Bilateral Gradual Semantics for Weighted Argumentation

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Abstract

Abstract argumentation is a reasoning model for evaluating arguments. Recently, *gradual semantics* has received considerable attention in *weighted argumentation*, which assigns an *acceptability degree* to each argument as its *strength*. In this paper, we aim to enhance gradual semantics by *non-reciprocally* incorporating the notion of *rejectability degree*. Such a setting offers a *bilateral* perspective on argument strength, enabling more comprehensive argument evaluations in practical situations. To this end, we first provide a set of principles for our semantics, taking both the acceptability and rejectability degrees into account, and propose three novel semantics conforming to the above principles. These semantics are defined as the *limits* of iterative sequences that always *converge* in any given weighted argumentation system, making them preferable for real-world applications.

Introduction

Abstract argumentation framework (AF) is a well-studied model for reasoning and decision-making in conflict situations (Dung 1995; Amgoud and Prade 2009; Bench-Capon and Dunne 2007). An AF is a directed graph whose nodes represent *arguments* and arrows represent *attacks* between arguments. Evaluating arguments is a central topic in abstract argumentation and commonly achieved through various *semantics*. For instance, the original *extension semantics* (Dung 1995) looks for sets of arguments that are jointly acceptable, and the justification status of arguments is classified as *sceptically/credulously accepted* and *rejected* (Baroni, Caminada, and Giacomin 2011). Similarly, the *labelling semantics* (Caminada and Gabbay 2009) assigns each argument a label from $\{accepted, rejected, undecided\}$. However, such *qualitative* settings may not always be sufficient in practical applications (Leite and Martins 2011; Polberg and Hunter 2018; Amgoud, Doder, and Vesic 2022), as an argument may have *quantitative* properties.

In recent years, the *gradual semantics* has emerged as a prominent approach for evaluating arguments (Amgoud et al. 2017; Amgoud and Doder 2018; Amgoud and David 2021; Amgoud, Doder, and Vesic 2022; Baroni, Rago, and Toni 2018; Oren et al. 2022; Prakken 2021). This method provides a fine-grained evaluation scheme which introduces the notion of *acceptability degree* to describe *argument strength*. It generally assigns each argument a numerical value as its acceptability degree that satisfies a set of desirable properties.

In this paper, we aim to enhance gradual semantics by *non-reciprocally* incorporating the notion of *rejectability degree*. Such a setting offers a *bilateral* perspective on argument strength, in which positive and negative strength are described *separately* and *non-interchangeably*. Bilateral evaluative processes that are non-reciprocal extensively exist in societal phenomena. Evidence in Cognitive Science suggests that in such situations, strength of positivity and negativity should be conceptualized and measured separately (Cacioppo and Berntson 1994; Cacioppo, Gardner, and Berntson 1997).

In the literature, gradual semantics assigns each argument an acceptability degree by aggregating a basic weight together with the acceptability degrees of its attackers. Our bilateral gradual semantics non-reciprocally introduces rejectability degree in the sense that on the one hand it additionally influences the acceptability degree of an argument, on the other hand it is solely determined by the acceptability degree of attackers. This is illustrated in Table 1.

Degree	Source of Stength
acceptability	• basic weight • acceptability degree of attackers • rejectability degree of attackers
rejectability	• acceptability degree of attackers

Table 1: Non-reciprocity of Bilateral Gradual Semantics

Simply speaking, we also assume acceptability degree and rejectablility degree are generally antagonistic, but *not* simply polar opposites. To see why non-reciprocal effects in evaluating arguments do exist, one may imagine scenarios in which safer arguments (i.e., with minor criticisms/lower rejectability degree) are preferred, even if other arguments have higher acceptability degree. For instance, a policy may be abandoned after polling if a threshold for the number of people who are against this policy is reached.

Consider the following policy polling example:

Scenario 1 ($S1$):
(a) Increasing corporate taxes to finance new infrastructure.
(x) Increasing corporate taxes slows economic growth.
Scenario 2 ($S2$):
(b) Increasing individual taxes to finance new infrastructure.
(y) Increasing individual taxes imposes a heavy burden.
In $S1$, argument x attacks argument a and in $S2$, argument y attacks argument b. Suppose in $S1$ there are $0.2M$ votes for x, $0.6M$ votes for a, the rest votes are 'neutral'. In $S2$, $0.6M$ votes for y, $0.8M$ votes for b, the rest are 'neutral'. The basic weights $w(x)$, $w(a)$, $w(y)$ and $w(b)$ are simplified to 0.2, 0.6, 0.6 and 0.8, respectively.

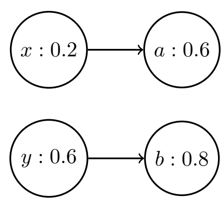


Figure 1: Policy Polling Scenario

Now we calculate argument strength under bilateral semantics. As x, y have no attackers, $acc(x) = w(x) = 0.2$, $rej(x) = 0$, $acc(y) = w(y) = 0.6$, $rej(y) = 0$. By our tailored h-categorizer based equations we have:

$$\begin{cases} acc(a) = & \frac{w(a)}{1+\frac{acc(x)}{1+rej(x)}} = \frac{0.6}{1+\frac{0.2}{1+0}} = \frac{1}{2} \\ rej(a) = & \frac{acc(x)}{1+acc(x)} = \frac{0.2}{1+0.2} = \frac{1}{6} \end{cases}$$

$$\begin{cases} acc(b) = & \frac{w(b)}{1+\frac{acc(y)}{1+rej(y)}} = \frac{0.8}{1+\frac{0.6}{1+0}} = \frac{1}{2} \\ rej(b) = & \frac{acc(y)}{1+acc(y)} = \frac{0.6}{1+0.6} = \frac{3}{8} \end{cases}$$

Note that a and b are indistinguishable in terms of acceptability degree ($\frac{1}{2}$). But b has a rejectability degree ($\frac{3}{8}$) higher than a ($\frac{1}{6}$), for b suffers a stronger attack from y compared to that suffered by a from x. As mentioned above, a policymaker may abandon b and choose a for the latter receives less attacks. It is worth mentioning that the gradual semantics in (Amgoud et al. 2017) also provides the same acceptability degree ($\frac{1}{2}$) for a and b, leaving them indistinguishable as well.

