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Edge-Binary Public Goods Games

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Abstract

Binary networked public goods games model situations in which players can choose whether to participate in an action at some cost which benefits players in their immediate vicinity within some typically social or infrastructural network. An important underlying assumption for this model is that participation in an action impacts *the entire* vicinity of participating players. However, there are numerous natural settings in which participation influences only a subset of the neighbors and is in fact more "interaction-specific". In this work, we introduce a type of game that is more appropriate in such settings. We initiate the investigation of these games, by studying the complexity of deciding existence of their Nash equilibria in general and with respect to well-motivated structural restrictions on the network. The outcome is a comprehensive understanding of the complexity of computing Nash equilibria with respect to any combination of three natural properties of the network structure.

Introduction

Binary networked public goods games (BNPGs) have been established as a natural and elementary model for situations in which players can choose whether to participate in some action at a certain cost which benefits the players in their closed neighborhood. This is useful for modeling herd immunity, crime reporting, anti-pollution efforts or information dissemination, investment in security or maintenance and more (a collection of according scenarios can be found in e.g. (Samuelson 1954; Foster and Rosenzweig 1995; Glaeser, Sacerdote, and Scheinkman 1996; Morison et al. 2024; Ding et al. 2024)). For a concrete example, while getting a vaccination may incur some kind of inconvenience or *cost* for a single individual, it benefits from individuals that it interacts with (including itself), i.e. individuals in its closed neighborhood in some social network, being vaccinated. Another exemplary application mentioned frequently – in particular in light of the COVID-19 pandemic – replaces the action of getting vaccinated with that of wearing a mask (Yu, Kempe, and Vorobeychik 2021; Yong and Choy 2021; Li et al. 2023; Papadimitriou and Peng 2023; Maiti and Dey 2024b,a). At surface level, these two situations share a lot of similarities, but there is an important fundamental difference: being vaccinated is indeed a binary choice in the sense that it benefits *every* individual in one's closed neighborhood, whereas wearing a mask is a binary choice that can be made per individual one interacts with.

Similar situations occur in other contexts previously used to motivate binary public goods games where choices to participate are often not truly binary in the sense that players participate in a way that potentially benefits their entire closed neighborhood: for example, there is no reason to assume and no empirical evidence suggesting that individuals report only all or no crimes or participate in all or no anti-pollution efforts in their area, disseminate information using all or none of their available channels, or secure and maintain all of their network connections. This cannot be captured by classic networked binary public goods games, which is why we propose a natural adaptation which allows agents to make binary choices about their participation impacting each edge in their closed neighborhood. We refer to such choices as *edge-binary* and, accordingly, call the games we introduce *edge-binary (networked) public goods games*.

Technical Contributions. Beside the conceptual contribution of introducing edge binary public goods games, we initiate the complexity-theoretic investigation of the arguably most immediate algorithmic problem arising for any type of game, namely the complexity of deciding the existence of Nash equilibria. We start by classifying its general complexity as NP-complete. In view of this hardness, our goal is to carry out a more detailed complexity investigation on well-studied and practically relevant graph classes to establish a detailed theoretical baseline for computational complexity of the problem. Specifically, we investigate the influence of *planarity*, the *maximum vertex degree* and the *treewidth* of the network on the complexity of the problem.
Planarity naturally occurs in many infrastructural networks and is a classic feature considered in urban studies (Lin and Ban 2013; Liang and Kang 2021; Barthelemy and Boeing 2024), but also is a seemingly coincidental and for that reason slightly surprising property of many techno-social networks (Bowden et al. 2011). A small **maximum vertex degree** is relevant for settings in which the participation of each individual benefits at most a small number of other individuals, as is frequently the case in road networks (Jepsen, Jensen, and Nielsen 2019), networks modeling non-digital

interactions (Valle et al. 2007) and server networks (Guo et al. 2013). **Treewidth** is a parameter intuitively defined to measure the structural similarity of a graph to a tree. Not only has it proven to be an algorithmically very useful and theoretically fundamental parameter which is increasingly investigated as a parameter for a wide variety of problems in computational social choice (Hébert-Johnson et al. 2024; Eiben et al. 2023; Gupta, Saurabh, and Zehavi 2022; Arrighi et al. 2021; Deligkas et al. 2021; Peters 2016) including binary networked public goods games (Papadimitriou and Peng 2023; Maiti and Dey 2024b), it is also conveniently small in graphs arising in various practical settings that are relevant to public goods games (Maniu, Senellart, and Jog 2019; Chatterjee, Goharshady, and Goharshady 2019; Goharshady and Mohammadi 2020).

We show that the general NP-hardness of the problem persists, even for planar graphs with a small constant maximum vertex-degree. On the other hand, for graphs of constant treewidth, we can obtain a polynomial-time algorithm. The degree of its polynomial runtime depends on the treewidth, and we show that under standard complexity assumptions, this cannot be avoided, even on planar graphs. To the best of our knowledge, this is the first problem in the area of algorithmic game theory for which this kind of hardness (called W-hardness) is established on planar instances. As such, it may serve as a suitable starting problem to reduce from to obtain similar hardness results for other problems involving utilities in planar graphs and hence be of broader interest. Finally, we prove that bounding the treewidth together with the maximum vertex degree results in polynomial-time solvability where the degree of the polynomial runtime does not depend on the treewidth or maximum vertex degree. This yields a comprehensive understanding of the problem's complexity with respect to any combination of planarity, maximum vertex degree and treewidth.

Having classified the complexity-theoretic behavior on practically-relevant graph classes, we will have found examples for the edge-binary version of the considered problem being as difficult or more difficult than classic player-binary BNPG. We conclude by highlighting a setting in which the edge-binary analogue is computationally easier than BNPG.

