptyset$.

Note that if $\mathbf{t}(a) \in \mathbf{T}_m$, then $\alpha_m = 1$, if $\mathbf{t}(a) \in \mathbf{T}_{m-1}$, then $\alpha_{m-1} = 2$, and if $\mathbf{t}(a) \in \mathbf{T}_1$, then $\alpha_1 = m$. We now illustrate the semantics using the running example.

Example 1 (Cont) Recall that $\text{types} = \{t_1, t_2, t_3\}$. Let:

$$\mathbf{T}_3 = \{t_2\} \succ \mathbf{T}_2 = \{t_3\} \succ \mathbf{T}_1 = \{t_1\}.$$

Note that $\alpha_1 = 3$, $\alpha_2 = 2$, and $\alpha_3 = 1$. Recall that $\mathbf{t}(d_1) = \mathbf{t}(d_2) = t_1$ and $\mathbf{t}(p_1) = \mathbf{t}(p_2) = t_2$.

- $\mathbf{V}_{\mathbf{H}}^{\text{THbs}}(d_2) = \mathbf{w}(d_2) = 1$,
- $\mathbf{V}_{\mathbf{H}}^{\text{THbs}}(p_1) = \mathbf{w}(p_1) = 0.7$
- $\mathbf{V}_{\mathbf{H}}^{\text{THbs}}(d_1) = \frac{\mathbf{w}(d_1)}{1 + \alpha_1 \mathbf{w}((d_2, d_1)) \mathbf{V}_{\mathbf{H}}^{\text{THbs}}(d_2)} = \frac{0.98}{1 + 3 \times 1} = 0.24$
- $\mathbf{V}_{\mathbf{H}}^{\text{THbs}}(p_2) = \frac{\mathbf{w}(p_2)}{1 + \alpha_3 \mathbf{w}((d_1, p_2)) \mathbf{V}_{\mathbf{H}}^{\text{THbs}}(d_1)} = \frac{1}{1 + 1 \times 0.245} = 0.8$.

Note that $\mathbf{V}_{\mathbf{H}}^{\text{THbs}}(p_2) > \mathbf{V}_{\mathbf{H}}^{\text{THbs}}(p_1)$.

The example demonstrates that the hybrid semantics THbs produces the expected outcome—namely, recommending picnic (pi) over museum (mu). This underscores the crucial role of argument types in argument-based reasoning.

Example 2 (Cont) Assume that $\text{types} = \{t_1, t_2, t_3\}$, $\mathbf{T}_3 = \{t_2\} \succ \mathbf{T}_2 = \{t_3\} \succ \mathbf{T}_1 = \{t_1\}$. So, $\alpha_1 = 3$, $\alpha_2 = 2$ and $\alpha_3 = 1$. Recall that $\mathbf{t}(a) = t_1$, $\mathbf{t}(b) = t_2$, $\mathbf{t}(a_1) = \mathbf{t}(a_2) = \mathbf{t}(b_1) = \mathbf{t}(b_2) = t_3$.

- $\mathbf{V}_{\mathbf{H}}^{\text{THbs}}(a_1) = \mathbf{V}_{\mathbf{H}}^{\text{THbs}}(a_2) = \mathbf{V}_{\mathbf{H}}^{\text{THbs}}(b_1) = \mathbf{V}_{\mathbf{H}}^{\text{THbs}}(b_2) = 1$.
- $\mathbf{V}_{\mathbf{H}}^{\text{THbs}}(a) = \frac{\mathbf{w}(a)}{1 + \alpha_1 \sum_{i=1,2} \mathbf{w}((a_i, a)) \mathbf{V}_{\mathbf{H}}^{\text{THbs}}(a_i)} = \frac{1}{1 + 3 \times 2} = 0.14$.
- $\mathbf{V}_{\mathbf{H}}^{\text{THbs}}(b) = \frac{\mathbf{w}(b)}{1 + \alpha_3 \sum_{i=1,2} \mathbf{w}((b_i, b)) \mathbf{V}_{\mathbf{H}}^{\text{THbs}}(b_i)} = \frac{1}{1 + 2} = 0.33$.

Note that even if the arguments a and b have the same initial weights and are targeted by equally strong attacks, the effects of those attacks differ due to the types of the arguments: the type $\mathbf{t}(b)$ is more robust to attacks than $\mathbf{t}(a)$.