Our bilateral semantics also contributes to the notation of *defense*, which plays a central role in classical semantics (Dung 1995; Baroni, Caminada, and Giacomin 2011) however remains not yet discussed in the context of gradual semantics. In classic semantics, an argument is accepted only if it is defended, i.e., all of its attackers are rejected. That is to say, a rejected attacker has no impact on weakening its target arguments. In this paper, the bilateral semantics naturally leads to a gradual version of 'defense': an attacker with a higher rejectability degree has less impact on weakening the acceptability of its target arguments.

The technical contributions of the paper are as follows. Firstly, we introduce *bilateral gradual semantics* to measure argument strength through the acceptability and rejectability degrees in *weighted argumentation graph* (WAG). Then we follow the well-studied *principle-based approach* (Amgoud, Doder, and Vesic 2022; van der Torre and Vesic 2017) to: (i) Present a set of desirable properties for semantics concerning acceptability and rejectability degrees, which generally serves as the guidelines for establishing semantics. (ii) Investigate the compatibility and links between the above properties. (iii) Define *iterative sequences* according to the desirable properties and prove their *convergence* for any WAG. (iv) Propose *AR-max-based semantics*, *AR-card-based semantics* and *AR-hybrid-based semantics*, which are defined as the *limits* of the corresponding iterative sequences. (v) Show that the three semantics satisfied *most* of desirable principles. Finally, the paper ends with discussions and conclusions.

Preliminaries

A *weighted argumentation graph* (WAG) (Amgoud et al. 2017) consists of a set of arguments and a set of associated attacks. Each argument is assigned a *basic weight* from the real interval $[0, 1]$.

Definition 1 (WAG) *A weighted argumentation graph (WAG) is a triple* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, *where* $\mathcal{A}$ *is a non-empty finite set of arguments,* w *is a function from* $\mathcal{A}$ *to* $[0, 1]$, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$.

Notations: Let $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ be a WAG and $a, b \in \mathcal{A}$. $w(a)$ denotes the basic weight of argument a. $(b, a) \in \mathcal{R}$ means that b attacks a. $Att_{\mathbf{G}}(a)$ denotes the set of all attackers of a, i.e., $Att_{\mathbf{G}}(a) = \{b \in \mathcal{A} \mid (b, a) \in \mathcal{R}\}$. We say that a is *non-attacked* if $Att_{\mathbf{G}}(a) = \varnothing$. We abbreviate $Att_{\mathbf{G}}(a)$ as $Att(a)$ when the context is clear. For $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$ and $\mathbf{G}' = \langle \mathcal{A}', w', R' \rangle$ s.t. $\mathcal{A} \cap \mathcal{A}' = \varnothing$, $\mathbf{G} \oplus \mathbf{G}' = \langle \mathcal{A} \cup \mathcal{A}', w^*, R \cup R' \rangle$ where for any $a \in \mathcal{A}$ (resp. $a \in \mathcal{A}'$), $w^*(a) = w(a)$ (resp. $w^*(a) = w'(a)$).

In the paper, arguments are evaluated through *bilateral gradual semantics* which assigns both acceptability and rejectability degrees to each argument. We use the real interval $[0, 1]$ as the scale for the acceptability degree and $[0, 1)$ as the scale for the rejectability degree. Dropping the maximal value from $[0, 1]$ will simplify the discussion in establishing principles for the rejectability degree.

Definition 2 (Bilateral gradual semantics) *A* bilateral gradual semantics *is a function* $\mathcal{S}$ *transforming any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ *to a function* $Deg^{\mathcal{S}}_{\mathbf{G}}$ *defined from* $\mathcal{A}$ *to* $[0, 1] \times [0, 1)$. *For any* $a \in \mathcal{A}$, $Deg^{\mathcal{S}}_{\mathbf{G}}(a) = (\sigma^+_{\mathbf{G}}(a), \sigma^-_{\mathbf{G}}(a))$ *where* $\sigma^+_{\mathbf{G}}(a)$ *and* $\sigma^-_{\mathbf{G}}(a)$ *represent the acceptability and rejectability degree of* a *respectively.*

When the context is clear, we simply write Deg (resp. σ^+, σ^-) instead of $Deg^{\mathcal{S}}_{\mathbf{G}}$ (resp. $\sigma^+_{\mathbf{G}}$, $\sigma^-_{\mathbf{G}}$). For convenience, we will direct our attention to the individual functions σ^+ and σ^-. The notions of *Isomorphism* and *Path* will be used to establish principles.

Definition 3 (Isomorphism) *Let* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$ *and* $\mathbf{G}' = \langle \mathcal{A}', w', R' \rangle$ *be two WAG. An* isomorphism *from* $\mathbf{G}$ *to* $\mathbf{G}'$ *is a bijective function* f *from* $\mathcal{A}$ *to* $\mathcal{A}'$ *s.t. (i)* $\forall a \in \mathcal{A}$, $w(a) = w'(f(a))$, *(ii)* $\forall a, b \in \mathcal{A}$, $(a, b) \in R$ *iff* $(f(a), f(b)) \in R'$.

Definition 4 (Path) *We say that there is a* path *from* a_1 *to* a_n *iff there is a sequence consisting of* $\{a_1, ..., a_n\}$ *s.t.* $(a_i, a_{i+1}) \in \mathcal{R}$ *for any* $i \in \{1, ..., n-1\}$.

Principles for Semantics

The desirable properties, known as *principles*, represent the conditions that a semantics usually needs to satisfy in practical applications. The principles help to understand the foundations of semantics, choose appropriate semantics for practical applications, and construct new semantics. Numerous studies have investigated the principles of gradual semantics that consider acceptability degree (Amgoud et al. 2017; Baroni, Rago, and Toni 2019).