Related Work. The behavior of agents or – as they are more commonly called in game theory literature, players – when contributing to and profiting from public goods has been an important branch in computational social choice and game theory; see (Ledyard 1994) for a classic survey, or (Buchholz and Sandler 2021) for an exposition of the increasing importance of theoretical models for strategic behavior relating to public goods.

On the other hand, game theoretic settings in which not each player is impacted by the actions of every other player but only those close to it in some social, infrastructural or interaction network are ubiquitous. *Graphical games* (see (Kearns 2007) for a textbook overview) are a general framework of multiplayer games which models this.

Networked public goods games (Bramoullé and Kranton 2007) constitute a special type of graphical games combining them with the setting of public goods games. Within networked public goods games, binary networked pubic goods games represent a particularly well-studied subclass; due to their elementary nature they offer a canonical starting point for an investigation of fundamental properties.

Various restrictions on the underlying network structure or the best-response functions of players have been used to obtain a better understanding of game theoretic, algorithmic and modeling questions around graphical games and variants of networked public goods games (Dilkina, Gomes, and Sabharwal 2007; Galeotti et al. 2010; Feldman et al. 2013; Wunder, Suri, and Watts 2013; Kempe, Yu, and Vorobeychik 2020; Yu, Kempe, and Vorobeychik 2021; Gilboa and Nisan 2022; Li et al. 2023). Topically, our technical contributions are closest to the extensive recent research on the complexity of deciding the existence of Nash equilibria in BNPGs or variants thereof under a wide variety of structural constraints (Yu et al. 2020; Papadimitriou and Peng 2023; Klimm and Stahlberg 2023; Maiti and Dey 2024b).

The edge-binary public goods games that we define also fall into the very general class of graphical games but are incomparable in terms of their theoretical behavior to BNPGs (a problem abbreviated as BNPG-PSNS), which is exemplified by the complexity of deciding the existence of Nash equilibria in some of our results. None of our results are easily transferred from BNPGs.

Note that if each binary edge choice has the same constant cost for every player, deciding the existence of Nash equilibria in our edge-binary public goods games can be viewed as a graph theoretic problem which is a directed version of the so called GENERAL FACTOR problem (Lovász 1972) (see the Conclusions for a more detailed explanation). While GENERAL FACTOR is well studied in graph and complexity theory, to the best of our knowledge its directed version has not yet been considered and previous algorithmic results on GENERAL FACTOR do not readily translate to the problem considered by us in the setting of uniform edge costs.

Problem Definition

In the following we use standard terminology from game theory (Nisan et al. 2007), graph theory (Diestel 2012) and parameterized complexity theory (Cygan et al. 2015). All graphs we consider will be simple, without loops and with the exception of the proof of Theorem 12 undirected. We use $vw = wv$ to denote an edge between vertices u and w. Moreover, for a graph G and $v \in V(G)$ we denote by $E(G)_v$ the set of edges that are incident to v, by $N_G(v)$ the neighborhood of v, by $\deg_G(v) := |E(G)_v|$ and by $\Delta_G := \max_{v \in V(G)} \deg_G(v)$ a vertex' degree and the graph's maximum vertex degree, respectively. When G is clear from the context, we omit it from the subscript. For two sets X, Y, we use X^Y to denote the set of all functions from Y to X.

A *nice tree decomposition* for a graph G is a pair (T, χ), where T is a rooted tree, χ is a function mapping each node of T to a subset of $V(G)$ satisfying: (1) $\cup_{t \in V(T)} \chi(t) = V(G)$; (2) for every $xy \in E(G)$, there exists $t \in V(T)$ such that $\{x, y\} \subseteq \chi(t)$; (3) for each $x \in V(G)$, the subgraph of

T induced by the vertex set $\{t \in V(T) \mid x \in \chi(t)\}$ is connected. Moreover, T has only four types of nodes: leaf, introduce, forget and join. A *leaf* node t is a node with no child and such that $\chi(t) = \emptyset$. An *introduce* node t has exactly one child t' and $\chi(t) = \chi(t') \cup \{v\}$ for a vertex $v \notin \chi(t')$. A *forget* node t has exactly one child t' and $\chi(t) = \chi(t') \setminus \{v\}$, for a vertex $v \in \chi(t')$. A *join* node t has exactly two children t_1 and t_2 and $\chi(t) = \chi(t_1) = \chi(t_2)$. The *width* of (T, χ) is $\max_{t \in V(T)} |\chi(t)| - 1$. The *treewidth* of G is the minimum width over all nice tree decompositions of G. The treewidth of G is denoted by $\mathrm{tw}(G)$.

A parameterized problem is *fixed-parameter tractable* (FPT) (resp. XP) if it admits an algorithm running in time $f(k)n^c$ (resp. $f(k)n^{g(k)}$), where n is the size of the input, k is the parameter and c is a constant.