Theorem 6. The semantics THbs satisfies all the principles.

Hybrid semantics in the set Sem are grounded in base semantics such as eHbs, pHbs, eMbs, and Ebs. In what follows, we analyze the worst-case complexity of computing these semantics. The convergence result (Theorem 2) provides a simple iterative algorithm to perform this computation. It also guarantees the existence of an upper bound B on the number of iterations required, where B depends on the functions \mathbf{f} , \mathbf{g} , and \mathbf{h} , as well as the target precision ϵ .

Theorem 7. Let $\mathbf{H} = \langle \mathcal{A}, \mathbf{t}, \mathcal{R}, \mathbf{w} \rangle \in \text{TAF}$ s.t. $N = |\mathcal{A}|$, $\mathbf{S} \in \{\text{eHbs}, \text{pHbs}, \text{eMbs}, \text{Ebs}\}$, B the bound of the number of iterations used in the computation of \mathbf{S} over \mathbf{H} . The complexity in the worst case of the computation of \mathbf{S} is $\mathcal{O}(B \times N^2)$.²

The bound B is an unknown parameter that cannot be determined in general, as it depends on various factors, including the topology of the graph.

² \mathcal{O} is the standard Landau notation commonly used in algorithmic complexity theory (see (Cormen, Leiserson, and Rivest 1990)).

We introduced hybrid semantics, which integrate multiple evaluation strategies tailored to different types of arguments. We proposed a broad family of semantics, some of which satisfy all the principles, including type-sensitivity. To our knowledge, no existing work in the literature has explored the integration of types into the evaluation of argument strength. This paper offers the first contribution in that direction. However, the combination of semantics has been tackled in two prior works, albeit for different purposes.

In (Bonzon et al. 2018), the authors advocate for maximizing the comparability of arguments with respect to their strength. To achieve this, they apply separately both a gradual semantics and an extension-based semantics to the same argumentation framework. They use the ranking induced by one semantics as a basis and resolve ties using the ranking provided by the other. However, this approach does not address the central problem raised in our work, as it disregards argument types. For instance, in Example 1, applying Hbs yields the ranking: $d_2 \succ p_1 \succ p_2 \succ d_1$. Since no ties arise, a second semantics is unnecessary. Nevertheless, as discussed in the introduction, this result is unsatisfactory because it evaluates both types of arguments using the same criteria. In Example 2, Hbs produces the scores: $\delta(a) = \delta(b) = \frac{1}{3}$, implying that a and b are equally strong. Moreover, the grounded extension is $\{a_1, a_2, b_1, b_2\}$, meaning that both a and b are rejected and therefore equally weak under grounded semantics. Hence, this approach fails to resolve the tie. In contrast, our hybrid semantics fully integrate different evaluation methods in a type-sensitive manner. In Example 2, this results in $\delta(a) < \delta(b)$, as the type of b is more resilient to attacks than the type of a .

The closest related work is the recent study by (Amgoud, Doder, and Lagasque-Schiex 2024) on gradual semantics for higher-order attacks. That paper proposes semantics that evaluate both arguments and attacks, some semantics treat them differently, but do not incorporate types. Our approach builds on a similar principle of nesting distinct evaluation functions within a unified semantics, while extending it to account for multiple argument types.

It enables a more nuanced assessment of arguments, which is particularly crucial in contexts where different types of arguments interact. For example, decision systems such as those proposed in (Amgoud, Dimopoulos, and Moraitis 2008; Hadidi, Dimopoulos, and Moraitis 2010; Dimopoulos, Maily, and Moraitis 2019) distinguish between two main types of arguments: deductive (or epistemic) arguments and practical arguments. However, both are evaluated using extension semantics, which, as illustrated in our running example, can lead to counter-intuitive outcomes.

In our setting, we assume a fixed total preorder over types. While this assumption is motivated by well-established comparisons, a fully specified ordering remains an open issue. As part of our future work, we aim to derive such a preorder through a theoretical analysis of the argument schemes presented in (Walton, Reed, and Macagno 2008).

Acknowledgments

This work was supported by the AI Interdisciplinary Institute ANITI, funded by the French program AI Cluster – ANR-23-IACL-0002.

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