In this section, we propose novel principles that simultaneously take into account the *rejectability degree*. Simply speaking, the acceptability degree of an argument is determined by its basic weight as well as the acceptability and rejectability degrees of its attackers. On the other hand, the rejectability degree of an argument is solely determined by the acceptability degree of its attackers. It turns out that such a non-reciprocal setting lays down a solid base for our principles and semantics. Furthermore, the setting is also coherent with the labeling semantics (Caminada and Gabbay 2009; Wang and Shen 2023).

We consider nearly two dozen principles. Some basic ones are modified or adopted from (Amgoud et al. 2017) (e.g., Anonymity, Resilience, Proportionality). In particular, we propose principles from a bilateral view to characterize rejectability degree, including: *R-Neutrality, R-Minimality, R-Strengthening, R-Strengthening Soundness, R-Counting, R-Reinforcement*. Principles *Weakened Defense* and *Strict Weakened Defense* are introduced to generalize the notion of defense for gradual semantics in a natural and intuitive way.

Anonymity states that the acceptability and rejectability degrees of an argument are independent of its identity.

Principle 1 (Anonymity) *A semantics* $\mathcal{S}$ *satisfies* Anonymity *iff, for any two WAG* $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$ *and* $\mathbf{G}' = \langle \mathcal{A}', w', R'\rangle$*, for any isomorphism f from* $\mathbf{G}$ *to* $\mathbf{G}'$*, it holds that:* $\forall a \in \mathcal{A}$, $\sigma^+_{\mathbf{G}}(a) = \sigma^+_{\mathbf{G}'}(f(a))$, $\sigma^-_{\mathbf{G}}(a) = \sigma^-_{\mathbf{G}'}(f(a))$.

Independence states that the acceptability and rejectability degrees of an argument should be independent of any argument that is not connected to it.

Principle 2 (Independence) *A semantics* $\mathcal{S}$ *satisfies* Independence *iff, for any two WAG* $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$ *and* $\mathbf{G}' = \langle \mathcal{A}', w', R'\rangle$ *s.t.* $\mathcal{A} \cap \mathcal{A}' = \varnothing$*, it holds that:* $\forall a \in \mathcal{A}$, $\sigma^+_{\mathbf{G}}(a) = \sigma^+_{\mathbf{G}'\oplus\mathbf{G}}(a)$, $\sigma^-_{\mathbf{G}}(a) = \sigma^-_{\mathbf{G}'\oplus\mathbf{G}}(a)$.

Directionality states that the acceptability and rejectability degrees of an argument a can depend on argument b only if there is a path from b to a.

Principle 3 (Directionality) *A semantics* $\mathcal{S}$ *satisfies* Directionality *iff, for any two WAG* $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$ *and* $\mathbf{G}' = \langle \mathcal{A}, w, R'\rangle$ *s.t.* $R' = R \cup \{(a, b)\}$, $\forall x \in \mathcal{A}$*, if there is no path from b to x, then* $\sigma^+_{\mathbf{G}}(x) = \sigma^+_{\mathbf{G}'}(x)$, $\sigma^-_{\mathbf{G}}(x) = \sigma^-_{\mathbf{G}'}(x)$.

Equivalence states that: (i) the acceptability degree of an argument depends on only the acceptability and rejectability degrees of its attackers, as well as its basic weight; (ii) the rejectability degree depends on only the acceptability degree of its attackers.

Principle 4 (Equivalence) *A semantics* $\mathcal{S}$ *satisfies* Equivalence *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$, $\forall a, b \in \mathcal{A}$,

- *if (i)* $w(a) = w(b)$*, (ii) there exists a bijective function f from* $Att(a)$ *to* $Att(b)$ *s.t.* $\forall x \in Att(a)$, $\sigma^+(x) = \sigma^+(f(x))$ *and* $\sigma^-(x) = \sigma^-(f(x))$*, then* $\sigma^+(a) = \sigma^+(b)$*;*
- *if there exists a bijective function f from* $Att(a)$ *to* $Att(b)$ *s.t.* $\forall x \in Att(a)$, $\sigma^+(x) = \sigma^+(f(x))$*, then* $\sigma^-(a) = \sigma^-(b)$.

Resilience states that if the basic weight of an argument is greater than 0, then its acceptability degree cannot be reduced to 0.

Principle 5 (Resilience) *A semantics* $\mathcal{S}$ *satisfies* Resilience *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$, $\forall a \in \mathcal{A}$*, if* $w(a) > 0$*, then* $\sigma^+(a) > 0$.

Proportionality states that the higher the basic weight of an argument, the higher its acceptability degree.

Principle 6 (Proportionality) *A semantics* $\mathcal{S}$ *satisfies* Proportionality *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$, $\forall a, b \in \mathcal{A}$*, if (i)* $w(a) > w(b)$*, (ii)* $\sigma^+(a) > 0$*, and (iii)* $Att(a) = Att(b)$*, then* $\sigma^+(a) > \sigma^+(b)$.

The following principles consider the impact of attackers on the acceptability and rejectability degrees of the arguments under attack. We say an argument a is *worthless* if $\sigma^+(a) = 0$ and *alive* if $\sigma^+(a) > 0$.

A-Neutrality (resp. *R-Neutrality*) states that a worthless attacker has no impact on the acceptability (resp. rejectability) degree of the arguments it attacks.

Principle 7 (A-Neutrality) *A semantics* $\mathcal{S}$ *satisfies* A-Neutrality *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$, $\forall a, b \in \mathcal{A}$*, if (i)* $w(a) = w(b)$*, (ii)* $Att(a) = Att(b) \setminus \{x\}$ *with* $x \in Att(b)$*, and (iii)* $\sigma^+(x) = 0$*, then* $\sigma^+(a) = \sigma^+(b)$.