Definition 1 (EBPG). *Given a graph G and non-decreasing $g_v : [2|N(v)|] \cup \{0\} \to \mathbb{R}$ which we refer to as* gain *functions and $c_v : N(v) \to \mathbb{R}^+$ which we refer to as* cost *functions for each $v \in V(G)$, we define the* edge binary public goods game *(EBPG for short) $\mathcal{I} = (G, \{g_v \mid v \in V(G)\}, \{c_v \mid v \in V(G)\})$ as follows. We consider $V(G)$ as the set of* players *and let each $v \in V(G)$ have* strategy *set $\{0,1\}^{E_v}$. We will say that a player v* chooses *an edge e whenever the v's strategy maps e to 1. The* utility *for player $w \in V(G)$ of a strategy profile $(x_v)_{v \in V(G)}$, i.e. each $(x_v)_{v \in V(G)}$ such that each $x_v \in \{0,1\}^{E_v}$, is defined as*

$$U_w((x_v)_{v \in V(G)}) = g_w(|\{v \in N(w) \mid x_w(vw) = 1\}| + |\{v \in N(w) \mid x_v(vw) = 1\}|) - \sum_{v \in N(w)} c_w(v) \cdot x_w(vw).$$

Intuitively, in EBPGs, the vertices are players which can choose arbitrary subsets of their incident edges at some additive cost and receive payoffs based on the number of edges chosen by them and their neighbors (a higher number is never worse) minus this cost.

Let us remark that our definition makes a difference for the utility of a player if an edge incident to that player is chosen by exactly one or both its endpoints. While it is conceivable that this need not be the case (see also a short remark in the Conclusion), we find this to be plausible in all applications mentioned in the introduction that motivated us to propose EBPGs. A natural side effect of this choice is that in networks in which every player is incident to precisely one edge an EBPG is immediately equivalent to the BNPG with the same gain and cost functions.

Definition 2 (Nash equilibrium). *Given a EBPG $\mathcal{I} = (G, \{g_v \mid v \in V(G)\}, \{c_v \mid v \in V(G)\})$, a strategy profile $(x_v)_{v \in V(G)}$ is a* (pure-strategy) Nash equilibrium *for $\mathcal{I}$ if there is no $w \in V(G)$ and strategy $x'_w \in \{0,1\}^{E_w}$ such that*

$$U_w((\tilde{x}_v)_{v \in V(G)}) > U_w((x_v)_{v \in V(G)}) \text{ for}$$

$$\tilde{x}_v = \begin{cases} x_v & \text{if } v \neq w \\ x'_w & \text{otherwise.} \end{cases}$$

If such w and x'_w exist, we call x'_w a Nash deviation *for w.*

We study the problem of deciding the existence of Nash equilibria in EBPGs.

EBPG-PSNS	
Instance	An EBPG $\mathcal{I}$
Question	Is there a Nash equilibrium for $\mathcal{I}$?

Theorem 3 ($\star$). EBPG-PSNS *is in* NP.

Inspired by analogous restrictions for classic BNPGs, we call an instance of EBPG-PSNS *homogeneous* if all players have equal gain functions, and *fully homogeneous* if all players have equal gain functions and all cost functions map uniformly to some constant.

In the case of the Binary Networked Public Goods games, Maiti and Dey showed that when the network is a path, a cycle, a biclique, or complete graph, and all players have the same utility functions, then such games always have a Nash equilibrium. Here we show that this is not the case for any of these four graph classes in our variant of the problem. In fact, we consider instances of EBPG-PSNS in which G is a path on three vertices (which is also a biclique) or a complete graph on three vertices (which is also a cycle). We show that even if the utility functions of the players are the same, we are not guaranteed to have a Nash equilibrium.

Observation 4. *There exists a fully homogeneous instance $(G, \{g_v\}, \{c_v\})$ of* EBPG-PSNS *that does not have a Nash equilibrium. Moreover, the graph G in this instance is a path (or a biclique) on three vertices.*

Proof. Let G be the graph with $V(G) = \{a, b, c\}$ and $E(G) = \{ab, bc\}$. We define $\{g_v\}$ and $\{c_v\}$ as follows. If $v \in \{a, b, c\}$, we let $c_v \equiv 1$ and

$$g_v(i) = \begin{cases} 0 & \text{if } i = 0 \\ 2 & \text{if } i \in \{1,2,3\} \\ 100 & \text{if } i = 4. \end{cases}$$

We now proceed to show that this instance has no Nash equilibrium. Suppose for a contradiction that there exists a Nash equilibrium $(x_v)_{v \in V(H)}$.

If both $x_a(ab) = 0$ and $x_c(bc) = 0$, then we conclude $x_b(ab) = 1$ or $x_b(bc) = 1$, otherwise b would have utility zero, and it would have an incentive to choose one of its incident edges to increase its gain to two and its cost to one. However, if $x_b(ab) = 1$, c has an incentive to choose its edge to b, as this would increase its utility from zero to one. This contradicts the assumption that we had an equilibrium in which $x_c(bc) = 0$. The same holds if $x_b(bc) = 1$.

If both $x_a(ab) = 1$ and $x_c(bc) = 1$, then we have that $x_b(ab) = x_b(bc) = 1$, otherwise b would have an incentive to choose these edges and increase its gain to 100 and its cost to two. However, in this case both a and c have an incentive to change their strategy, as this would keep the gain at two, and reduce the cost to zero, a contradiction.

Finally, assume $x_a(ab) = 1$ and $x_c(bc) = 0$ (the case $x_a(ab) = 0$ and $x_c(bc) = 1$ is symmetric). Then $x_b(ab) = x_b(bc) = 0$, as b already has gain two, and cannot improve its gain to 100. However, in this case, c has an incentive to choose the edge bc as this would improve its utility from zero to one, a contradiction with our assumption $x_c(bc) = 0$. $\square$

$\phi = (x_1 \vee x_2 \vee x_3) \wedge (x_2 \vee x_4 \vee x_5) \wedge (x_2 \vee x_3 \vee x_4) \wedge (x_1 \vee x_4 \vee x_5)$

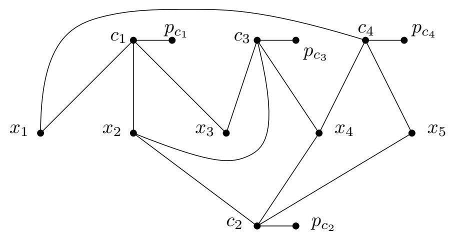

Figure 1: A graph G constructed from ϕ as in Theorem 7.

