Enhance Diversified Top-k MaxSAT Solving by Incorporating New Strategy for Generating Diversified Initial Assignments (Student Abstract)

Jiaxin Liang[1], Junping Zhou[1*], Minghao Yin[1, 2*]

[1]School of Information Science and Technology, Northeast Normal University, Changchun, China
[2]Key Laboratory of Applied Statistics of MOE, Northeast Normal University, Changchun, China
{liangjx348, zhoujp877, ymh}@nenu.edu.cn

Abstract

The Diversified Top-k MaxSAT (DTKMS) problem is an extension of MaxSAT. The objective of DTKMS is to find k feasible assignments of a given formula, such that each assignment satisfies all hard clauses and the k assignments together satisfy the maximum number of soft clauses. This paper presents a local search algorithm, DTKMS-DIA, which incorporates a new approach to generating initial assignments. Experimental results indicate that DTKMS-DIA can achieve attractive performance on 826 instances compared with state-of-the-art solvers.

Introduction

Given a propositional formula expressed in the Conjunctive Normal Form (CNF), the diversified top-k MaxSAT (DTKMS) problem is to find k assignments such that each assignment satisfies all hard clauses and the k assignments together satisfy the maximum number of soft clauses. As a variant of MaxSAT, DTKMS is considered to be a bi-standard optimization problem balancing relevancy and diversity of results. DTKMS has a wide range of practical applications, such as social networks, network routing, and combinatorial auctions. Recently, Zhou et al. proposed a local search algorithm for DTKMS, called LS-DTKMS (Zhou et al. 2023). They focused on designing the heuristic process but allocated limited attention to generating the appropriate initial assignments, which is known as a major process affecting the performance of local search algorithms.

This work aims to enhance the performance of local search algorithms for DTKMS by developing a new diversified initial assignment generation strategy. This strategy works in the form of unit propagation driven process, which is based on a decimation approach for MaxSAT (Cai, Luo, and Zhang 2017). The strategy assigns variables iteratively in a controlled way instead of generating a random assignment. Specifically, we first assign variables in unit clauses in a set of selection order to perform unit propagation. When unit propagation cannot be applied, we design a heuristic to select unassigned variables. The two processes, unit propagation and the heuristic, work alternately until an initial assignment is generated. With this strategy, our algorithm, DTKMS-DIA, is able to perform a broader exploration of the solution space and achieve a good result.

*Corresponding author

Diversified Top-k MaxSAT Problem

Given a set of n Boolean *variables*, $X = \{x_1, x_2, ..., x_n\}$, a *literal* is either a variable itself x_i or its negation $\neg x_i$. A *clause* is a disjunction of literals, i.e., $c_j = l_1 \vee l_2 \vee \ldots \vee l_{n_i}$. A clause containing one literal is called a *unit clause*. A CNF formula F is a conjunction of clauses, i.e., $F = c_1 \wedge c_2 \wedge \ldots \wedge c_m$. An *assignment* α for F is a mapping that assigns each variable to *true* or *false*. Given an assignment α, a clause is satisfied if at least one of its literals is *true*. Given a formula $F = Hard \cup Soft$, whose clauses are divided into hard clauses and soft clauses, the *Maximum Satisfiability (MaxSAT)* problem is to find an assignment that satisfies all hard clauses and maximizes the number of satisfied soft clauses. An assignment α is called a *feasible assignment iff* it satisfies all hard clauses of F. The soft clauses satisfied by α is denoted by $sat(\alpha)$. Given a set of feasible assignments $S = \{\alpha_1, \alpha_2, \cdots, \alpha_k\}$ of a MaxSAT formula F, $sat(S)$ is the total soft clauses satisfied by S, i.e., $sat(S) = sat(\alpha_1) \cup sat(\alpha_2) \cup \cdots \cup sat(\alpha_k)$. Given a formula $F = Hard \cup Soft$ and a positive integer k, the *Diversified Top-k MaxSAT (DTKMS)* problem aims at finding a set S with at most k feasible assignments such that these feasible assignments in S satisfy the maximum number of soft clauses of F, i.e., $|sat(S)|$ is maximized. Given a DTKMS formula F, we define S as the *solution* of F, the clauses in $sat(S)$ as the *solution-satisfied soft clauses*, and the clauses in $Soft \setminus sat(S)$ as the *solution-falsified soft clauses*.