Principle 8 (R-Neutrality) *A semantics* $\mathcal{S}$ *satisfies* R-Neutrality *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$, $\forall a, b \in \mathcal{A}$*, if (i)* $Att(a) = Att(b) \setminus \{x\}$ *with* $x \in Att(b)$*, and (ii)* $\sigma^+(x) = 0$*, then* $\sigma^-(a) = \sigma^-(b)$.

A-Maximality states that the acceptability degree of a non-attacked argument is equal to its basic weight, and *R-Minimality* states that the rejectability degree of a non-attacked argument is equal to 0.

Principle 9 (A-Maximality) *A semantics* $\mathcal{S}$ *satisfies* A-Maximality *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$, $\forall a \in \mathcal{A}$*, if* $Att(a) = \varnothing$*, then* $\sigma^+(a) = w(a)$.

Principle 10 (R-Minimality) *A semantics* $\mathcal{S}$ *satisfies* R-Minimality *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$, $\forall a \in \mathcal{A}$*, if* $Att(a) = \varnothing$*, then* $\sigma^-(a) = 0$.

A-Weakening states that an alive attacker weakens the acceptability degree of an argument to be less than its basic weight, and *R-Strengthening* states that an alive attacker strengthens the rejectability degree to be greater than 0.

Principle 11 (A-Weakening) *A semantics $\mathcal{S}$ satisfies* A-Weakening *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$*,* $\forall a \in \mathcal{A}$*, if* $w(a) > 0$ *and* $\exists b \in Att(a)$ *s.t.* $\sigma^+(b) > 0$*, then* $\sigma^+(a) < w(a)$.

Principle 12 (R-Strengthening) *A semantics $\mathcal{S}$ satisfies* R-Strengthening *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$*,* $\forall a \in \mathcal{A}$*, if* $\exists b \in Att(a)$ *s.t.* $\sigma^+(b) > 0$*, then* $\sigma^-(a) > 0$.

A-Weakening Soundness states that alive attacks are the only source of losing acceptability degree, and *R-Strengthening Soundness* states that alive attacks are the only source of obtaining rejectability degree.

Principle 13 (A-Weakening Soundness) *A semantics $\mathcal{S}$ satisfies* A-Weakening Soundness *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$*,* $\forall a \in \mathcal{A}$ *s.t.* $w(a) > 0$*, if* $\sigma^+(a) < w(a)$*, then* $\exists b \in Att(a)$ *s.t.* $\sigma^+(b) > 0$.

Principle 14 (R-Strengthening Soundness) *A semantics $\mathcal{S}$ satisfies* R-Weakening Soundness *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$*,* $\forall a \in \mathcal{A}$*, if* $\sigma^-(a) > 0$*, then* $\exists b \in Att(a)$ *s.t.* $\sigma^+(b) > 0$.

A-Counting states that adding an alive attacker leads to a decrease in the acceptability degree of the arguments it attacks, and *R-Counting* states that adding an alive attacker leads to an increase in the rejectability degree of the arguments it attacks.

Principle 15 (A-Counting) *A semantics $\mathcal{S}$ satisfies* A-Counting *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$*,* $\forall a, b \in \mathcal{A}$*, if (i)* $w(a) = w(b)$*, (ii)* $\sigma^+(a) > 0$*, (iii)* $Att(a) = Att(b) \setminus \{x\}$ *with* $x \in Att(b)$*, and (iv)* $\sigma^+(x) > 0$*, then* $\sigma^+(a) > \sigma^+(b)$.

Principle 16 (R-Counting) *A semantics $\mathcal{S}$ satisfies* R-Counting *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$*,* $\forall a, b \in \mathcal{A}$*, if (i)* $Att(a) = Att(b) \setminus \{x\}$ *with* $x \in Att(b)$*, and (ii)* $\sigma^+(x) > 0$*, then* $\sigma^-(a) < \sigma^-(b)$.

A-Reinforcement states that increasing an attacker's acceptability degree leads to a decrease in the acceptability degree of the arguments it attacks, and *R-Reinforcement* states that increasing an attacker's acceptability degree leads to an increase in the rejectability degree of the arguments it attacks.

Principle 17 (A-Reinforcement) *A semantics $\mathcal{S}$ satisfies* A-Reinforcement *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$*,* $\forall a, b \in \mathcal{A}$*, if (i)* $w(a) = w(b)$*, (ii)* $\sigma^+(a) > 0$*, (iii)* $Att(a) \setminus \{x\} = Att(b) \setminus \{y\}$ *with* $x \in Att(a)$ *and* $y \in Att(b)$*, (iv)* $\sigma^+(x) < \sigma^+(y)$ *and* $\sigma^-(x) = \sigma^-(y)$*, then* $\sigma^+(a) > \sigma^+(b)$.

Principle 18 (R-Reinforcement) *A semantics $\mathcal{S}$ satisfies* R-Reinforcement *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$*,* $\forall a, b \in \mathcal{A}$*, if (i)* $Att(a) \setminus \{x\} = Att(b) \setminus \{y\}$ *with* $x \in Att(a)$ *and* $y \in Att(b)$*, (ii)* $\sigma^+(x) < \sigma^+(y)$*, then* $\sigma^-(a) < \sigma^-(b)$.

Principles *Weakened Defense* and *Strict Weakened Defense* are provided to refine the classical concept of *defense* for bilateral gradual semantics. The Weakened Defense principle specifically applies to situations where arguments have only one attacker, stating that increasing the rejectability of the attacker leads to an increase in the acceptability of its target arguments. Its strict version extends to a general case, stating that increasing the rejectability of any attacker leads to an increase in the acceptability of its target arguments.

Principle 19 (Weakened Defense) *A semantics $\mathcal{S}$ satisfies* Weakened Defense *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$*,* $\forall a, b \in \mathcal{A}$*, if (i)* $w(a) = w(b)$*, (ii)* $\sigma^+(a) > 0$*, (iii)* $Att(a) = \{x\}$ *and* $Att(b) = \{y\}$*, (iv)* $\sigma^-(x) > \sigma^-(y)$ *and* $\sigma^+(x) = \sigma^+(y)$*, then* $\sigma^+(a) > \sigma^+(b)$.