Observation 5 ($\star$)**.** *There exists a fully homogeneous instance $(G, \{g_v\}, \{c_v\})$ of* EBPG-PSNS *that does not have a Nash equilibrium. Moreover, the graph G in this instance is a complete graph (and a cycle) on three vertices.*

NP-Hardness

In this section, we consider the hardness of computing Nash equilibria in Edge Binary Public Goods games.

To show that EBPG-PSNSis NP-complete on planar graphs of maximum degree four, we will give a reduction from a variant of the well known NAE-3SAT problem. This variant which we refer to as RESTRICTED PLANAR POSITIVE NOT-ALL-EQUAL 3-SAT (RPP-NAE-3SAT) is known to be NP-complete (Dehghan 2016). RPP-NAE-3SAT asks whether there is a satisfying assignment of a given planar positive 3CNF-formula φ which does not set all literals of any single clause to true and which sets all variables in a given set P to true. Recall a formula is *positive* if all its variables appear only as positive literals, and it is *planar* if its clause-vertex incidence graph is planar.

We strengthen the NP-hardness of RPP-NAE-3SAT to when variables each appear in at most three clauses.

Theorem 6 ($\star$)**.** RPP-NAE-3SAT *is* NP*-hard even if every variable appears in at most three clauses.*

We are now ready to show EBPG-PSNS is NP-hard, even on planar graphs of constant maximum degree.

Theorem 7. EBPG-PSNS *is* NP*-hard, even on planar graphs of maximum degree four.*

Proof. We reduce from the variant of RPP-NAE-3SAT in which each variable appears in at most three clauses. By Theorem 6 this problem is NP-complete. Given an instance (φ, P) of RPP-NAE-3SAT we describe an equivalent instance $(G, \{g_v \mid v \in V(G)\}, \{c_v \mid v \in V(G)\})$ of EBPG-PSNS as follows. Let G_φ be the planar vertex-clause incidence graph associated with φ. We attach to each clause c a new pendant vertex p_c and let the resulting graph be G. See Figure 1. Note that by construction and the fact that G_φ is planar, G is planar. Moreover, note that G has degree at most four, since the vertices corresponding to clauses containing three literals have degree at four, and since every variable is contained in at most three clauses, the vertices of G corresponding to variables of φ have degree at most three.

For $v \in V(G)$, we define g_v and c_v as follows.

If $v \notin P$ is a variable of φ, we let $c_v \equiv 1$ and

$$g_v(i) = \begin{cases} 1 & \text{if } i < |N_G(v)| \\ |N_G(v)| + 1 & \text{if } i \geq |N_G(v)|. \end{cases}$$

If $v \in P$ is a variable of φ, we let $c_v \equiv 1$ and $g_v(i) = 2i$. If c is a clause of φ, we let

$$c_v(w) = \begin{cases} 1 & \text{if } w = p_c \\ 5 & \text{otherwise} \end{cases}$$

$$\text{and } g_c(i) = \begin{cases} 0 & \text{if } i = 0 \\ 2 & \text{if } 1 \leq i \leq 3 \\ 4 & \text{if } i \geq 4. \end{cases}$$

If $v = p_c$ for a clause c of φ, we let $c_v(w) \equiv 1$ and

$$g_v(i) = \begin{cases} 0 & \text{if } i < 2 \\ 2 & \text{if } i \geq 2. \end{cases}$$

Claim 1 ($\star$)**.** *If there is a truth assignment for the variables of φ that witnesses that (φ, P) is a yes-instance of* RPP-NAE-3SAT *then letting each variable that is set to true by this assignment choose all its incident edges and leaving all other edges unchosen by all other players describes a Nash equilibrium for $(G, \{g_v\}, \{c_v\})$.*

Claim 2 ($\star$)**.** *If there is a Nash equilibrium $(x_w)_{w \in V(G)}$ for $(G, \{g_v\}, \{c_v\})$ then each variable v of φ chooses all or non of its incident edges under x_v, and setting precisely the variables to true which choose all their incident edges describes an assignment witnessing that (φ, P) is a yes-instance of* RPP-NAE-3SAT.

From the claims above, we conclude: (φ, P) is a yes-instance if and only if $(G, \{g_v\}, \{c_v\})$ is. ☐

Finally, in sharp contrast with the complexity of BNPG-PSNS that has been shown to be polynomial-time solvable on complete graphs (Yu et al. 2020), we show that EBPG-PSNS remains NP-hard when restricted to complete graphs.

Theorem 8 ($\star$)**.** EBPG-PSNS *is* NP*-hard, even on complete graphs.*

Parameterizing by Treewidth

In this section, we consider the tractability of EBPG-PSNS on graphs of bounded treewidth. We first show an XP-algorithm for the problem parameterized by treewidth.

It will be useful to distinguish certain kinds of deviations.

Definition 9. *Given a strategy $(x_v)_{v \in V(G)}$ for an EBPG $\mathcal{I} = (G, \{g_v\}, \{c_v\})$, a Nash deviation x'_w for $w \in V(G)$ is* gain-preserving *if $|\{e \mid x_w(e) = 1\}| = |\{e \mid x'_w(e) = 1|$.*

Observe that for a Nash deviation for a player to be gain-preserving, it has to strictly decrease its cost.