The process of unit propagation working on F is as follows. If a unit clause l is contained in F, assign l to $true$, remove the clauses containing l from F and $\neg l$ from the remaining clauses. The above operation is executed iteratively until no more unit clause remains.

DTKMS-DIA Algorithm for DTKMS

In this section, we propose the algorithm, DTKMS-DIA, which alternatively performs solution construction and solution updating procedures until a given timelimit is reached. The solution construction procedure generates k feasible assignments to build a solution S. Then, the solution updating procedure is executed. This procedure maintains the solution S by constantly replacing an old feasible assignment with a new feasible assignment so that the solution S satisfies a larger number of soft clauses. Indeed, both procedures benefit from a feasible assignment generation algorithm and prefer the algorithm to return diversified feasible assignments. To obtain such assignments, we develop a new diversified initial assignment generation strategy, which is a primary influence factor on the efficiency of local search. This strategy works in the form of unit propagation driven process, assigning variables one by one until a complete assignment is produced. If there exist unit clauses, it selects a unit clause in priority order: hard unit clauses, solution-falsified soft unit clauses, and solution-satisfied soft unit clauses. Then the unit propagation is performed. When a conflict is reached during the unit propagation, the variable contained by the selected unit clause is reassigned to the value in the last feasible assignment. If there is no unit clauses, the strategy selects an unassigned variable according to the following heuristic and assigns it to a random value. In detail, the heuristic chooses m variables (m is an integer parameter) randomly from the set of unassigned variables, and returns the one with the greatest $score(x)$, where $score(x) = \lambda_1 \times pre\_order(x) + \lambda_2 \times num(x)$ ($\lambda_1 < 1, \lambda_2 < 1$). Here, $num(x)$ denotes the occurrence of the variable x in the solution-falsified soft clauses and $pre\_order(x)$ represents the assigning order of the variable x in the last initial assignment. The larger the $pre\_order(x)$, the later x is assigned. In general, the diversified initial assignment generation strategy uses unit propagation to guarantee that the initial assignments are determined in a controlled way, and the heuristic to guide the exploration in the trend of the diversity. After generating initial assignments, they are improved by local search so that feasible assignments in different search areas are found.

Experimental Evaluation

We evaluate DTKMS-DIA on 826 instances from the main unweighted track in MSE 2023, the benchmarks from the top-k track in MSE 2020, and an application of DTKMS, called Diversified Top-k Clique Search (DTKCS) (Wu et al. 2020), whose benchmarks are from DIMACS graph[1]. To ensure DTKMS-DIA can solve DTKCS instances, we model them into DTKMS instances using the usual encoding (Li and Quan 2010). In the experiment, we compare DTKMS-DIA with the state-of-the-art local search solver LS-DTKMS (Zhou et al. 2023), which is the best DTKMS solver so far. We set the parameter k to 2, 3, 4, 5, and thus obtained 826×4=3304 DTKMS instances. Each instance was executed by each solver ten times, and the average results are summarized in Table 1. We report the number of instances in each benchmark set (in brackets), the number of instances where both solvers found the same number of satisfying soft clauses (#Equal), and the number of instances where a solver found a better solution with more satisfied soft clauses than the other (#Better). As seen from the table, DTKMS-DIA is overall the best solver on these benchmarks.

[1]https://github.com/LyreRabbit/DTKMS

Benchmarks		LS-DTKMS		DTKMS-DIA	
	k	#Better	#Equal	#Better	#Equal
DTKCS (21)	2	2	6	**13**	6
	3	0	7	**14**	7
	4	0	10	**11**	10
	5	0	9	**12**	9
Unweighted Top-k (73)	2	0	50	**23**	50
	3	0	53	**20**	53
	4	0	53	**20**	53
	5	0	56	**17**	56
Unweighted MSE23 (732)	2	90	40	**602**	40
	3	72	162	**498**	162
	4	61	417	**254**	417
	5	10	544	**178**	544
Total		235	1407	**1662**	1407

Table 1: Summary of the comparison of DTKMS-DIA and LS-DTKMS, where the timelimit is 600 seconds.

Conclusion

We propose a local search algorithm for DTKMS problem, called DTKMS-DIA, which contains a new diversified initial assignment generation strategy. The experimental results demonstrate that DTKMS-DIA exhibits higher efficiency due to its ability to generate better solutions on more instances. In the future, we will develop more optimized techniques and conduct comprehensive studies into the real-world applications of DTKMS.

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