Principle 20 (Strict Weakened Defense) *A semantics $\mathcal{S}$ satisfies* Strict Weakened Defense *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$*,* $\forall a, b \in \mathcal{A}$*, if (i)* $w(a) = w(b)$*, (ii)* $\sigma^+(a) > 0$*, (iii)* $Att(a) \setminus \{x\} = Att(b) \setminus \{y\}$ *with* $x \in Att(a)$ *and* $y \in Att(b)$*, (iv)* $\sigma^-(x) > \sigma^-(y)$ *and* $\sigma^+(x) = \sigma^+(y)$*, then* $\sigma^+(a) > \sigma^+(b)$.

The last three principles offer three strategies when confronted with a question (Fig. 2): which aspect is more significant – the quality of attackers or the quantity of attackers?

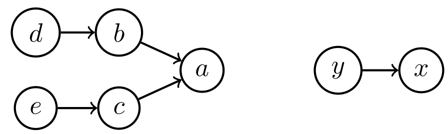


Figure 2: Let all the arguments be assigned the basic weight of 1. The argument x has a strong attacker, whereas the argument a has two weak attackers (each one is attacked). Which one is more acceptable and which one is more rejectable?

Quality Precedence (QP) prioritizes the quality of attackers, which states that (i) the greater acceptability and the lower rejectability of the *strongest* attacker of an argument, the lower its acceptability, and (ii) the greater acceptability of the *strongest* attacker of an argument, the greater its rejectability.

Principle 21 (Quality Precedence) *A semantics $\mathcal{S}$ satisfies* Quality Precedence *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$*,* $\forall a, b \in \mathcal{A}$*,*

- *if (i)* $w(a) = w(b)$*, (ii)* $\sigma^+(a) > 0$*, and (iii)* $\exists y \in Att(b)$ *s.t.* $\forall x \in Att(a)$*,* $\sigma^+(x) < \sigma^+(y)$ *and* $\sigma^-(x) > \sigma^-(y)$*, then* $\sigma^+(a) > \sigma^+(b)$*;*
- *if* $\exists y \in Att(b)$ *s.t.* $\forall x \in Att(a)$*,* $\sigma^+(x) < \sigma^+(y)$*, then* $\sigma^-(a) < \sigma^-(b)$.

Cardinality Precedence (CP) gives more importance to the quantity of attackers. It states that the greater the number of alive attackers of an argument, the lower its acceptability and the greater its rejectability.

Principle 22 (Cardinality Precedence) *A semantics $\mathcal{S}$ satisfies* Cardinality Precedence *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$*,* $\forall a, b \in \mathcal{A}$*,*

- *if (i)* $w(a) = w(b)$*, (ii)* $\sigma^+(a) > 0$*, and (iii)* $|\{x \in Att(a) | \sigma^+(x) > 0\}| < |\{y \in Att(b) | \sigma^+(y) > 0\}|$*, then* $\sigma^+(a) > \sigma^+(b)$*;*
- *if* $|\{x \in Att(a) | \sigma^+(x) > 0\}| < |\{y \in Att(b) | \sigma^+(y) > 0\}|$*, then* $\sigma^-(a) < \sigma^-(b)$.

Compensation concerns both the quality and quantity of attackers.

Principle 23 (Compensation) *A semantics $\mathcal{S}$ satisfies* Compensation *iff it violates both Quality Precedence and Cardinality Precedence.*

Links Between Principles

In this section we briefly present some links between the principles. Some principles are incompatible, i.e., they cannot be satisfied at the same time under a given semantics.

Proposition 1 *The following properties hold:*

1. *A-Maximality, QP, CP are incompatible.*
2. *Compensation is not compatible with QP or CP.*
3. *Independence, Directionality, Equivalence, Resilience, A-Maximality, R-Minimality, A-Reinforcement, R-Reinforcement, A-Weakening, R-Strengthening, A-Counting, R-Counting, Weakened Defense and QP are incompatible.*
4. *CP (resp. Compensation) is compatible with principles 1-20.*

Implications between principles are presented as below.

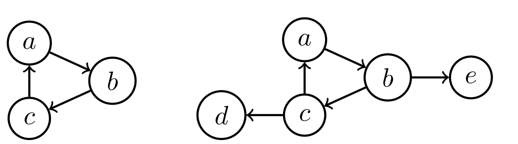
Proposition 2 *Let $\mathcal{S}$ be a semantics which satisfies Directionality, and Independence. Then:*

1. *If $\mathcal{S}$ satisfies A-Maximality, A-Neutrality, then it also satisfies A-Weakening Soundness.*
2. *If $\mathcal{S}$ satisfies R-Minimality, R-Neutrality, then it also satisfies R-Strengthening Soundness.*
3. *If $\mathcal{S}$ satisfies Equivalence, A-Maximality, A-Neutrality, A-Reinforcement, then it also satisfies A-Counting and A-Weakening.*
4. *If $\mathcal{S}$ satisfies Equivalence, A-Maximality, R-Neutrality, R-Reinforcement, then it also satisfies R-Counting and R-Strengthening.*

A number of properties concerning the acceptability degree can be found in (Amgoud et al. 2017). Here we present properties regarding the rejectability degree derived from our principles. To begin with, a set of arguments that are not attacked by any other arguments keeps their acceptability and rejectability degrees unchanged in any graph whenever the semantics satisfies Independence and Directionality.