Theorem 10. EBPG-PSNS *parameterized by the treewidth of its input graph is in* XP*. Specifically there is an algorithm for* EBPG-PSNS *with time complexity $|V(G)|\Delta^{\mathcal{O}(\mathrm{tw}(G))}2^{\mathcal{O}(\mathrm{tw}(G)^3)}$, where Δ denotes the maximum vertex degree in the input graph.*

Proof. Let $(G, \{g_v \mid v \in V(G)\}, \{c_v \mid v \in V(G)\})$ be an instance of EBPG-PSNS. We can compute a nice tree decomposition (T, χ) of G with at most $\mathcal{O}(n)$ nodes whose width is in $\mathcal{O}(\text{tw}(G))$ in time $2^{\mathcal{O}(\text{tw}(G))}|V(G)|$ (Korhonen 2021). We give a dynamic program (DP) that works in a leaves-to-root fashion along T and stores and updates the following information.

We consider tuples (t, i, j, m, x^t) of the following form

- $t \in V(T)$ will mark the current progress in our DP.
- $i, j : \chi(t) \to [|N_{G_t}[\chi(t)]|]$ will track the number of edges already chosen among $\{vw \mid w \in N_{G_t}(v)\}$ by the players $v \in \chi(t)$ and $w \in N_{G_t}(v)$ respectively.
- $m : \chi(i) \to \{e \in E(G_t) \mid e \cap \chi(t) \neq \emptyset\} \cup \{\emptyset\}$ will track a most costly edge that each player in $\chi(t)$ has chosen among $E(G_t)$.
- $x^t = (x^t_v)_{v \in \chi(t)}$ where for each $v \in \chi(t)$, $x^t_v \in \{0,1\}_v^{E(G[\chi(t)])}$, i.e. x^t describes the restriction of a strategy profile to $G[\chi(t)]$.

We make the described intention of storing such tuples formal by targeting to store such tuples (t, i, j, m, x^t) in a way that for each $t \in V(T)$:

P1) If there is a Nash equilibrium x for $(G, \{g_v \mid v \in V(G)\}, \{c_v \mid v \in V(G)\})$ then we store a tuple (t, i, j, m, x^t) where for each $v \in \chi(t)$, $i(v) = |\{vw \in E(G_t) \mid x_v(vw) = 1\}|$, $j(v) = |\{vw \mid x_w(vw) = 1\}|$, $m(v) \in E(G_t) \in \operatorname{argmax}\{c(e) \mid x_v(e) = 1\}$ and $x^t(v) = x\big|_{E(G[\chi(t)])}$; and

P2) If we store a tuple (t, i, j, m, x^t) then there is a Nash equilibrium x for $(G_t, \{g_v \mid v \in V(G_t) \setminus \chi(t)\} \cup \{g_v \equiv 0 \mid v \in \chi(t)\}, \{c_v\big|_{E(G_t)} \mid v \in V(G_t) \setminus \chi(t)\} \cup \{c_v \equiv 0 \mid v \in \chi(t)\})$ which has no gain-preserving Nash deviation when considered as a strategy for $(G_t, \{g_v \mid v \in V(G_t)\}, \{c_v\big|_{E(G_t)} \mid v \in V(G_t)\})$, and for each $v \in \chi(t)$, $i(v) = |\{vw \in E(G_t) \mid x_v(vw) = 1\}|$, $j(v) = |\{vw \mid x_w(vw) = 1\}|$, $m(v) \in E(G_t) \in \operatorname{argmax}\{c(e) \mid x_v(e) = 1\}$ and $x^t(v) = x\big|_{E(G[\chi(t)])}$.

The following claims postulate procedures for each dynamic programming step depending on the node type; of these we present the introduce node in full detail.

Claim 3 ($\star$)**.** *Let t be a leaf node (this implies $\chi(t) = \emptyset$). A set $\mathcal{S}$ of tuples (t, i, j, m, x^t) satisfying P1) and P2) for each $\tilde{t} \in V(T_t)$ can be computed in constant time.*

Claim 4. *Let t be an introduce node with child t' and $\{u\} = \chi(t) \setminus \chi(t')$. Given a set $\mathcal{S}'$ of tuples $(t', i', j', m', x^{t'})$ satisfying P1) and P2) for each $\tilde{t} \in V(T_{t'})$, we can compute a set $\mathcal{S}$ of tuples (t, i, j, m, x^t) satisfying P1) and P2) for t in time $|\mathcal{S}'|2^{\mathcal{O}(|\chi(t)|)}$.*

Consider any choice of $F_u \subseteq E(G[\chi(t)])_u$ and $F_{N(u)} \subseteq E(G[\chi(t)])_u$ with the property that whenever $e \in F_u$ then $\{e' \in E_{G[\chi(t)]}(u) \mid c_u(e') < c_u(e)\} \subseteq F_u$, and whenever for some $uw \in E(G[\chi(t)])_u$, $c_w(uw) < m'(w)$ then $uw \in F_{N(u)}$(this condiction will preclude the existence of gain-preserving deviations for w). For each tuple $(t', i', j', m', x^{t'}) \in \mathcal{S}'$ and F_u and $F_{N(u)}$ as above, we let

$$i(u) = |F_u| \text{ and } i(w) = \begin{cases} i'(w) + 1 & \text{if } uw \in F_{N(u)} \\ i'(w) & \text{otherwise,} \end{cases}$$

$$j(v) = |F_{N(u)}| \text{ and } j(w) = \begin{cases} j'(w) + 1 & \text{if } uw \in F_v \\ j'(w) & \text{otherwise,} \end{cases}$$

$m(u)$ be arbitrary in $\operatorname{argmax}\{c_u(e) \mid e \in F_u\}$ or $\emptyset$ if $F_u = \emptyset$ and

$$m(w) = \begin{cases} uw & \text{if } uw \in F_{N(u)} \wedge (m'(w) = \emptyset \vee \\ & \qquad c_w(uw) > c_w(m'(w))) \\ m'(w) & \text{otherwise,} \end{cases}$$

$x^t_u = \mathbb{1}_{F_u}$ and

$$x^t_w(e) = \begin{cases} x^{t'}_w(e) & \text{if } e \in E(G[\chi(t)])_w \setminus E(G[\chi(t)])_u \\ 1 & \text{if } e = uw \wedge uw \in F_{N(u)} \\ 0 & \text{otherwise.} \end{cases}$$

Let $\mathcal{S}$ be the set of tuples (t, i, j, m, x^t) arising in this way.

