

(1+1) Genetic Programming with Functionally Complete Instruction Sets Can Evolve Boolean Conjunctions and Disjunctions with Arbitrarily Small Error

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Abstract

Recently it has been proven that simple GP systems can efficiently evolve a conjunction of n variables if they are equipped with the minimal required components. In this paper, we make a considerable step forward by analysing the behaviour and performance of a GP system for evolving a Boolean conjunction or disjunction of n variables using a complete function set that allows the expression of any Boolean function of up to n variables. First we rigorously prove that a GP system using the complete truth table to evaluate the program quality, and equipped with both the AND and OR operators and positive literals, evolves the exact target function in $O(\ell n \log_2 n)$ iterations in expectation, where $\ell \geq n$ is a limit on the size of any accepted tree. Additionally, we show that when a polynomial sample of possible inputs is used to evaluate the solution quality, conjunctions or disjunctions with any polynomially small generalisation error can be evolved with probability $1 - O\left(\frac{\log_2(n)}{n}\right)$. The latter result also holds if GP uses AND, OR and positive and negated literals, thus has the power to express any Boolean function of n distinct variables. To prove our results we introduce a super-multiplicative drift theorem that gives significantly stronger runtime bounds when the expected progress is only slightly super-linear in the distance from the optimum.

References

Doerr, B.; Lissovoi, A.; and Oliveto, P. S. 2023. (1+ 1) genetic programming with functionally complete instruction sets can evolve Boolean conjunctions and disjunctions with arbitrarily small error. *Artificial Intelligence*, 319: 103906.