Proposition 3 *If a semantics $\mathcal{S}$ satisfies Independence and Directionality, then for any two WAG $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$ and $\mathbf{G}' = \langle \mathcal{A}', w', R'\rangle$, if $\mathcal{A} \subseteq \mathcal{A}'$, for any $a \in \mathcal{A}$, $w(a) = w'(a)$, and $R' \cap (\mathcal{A}' \times \mathcal{A}) = R$, then for any $a \in \mathcal{A}$, $\sigma^+_{\mathbf{G}}(a) = \sigma^+_{\mathbf{G}'}(a)$ and $\sigma^-_{\mathbf{G}}(a) = \sigma^-_{\mathbf{G}'}(a)$.*

Example 1 *Consider two WAG depicted as follows and let all the arguments be assigned the basic weight of 1.*



(a) $\mathbf{G}_1$ (b) $\mathbf{G}_2$

Let $\mathcal{S}$ be a semantics which satisfies Independence and Directionality. Since $\forall x \in \{a, b, c\}$ is not attacked by $\{d, e\}$, we have $\sigma^+_{\mathbf{G}_1}(x) = \sigma^+_{\mathbf{G}_2}(x)$ and $\sigma^-_{\mathbf{G}_1}(x) = \sigma^-_{\mathbf{G}_2}(x)$.

If an argument is only attacked by worthless arguments, then its rejectability degree is 0, whenever the semantics satisfies Independence, Directionality, Equivalence, R-Minimality, and R-Neutrality.

Proposition 4 *If a semantics $\mathcal{S}$ satisfies Independence, Directionality, Equivalence, R-Minimality, and R-Neutrality, then for any WAG $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$, $\forall a \in \mathcal{A}$, if for any $x \in Att(a)$, $\sigma^+(x) = 0$, then $\sigma^-(a) = 0$.*

Worthless attackers have no effect on the rejectability degree of their target arguments whenever the semantics satisfies Independence, Directionality, Equivalence, and R-Neutrality.

Proposition 5 *If a semantics $\mathcal{S}$ satisfies Independence, Directionality, Equivalence, and R-Neutrality, then for any WAG $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$, $\forall a, b \in \mathcal{A}$, if $Att(a) \subsetneq Att(b)$ and for any $x \in Att(b) \setminus Att(a)$, $\sigma^+(x) = 0$, then $\sigma^-(a) = \sigma^-(b)$.*

Semantics and their Properties

In this section, we propose bilateral gradual semantics which correspond to three strategies: QP, CP, and Compensation. As pointed out in (Gabbay and Rodrigues 2015), the best way of defining gradual semantics is to introduce *iterative functions* that take WAG as inputs and produce sequences of values that eventually converge. We construct our semantics based on the well-studied h-categorizer function family (Besnard and Hunter 2001; Pu et al. 2014; Amgoud et al. 2017; Amgoud, Doder, and Vesic 2022). The resulting functions produce iterative sequences that *always* converge for any WAG, not just restricted to *acyclic* ones.

AR-Max-Based Semantics

The *AR-max-based semantics* (ARM) satisfies Quality Precedence, prioritizing the quality of attackers over their quantity.

Definition 5 *Let $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$ be a WAG and an argument $a \in \mathcal{A}$. An iterative sequence $\{F^i(a)\}_{i\in\mathbb{N}}$ is defined as $F^i(a) = (f^i(a), g^i(a))$ where $f^i : \mathcal{A} \to [0, 1]$ and $g^i : \mathcal{A} \to [0, 1)$ such that*

$$f^i(a) = \begin{cases} w(a), & i = 0, \\ \frac{w(a)}{1+\max\limits_{b\in Att(a)} \frac{f^{i-1}(b)}{1+g^{i-1}(b)}}, & i \geq 1. \end{cases}$$

$$g^i(a) = \begin{cases} 0, & i = 0, \\ \frac{\max\limits_{b\in Att(a)} f^{i-1}(b)}{1+\max\limits_{b\in Att(a)} f^{i-1}(b)}, & i \geq 1. \end{cases}$$

By convention, $\max\limits_{b\in Att(a)} \frac{f^{i-1}(b)}{1+g^{i-1}(b)} = 0$ and $\max\limits_{b\in Att(a)} f^{i-1}(b) = 0$ if $Att(a) = \varnothing$.

Theorem 1 *The sequence $\{F^i(a)\}_{i\in\mathbb{N}}$ converges for any $a \in \mathcal{A}$ as i approaches infinity.*

Now we define the ARM semantics according to the *limit* of the above iterative sequence.

Definition 6 (ARM semantics) *The* AR-max-based semantics *is a function ARM transforming any WAG $\mathbf{G} = \langle \mathcal{A}, w, R\rangle$ into a function $Deg^{ARM}_{\mathbf{G}}$ defined from $\mathcal{A}$ to $[0, 1] \times [0, 1)$ s.t. $\forall a \in \mathcal{A}$, $Deg^{ARM}_{\mathbf{G}}(a) = (\sigma^+(a), \sigma^-(a))$ with $\sigma^+(a) = \lim\limits_{i\to\infty} f^i(a)$, $\sigma^-(a) = \lim\limits_{i\to\infty} g^i(a)$.*

Example 2 *Applying the ARM semantics to the WAG depicted in Figure 2, we have* $\sigma^+(a) = \frac{3}{4}$, $\sigma^-(a) = \frac{1}{3}$, $\sigma^+(x) = \frac{1}{2}$, $\sigma^-(z) = \frac{1}{2}$. *So* $\sigma^+(a) > \sigma^+(x)$ *and* $\sigma^-(a) < \sigma^-(x)$. *It illustrates that the ARM semantics prioritizes the quality of attackers over their quantity.*

The acceptability and rejectability degrees assigned to the arguments by the ARM semantics satisfy the equations defined in Definition 5, stated as below.

Theorem 2 *Let* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$ *be a WAG. Then for any* $a \in \mathcal{A}$, $Deg_{\mathbf{G}}^{ARM}(a) = (\sigma^+(a), \sigma^-(a))$ *where*

$$\sigma^+(a) = \frac{w(a)}{1 + \max\limits_{b \in Att(a)} \frac{\sigma^+(b)}{1+\sigma^-(b)}},$$

$$\sigma^-(a) = \frac{\max\limits_{b \in Att(a)} \sigma^+(b)}{1 + \max\limits_{b \in Att(a)} \sigma^+(b)}.$$

Since the ARM semantics focuses on the *strongest* attacker, it violates Strict Weakened Defense, A-Counting, R-Counting, A-Reinforcement, and R-Reinforcement. The theorem below says that the ARM semantics satisfies *all* the rest principles that are compatible with QP.