It is easy to see that $\mathcal{S}$ can be constructed in the claimed time complexity.

We now show that with this construction P1) and P2) hold for t.

P1) Assume that there is a Nash equilibrium x for $(G, \{g_v \mid v \in V(G)\}, \{c_v \mid v \in V(G)\})$. Because of P1) for t' this means that there is $(t', i', j', m', x^{t'}) \in \mathcal{S}'$ where for each $v \in \chi(t')$, $i'(v) = |\{vw \in E(G_{t'}) \mid x_v(vw) = 1\}|$, $j'(v) = |\{vw \mid x_w(vw) = 1\}|$, $m'(v) \in E(G_{t'}) \cap \operatorname{argmax}\{c(e) \mid x_v(e) = 1\}$ and $x^{t'}(v) = x\big|_{E(G[\chi(t')])}$. The tuple (t, i, j, m, x^t) that is added to $\mathcal{S}$ by starting from the $(t', i', j', m', x^{t'})$ above and choosing $F_u = \{e \in E(G[\chi(t)] \mid x_u(e) = 1\}$ and $F_{N(u)} = \{uw \in E(G[\chi(t)] \mid x_w(uw) = 1\}$ by construction satisfies that for each $v \in \chi(t)$, $i(v) = |\{vw \in E(G_t) \mid x_v(vw) = 1\}|$, $j(v) = |\{vw \mid x_w(vw) = 1\}|$, $m(v) \in E(G_t) \in \operatorname{argmax}\{c(e) \mid x_v(e) = 1\}$ and $x^t(v) = x\big|_{E(G[\chi(t)])}$, thereby verifying P1).

P2) Consider $(t, i, j, m, x^t) \in \mathcal{S}$. By construction (t, i, j, m, x^t) arose from some $(t', i', j', m', x^{t'}) \in \mathcal{S}'$ with some choice of $F_u \subseteq E(G[\chi(t)])_u$ and $F_{N(u)} \subseteq E(G[\chi(t)])_u$ such that whenever $e \in F_u$ then $\{e' \in E_{G[\chi(t)]}(u) \mid c_u(e') < c_u(e)\} \subseteq F_u$, and whenever for some $uw \in E(G[\chi(t)])_u$, $c_w(uw) < m'(w)$ then $uw \in F_{N(u)}$. By P2) for t' there is a Nash equilibrium x' for $(G_{t'}, \{g_v \mid v \in V(G_{t'}) \setminus \chi(t')\} \cup \{g_v \equiv 0 \mid v \in \chi(t')\}, \{c_v\big|_{E(G_{t'})} \mid v \in V(G_{t'}) \setminus \chi(t')\} \cup \{c_v \equiv 0 \mid v \in \chi(t')\})$ which has no gain-preserving Nash deviation when considered as a strategy for $(G_{t'}, \{g_v \mid v \in V(G_{t'})\}, \{c_v\big|_{E(G_{t'})} \mid v \in V(G_{t'})\})$ such that for each $v \in \chi(t')$, $i'(v) = |\{vw \in E(G_{t'}) \mid x'_v(vw) = 1\}|$,

$j'(v) = |\{vw \mid x'_w(vw) = 1\}|$, $m'(v) \in E(G_{t'}) \in \operatorname{argmax}\{c(e) \mid x'_v(e) = 1\}$ and $x^{t'}(v) = x'|_{E(G[\chi(t')])}$.

We extend and modify x' to a strategy for $(G_t, \{g_v \mid v \in V(G_t) \setminus \chi(t)\} \cup \{g_v \equiv 0 \mid v \in \chi(t)\}, \{c_v|_{E(G_t)} \mid v \in V(G_t) \setminus \chi(t)\} \cup \{c_v \equiv 0 \mid v \in \chi(t)\})$ by setting $x_u = \mathbb{1}_{F_u}$ and for $w \in V(G_t) \setminus \{u\}$, letting $x_w(e) = 1$ if and only if $x'_w(e) = 1$ or $e = uw \in F_{N(u)}$.

It is straightforward to verify that in this way for each $v \in \chi(t)$, $i(v) = |\{vw \in E(G_t) \mid x_v(vw) = 1\}|$, $j(v) = |\{vw \mid x_w(vw) = 1\}|$, $m(v) \in E(G_t) \in \operatorname{argmax}\{c(e) \mid x_v(e) = 1\}$ and $x^t(v) = x|_{E(G[\chi(t)])}$.

Assume for contradiction that x is not a Nash equilibrium and hence there is a Nash deviation for some $v \in V(G_t)$. Because gain and cost functions are explicitly set to zero for $\chi(t)$, it must hold that $v \in V(G_t) \setminus \chi(t) = V(G_{t'}) \setminus \chi(t')$. However when modifying x' to x no changes were made that influence edges incident to such players meaning the same Nash deviation was present in x' contradicting the fact that x' is a Nash equilibrium.