Theorem 3 *The ARM semantics violates Strict Weakened Defense, A-Counting, R-Counting, A-Reinforcement, R-Reinforcement, CP and Compensation. It satisfies all the remaining principles.*

AR-Card-Based Semantics

The *AR-card-based semantics* (ARC) satisfies Cardinality Precedence, giving more importance to the quantity of attackers. Moreover, the semantics considers only *founded* attackers, i.e., attackers with basic weight greater than 0. This is because unfounded arguments are worthless and their attacks are ineffective.

Definition 7 *Let* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$ *be a WAG and* $a \in \mathcal{A}$. *The argument* a *is* founded *iff* $w(a) > 0$. *It is* unfounded *otherwise. Let* $Att^*_{\mathbf{G}}(a)$ *(simplified as* $Att^*(a)$*) denote the set of the founded attackers of* a.

Now we introduce the following iterative sequences for AR-card-based semantics.

Definition 8 *Let* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$ *be a WAG and an argument* $a \in \mathcal{A}$. *An iterative sequence* $\{F^i(a)\}_{i \in \mathbb{N}}$ *is defined as* $F^i(a) = (f^i(a), g^i(a))$ *where* $f^i : \mathcal{A} \to [0,1]$ *and* $g^i : \mathcal{A} \to [0,1)$ *such that*

$$f^i(a) = \begin{cases} w(a), & i = 0, \\ \frac{w(a)}{1+|Att^*(a)|+\frac{1}{n}\cdot \sum\limits_{b\in Att^*(a)} \frac{f^{i-1}(b)}{1+g^{i-1}(b)}}, & i \geq 1. \end{cases}$$

$$g^i(a) = \begin{cases} 0, & i = 0, \\ \frac{|Att^*(a)|+\frac{1}{n}\cdot \sum\limits_{b\in Att^*(a)} f^{i-1}(b)}{1+|Att^*(a)|+\frac{1}{n}\cdot \sum\limits_{b\in Att^*(a)} f^{i-1}(b)}, & i \geq 1. \end{cases}$$

with $n = |\mathcal{A}|$. *By convention,* $\sum\limits_{b\in Att^*(a)} \frac{f^{i-1}(b)}{1+g^{i-1}(b)} = 0$ *and* $\sum\limits_{b\in Att^*(a)} f^{i-1}(b) = 0$ *if* $Att^*(a) = \varnothing$.

Theorem 4 *The sequence* $\{F^i(a)\}_{i\in\mathbb{N}}$ *converges for any* $a \in \mathcal{A}$ *as* i *approaches infinity.*

The ARC semantics is defined according to the *limit* of the above iterative sequence.

Definition 9 (ARC semantics) *The* AR-card-based semantics *is a function* ARC *transforming any WAG* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$ *into a function* $Deg_{\mathbf{G}}^{ARC}$ *defined from* $\mathcal{A}$ *to* $[0,1] \times [0,1)$ *s.t.* $\forall a \in \mathcal{A}$, $Deg_{\mathbf{G}}^{ARC}(a) = (\sigma^+(a), \sigma^-(a))$ *with* $\sigma^+(a) = \lim\limits_{i\to\infty} f^i(a)$, $\sigma^-(a) = \lim\limits_{i\to\infty} g^i(a)$.

Example 3 *Applying the ARC semantics to the WAG depicted in Figure 2, we have* $\sigma^+(a) = \frac{5}{16}$, $\sigma^-(a) = \frac{7}{10}$, $\sigma^+(x) = \frac{1}{3}$, $\sigma^-(x) = \frac{2}{3}$. *So* $\sigma^+(a) < \sigma^+(x)$ *and* $\sigma^-(a) > \sigma^-(x)$. *It illustrates that the ARC semantics prioritizes the quantity of attackers over their quality.*

The acceptability and rejectability degrees assigned to the arguments by the ARC semantics satisfy the equations defined in Definition 8, stated as below.

Theorem 5 *Let* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$ *be a WAG and* $n = |\mathcal{A}|$. *Then for any* $a \in \mathcal{A}$, $Deg_{\mathbf{G}}^{ARC}(a) = (\sigma^+(a), \sigma^-(a))$ *where*

$$\sigma^+(a) = \frac{w(a)}{1 + |Att^*(a)| + \frac{1}{n}\cdot \sum\limits_{b\in Att^*(a)} \frac{\sigma^+(b)}{1+\sigma^-(b)}}$$

$$\sigma^-(a) = \frac{|Att^*(a)| + \frac{1}{n}\cdot \sum\limits_{b\in Att^*(a)} \sigma^+(b)}{1 + |Att^*(a)| + \frac{1}{n}\cdot \sum\limits_{b\in Att^*(a)} \sigma^+(b)}.$$

The following theorem says that the ARC semantics satisfies *all* the principles that are compatible with CP.

Theorem 6 *The ARC semantics satisfies all the principles except QP and Compensation.*

AR-Hybrid-Based Semantics

The *AR-hybrid-based semantics* (ARH) satisfies Compensation, taking both the quantity and quality of attackers into account.

Definition 10 *Let* $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$ *be a WAG and an argument* $a \in \mathcal{A}$. *An iterative sequence* $\{F^i(a)\}_{i\in\mathbb{N}}$ *is defined as* $F^i(a) = (f^i(a), g^i(a))$ *where* $f^i : \mathcal{A} \to [0,1]$ *and* $g^i : \mathcal{A} \to [0,1)$ *such that*

$$f^i(a) = \begin{cases} w(a), & i = 0, \\ \frac{w(a)}{1+|Att^*(a)|+ \sum\limits_{b\in Att^*(a)} \frac{f^{i-1}(b)}{1+g^{i-1}(b)}}, & i \geq 1. \end{cases}$$

$$g^i(a) = \begin{cases} 0, & i = 0, \\ \frac{|Att^*(a)|+ \sum\limits_{b\in Att^*(a)} f^{i-1}(b)}{1+|Att^*(a)|+ \sum\limits_{b\in Att^*(a)} f^{i-1}(b)}, & i \geq 1. \end{cases}$$

Theorem 7 *The sequence* $\{F^i(a)\}_{i\in\mathbb{N}}$ *converges for any* $a \in \mathcal{A}$ *as* i *approaches infinity.*

Definition 11 (ARH semantics) *The* AR-hybrid-based semantics *is a function ARH transforming any WAG $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$ into a function $Deg^{ARH}_{\mathbf{G}}$ defined from $\mathcal{A}$ to $[0,1] \times [0,1)$ s.t. $\forall a \in \mathcal{A}$, $Deg^{ARH}_{\mathbf{G}}(a) = (\sigma^+(a), \sigma^-(a))$ with $\sigma^+(a) = \lim_{i \to \infty} f^i(a)$, $\sigma^-(a) = \lim_{i \to \infty} g^i(a)$.*

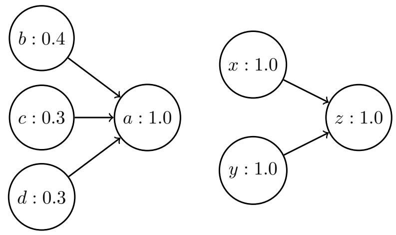


Figure 3: ARH Semantics Illustration

Example 4 *Applying the ARH semantics to the WAG depicted in Figure 3, we have (i) $\exists x \in Att(z)$ s.t. for any $b \in Att(a)$, $\sigma^+(x) > \sigma^+(a)$, (ii) $|\{x \in Att(a)|\sigma^+(x) > 0\}| < |\{y \in Att(b)|\sigma^+(y) > 0\}|$. But $\sigma^-(a) = \sigma^-(z) = \frac{4}{5}$. Therefore, the ARH semantics satisfies Compensation.*

The acceptability and rejectability degrees assigned to the arguments by the ARH semantics satisfy the equations defined in Definition 10, stated as below.

Theorem 8 *Let $\mathbf{G} = \langle \mathcal{A}, w, R \rangle$ be a WAG. Then for any $a \in \mathcal{A}$, $Deg^{ARH}_{\mathbf{G}}(a) = (\sigma^+(a), \sigma^-(a))$ where*

$$\sigma^+(a) = \frac{w(a)}{1 + |Att^*(a)| + \sum\limits_{b \in Att^*(a)} \frac{\sigma^+(b)}{1+\sigma^-(b)}}$$

$$\sigma^-(a) = \frac{|Att^*(a)| + \sum\limits_{b \in Att^*(a)} \sigma^+(b)}{1 + |Att^*(a)| + \sum\limits_{b \in Att^*(a)} \sigma^+(b)}.$$

Theorem 9 says that the ARH semantics satisfies *all* the principles that are compatible with Compensation.

Theorem 9 *The ARH semantics satisfies all the principles except QP and CP.*

Comparisons for principles under ARM, ARC and ARH are summaried in Table 2.

	ARM	ARC	ARH
Anonymity	✓	✓	✓
Independence	✓	✓	✓
Directionality	✓	✓	✓
Equivalence	✓	✓	✓
Resilience	✓	✓	✓
Proportionality	✓	✓	✓
A-Neutrality	✓	✓	✓
R-Neutrality	✓	✓	✓
A-Maximality	✓	✓	✓
R-Minimality	✓	✓	✓
A-Weakening	✓	✓	✓
R-Strengthening	✓	✓	✓
A-Weakening soundness	✓	✓	✓
R-Strengthening soundness	✓	✓	✓
A-Counting	✗	✓	✓
R-Counting	✗	✓	✓
A-Reinforcement	✗	✓	✓
R-Reinforcement	✗	✓	✓
Weakened Defense	✓	✓	✓
Strict Weakened Defense	✗	✓	✓
Quality Precedence	✓	✗	✗
Cardinality Precedence	✗	✓	✗
Compensation	✗	✗	✓

Table 2: Principles under ARM, ARC and ARH semanatics

Discussion and Conclusion

The gradual semantics has received considerable attention in the literature and a plethora of semantics have been proposed, e.g., trust-based semantics (da Costa Pereira, Tettamanzi, and Villata 2011), iterative-based semantics (Gabbay and Rodrigues 2015), weighted max-based, card-based and h-categorizer semantics (Amgoud et al. 2017; Amgoud, Doder, and Vesic 2022), QuAD semantics (Baroni et al. 2015), DF-QuAD semantics (Rago et al. 2016). However, to the best of our knowledge, none of the existing gradual semantics considers the rejectability degree for argument evaluation. In the probabilistic approach (Hunter, Polberg, and Thimm 2020; Polberg and Hunter 2018), the authors proposed that the acceptability of an argument can be expressed by the degree of believed and disbelieved. Nonetheless, the foundational approach to establishing semantics diverges considerably from our framework.

In this paper, we formalized a comprehensive framework to evaluate arguments through the acceptability and rejectability degrees. This novel approach provides new insight into argument strength and gives us a deeper understanding of the status of arguments. We then proposed a set of desirable properties that take into account both acceptability and rejectability degrees. Furthermore, we provided three novel semantics that satisfy most of the desirable properties: AR-max-based semantics, AR-card-based semantics and AR-hybrid-based semantics.

We believe bilateral gradual semantics can be applied to many areas including argumentative decision-making (Amgoud and Prade 2009), argumentative explainable AI (Čyras et al. 2021), etc., as considering both the positive and negative strength is a common approach in practice. In future work, we shall introduce bilateral gradual semantics to other argumentation systems, such as weighted bipolar argumentation graphs (Amgoud and Ben-Naim 2018) and SETAF (Nielsen and Parsons 2006).

Acknowledgements

This work was partially supported by the National Key Research Institutes Project (19JJD720002) and the National Social Science Fund of China (20BZX106).

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