Finally, the fact that there is no gain-preserving Nash deviation when considering x as a strategy for $(G_t, \{g_v \mid v \in V(G_t)\}, \{c_v|_{E(G_t)} \mid v \in V(G_t)\})$ is ensured by the required conditions on F_u and $F_{N(u)}$ as well as that x' had no gain preserving deviation as a strategy for $(G_{t'}, \{g_v \mid v \in V(G_{t'})\}, \{c_v|_{E(G_{t'})} \mid v \in V(G_{t'})\})$: A gain preserving deviation for u would have to select some edges which are of lower cost for u but equal number which is impossible as F_u optimal under this cardinality. Similarly, a gain preserving deviation for a neighbor w of u would have to select an edge set of lower cost but equal cardinality for that neighbor which is impossible because $F_{N(u)} \cap E(G_t)_w \cup {x'_w}^{-1}(1)$ is by downward-closed in terms of c_w with maximum value $m(w)$. Gain preserving deviations for all other players are precluded because they would also be gain preserving deviations in x' for $(G_{t'}, \{g_v \mid v \in V(G_{t'})\}, \{c_v|_{E(G_{t'})} \mid v \in V(G_{t'})\})$.

Overall, we found x as desired to show P2). $\lhd$

Claim 5 ($\star$)**.** *Let t be a forget node with child t' and $\{u\} = \chi(t') \setminus \chi(t)$. Given a set $\mathcal{S}'$ of tuples $(t', i', j', m', x^{t'})$ satisfying P1) and P2), we can compute a set $\mathcal{S}$ of tuples (t, i, j, m, x^t) satisfying P1) and P2) in time $\mathcal{O}(|\mathcal{S}'||E(G)_v|^2)$.*

Claim 6 ($\star$)**.** *Let t be a join node with children t_1 and t_2 (this implies $\chi(t) = \chi(t_1) = \chi(t_2)$). Given sets $\mathcal{S}_1$ of tuples $(t_1, i_1, j_1, m_1, x^{t_1})$ and $\mathcal{S}_2$ of tuples $(t_2, i_2, j_2, m_2, x^{t_2})$ satisfying P1) and P2), we can compute a set $\mathcal{S}$ of tuples (t, i, j, m, x^t) satisfying P1) and P2) in time $|\mathcal{S}_1||\mathcal{S}_2|\mathcal{O}(\max\{|E(G[\chi(t)])|, |\chi(t)|\})$.*

Based on the above claims we can canonically formulate a dynamic programming procedure which runs in time $|V(G)|\Delta^{\mathcal{O}(\operatorname{tw}(G))}2^{\mathcal{O}(\operatorname{tw}(G)^3)}$, where Δ is the maximum vertex degree in G, and stores tuples in a way adhering to P1) and P2). Considering tuples whose first entry is the root of T this is a set which is non-empty if and only if $(G, \{g_v \mid v \in V(G)\}, \{c_v \mid v \in V(G)\})$ is a yes-instance of EBPG-PSNS. This follows from P1) and P2) at the root r of T together with the fact that $G_r = G$ and $\chi(r) = \emptyset$. $\square$

From the running time of the above algorithm we immediately get a fixed-parameter tractability result using the maximum vertex degree as additional parameter.

Corollary 11. EBPG-PSNS *parameterized by the treewidth and maximum vertex degree of its input graph is in* FPT.

Under standard complexity assumptions, including the maximum vertex degree in the parameterization cannot be avoided when targeting an FPT-algorithm, in the sense that EBPG-PSNS is unlikely to admit an algorithm with an FPT running time when parameterized by treewidth alone. This is true, even when we consider planar networks.

Theorem 12 ($\star$)**.** EBPG-PSNS *is* W*-hard parameterized by the treewidth of the input graph, even when restricting to planar input graphs.*

Setting with Increased Tractability of EBPG-PSNS

Taking a step back and comparing the general complexity-theoretic behavior of EBPG-PSNS in contrast to deciding the existence of Nash equilibria in BNPGs, our investigation found two kinds of scenarios: (1) The computational complexity is the same, e.g. both BNPG-PSNS and EBPG-PSNS are in XP and W-hard when parameterized by the treewidth of the instance and FPT with respect to the combination of treewidth and maximum vertex degree. (2) EBPG-PSNS is computationally harder than BNPG-PSNS, e.g. BNPG-PSNS is polynomial-time solvable on cliques but EBPG-PSNS is NP-hard on cliques. We conclude with an example of a setting which exhibits the third type of behavior, i.e. hardness for BNPG-PSNS and tractability for the corresponding EBPG-PSNS problem.

Analogously to the setting for BNPG-PSNS, where the number of participating players has been used as parameter, we can use the number of chosen edges as parameter. Here, we discover contrasting complexity behavior: Deciding the existence of Nash equilibria in fully homogeneous BNPGs is known to be W-hard with respect to the numbers of participating players (Maiti and Dey 2024b). This rules out the existence of a fixed-parameter algorithm for this parameterization under standard complexity assumptions. On the other hand, we give a fixed-parameter algorithm for EBPG-PSNS on fully homogeneous instances parameterized by the number of chosen edges. Formally, we consider the following parameterized problem:

EBPG-PSNS parameterized by κ	
Instance	An EBPG $\mathcal{I}$
Question	Is there a Nash equilibrium x for $\mathcal{I}$ in which $\sum_{v \in V(G)} |x^{-1}(1)| \leq \kappa$?
Parameter	κ

Theorem 13. EBPG-PSNS *parameterized by κ is in* FPT *when restricted to fully homogeneous instances.*

Proof. We can check in polynomial time whether for a player v there is a lexicographically minimal with respect to c_v set of incident edges E^* such that $g_v(|E^*|)-c_v(E^*) > 0$. We call such players *needy*. If no edge incident to an arbitrary needy player v is chosen by any player (including v itself), v has a Nash deviation by choosing all edges in E^*. Hence, if there are more than 2κ needy v, choosing κ edges cannot suffice and we can correctly output 'no'.

Claim 7 ($\star$)**.** *If there is a Nash equilibrium in which at most κ edges are chosen, there is one in which players at distance more than κ from the set of needy players choose no edges.*

Hence, we can delete all players with a larger distance to the set of all needy players and obtain an equivalent instance. So far, we did not make use of the fact that we restrict to homogeneous instances. Let us do so now, by observing more carefully what being needy means in fully homogeneous instances: Firstly, the cost of an edge set witnessing the neediness of a player only depends on its size, i.e. for any player v and sets of incident edges E_1^* and E_2^* with $|E_1^*| = |E_2^*|$, $g_v(|E_1^*|) - c_v(E_1^*) = g_v(|E_1^*|) - c \cdot |E_1^*| = g_v(|E_2^*|) - c \cdot |E_2^*| = g_v(|E_1^*|) - c_v(E_1^*)$ where c denotes the constant uniform edge cost. Moreover, because the gain functions of all players are equal, if there is a player v whose neediness is witnessed by E^*, the only way for another player w not to be needy is w having fewer than $|E^*|$ incident edges.

For a needy player v, let us call $\ell \in [\deg_G(v)]$ its *threshold* if it is minimum with the property that for all $\ell' \in [\deg_G(v)]$, $g_v(\ell) - \ell \cdot c \geq g_v(\ell') - \ell' \cdot c$. If there is a needy player v with threshold greater than κ, we can correctly output 'no' as that player will always want to deviate to reach at least that threshold (edges to v chosen by its neighbors only only can increase the number of edges which are desirable for v to be chosen incident to it) and hence no Nash equilibrium with at most κ chosen edges exists.

Combining the previous observations, we can assume that each needy player has a threshold of at most κ and each non-needy player has degree at most κ. Hence, in the instance we consider at this point removing all needy players results in connected components whose diameter and maximum vertex degree, and hence size, is bounded in κ. We can consider two connected components of the instance minus the needy players as equivalent if they each induce the same graph together with the needy players. The size and number of graphs of diameter and maximum vertex degree κ is bounded in κ and connecting at most 2κ additional vertices still results in a number of possibilities that is bounded in κ. Overall, we never need to consider more than κ components of the same equivalence class and can delete all superfluous ones resulting in an equivalent instance of size bounded in κ. We can solve EBPG-PSNS on this equivalent instance by brute force to obtain an overall FPT-runtime parameterized by κ. □

Conclusion

We introduced EBPGs as an adaptation of BNPGs which can easily capture situations in which choices of a player do not impact the entire neighborhood. This addresses modeling weaknesses of BNPGs in settings which have previously been used to motivate them. Beyond this, we initiated the complexity-theoretic investigation of EBPG-PSNS, classifying its complexity under well-motivated structural assumptions and uncovering cases of contrasting complexity behavior compared to BNPG-PSNS.

A more detailed complexity-theoretic investigation of EBPG-PSNS which gives a more complete picture with respect to different graph parameters or best-response patterns similarly to what has already been achieved for BNPG-PSNS is an interesting direction for future work. Similarly, we can imagine that adapting enhancements to BNPGs which account e.g. for the effects of altruism (Li et al. 2023) may be worthwhile. On the flip side of this, our work poses planarity as a relevant structural restriction for EBPG-PSNS. Its impact on BNPG-PSNS is equally relevant and has not yet been studied in previous literature. Another concrete question left open by us is whether Theorem 13 also holds for *not* fully homogeneous instances.

Besides evening our understanding of EBPG and BNPG, there is of course also the matter of defining and studying a continuous or at least non-binary version of EBPG– or from a different perspective, an edge version of (non-binary) networked public goods games. Note also that we made the choice to count edges chosen by both its endpoints as contributing to the gain-function twice. It would be interesting whether a different choice such as making them count only once or halving the cost for such edges have a large effect on the behavior or difficulty of finding Nash equilibria. Computing mixed Nash equilibria which must exist by Nash's theorem is another problem our work does not yet address.

Having focused on the theoretical groundwork, we consider it an extremely interesting and valuable research direction to conduct an empirical analysis of the extent to which EBPGs are a more appropriate model than BNPGs in settings in which we would expect them to be.

On a more graph-theoretic note, as remarked in the Introduction, in the case in which each binary edge choice has the same constant cost for every player, EBPG-PSNS can be reformulated as a directed version of GENERAL FACTOR. GENERAL FACTOR asks to decide the existence of a subgraph of a given graph in which the degree of each vertex in the subgraph is constrained to be in some given permissible set. This can be adapted to directed graphs by deciding the existence of a directed subgraph in which the in- and out- degrees of vertices are in permissible sets. To encode EBPG-PSNS, $(i, o) \in [\deg^{\text{in}}(v)] \times [\deg^{\text{out}}(v)]$ would be in the permissible set for a player v if and only if v's best-response to i edges towards v being chosen by players in the neighborhood of v is choosing o edges (whose identity is irrelevant because of the assumed constant cost). In this way, a subgraph satisfying these constraints would straightforwardly correspond to a solution for EBPG-PSNS; a player chooses an edge if and only if the corresponding edge directed away from that agent is in the subgraph and vice versa. We are unaware of whether a directed version of GENERAL FACTOR has been studied but we view it as natural and worth investigating, in particular in view of its connection with EBPGs.

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