

# Effective Integration of Weighted Cost-to-Go and Conflict Heuristic within Suboptimal CBS

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## Abstract

Conflict-Based Search (CBS) is a popular multi-agent path finding (MAPF) solver that employs a low-level single agent planner and a high-level constraint tree to resolve conflicts. The vast majority of modern MAPF solvers focus on improving CBS by reducing the size of this tree through various strategies with few methods modifying the low level planner. Typically low level planners in existing CBS methods use an *unweighted* cost-to-go heuristic, with suboptimal CBS methods also using a conflict heuristic to help the high level search. In this paper, we show that, contrary to prevailing CBS beliefs, a *weighted* cost-to-go heuristic can be used effectively alongside the conflict heuristic in two possible variants. In particular, one of these variants can obtain large speedups, 2-100x, across several scenarios and suboptimal CBS methods. Importantly, we discover that performance is related not to the weighted cost-to-go heuristic but rather to the *relative conflict heuristic weight's* ability to effectively balance low-level and high-level work. Additionally, to the best of our knowledge, we show the first theoretical relation of prioritized planning and bounded suboptimal CBS and demonstrate that our methods are their natural generalization.

## Introduction

Multi-Agent Path Finding (MAPF) is the problem of computing collision-free paths for a team of agents in a known environment while minimizing a measure of their travel times. This is required for several real-world tasks such as the smooth operation of automated warehouses (Li et al. 2020b), robot soccer (Biswas et al. 2014), collaborative manufacturing (Sun and Mills 2002), coverage (Kusnur et al. 2021), and others. MAPF is a challenging problem and is shown to be NP-complete (Ratner and Warmuth 1986).

Prioritized Planning (PP) (Erdmann and Lozano-Perez 1987) is a fast multi-agent planning approach that sequentially plans agents avoiding earlier agents with better “priority”, and has been applied to several domains (Wu, Bhat-tacharya, and Prorok 2020; Čáp et al. 2015; Velagapudi, Sycara, and Scerri 2010). However PP provides no guarantees on completeness or bounded suboptimality.

Conflict-Based Search (CBS) is a popular complete and optimal MAPF solver that employs a low-level single agent

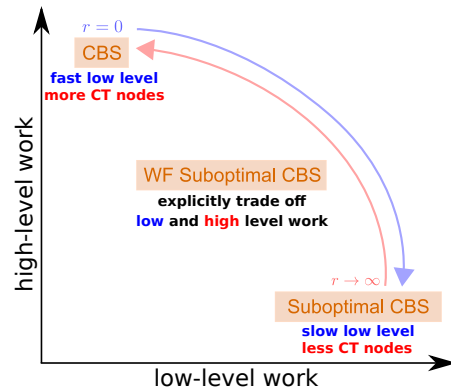


Figure 1: CBS’s original low-level planner finds shortest paths ignoring additional conflicts created, resulting in a large amount of constraint tree (CT) nodes for the high-level search to resolve the conflicts. Suboptimal CBS (ECBS, EECBS) utilize a low-level focal search that minimizes future conflicts alongside path length, resulting in a slower low level planner but substantially reducing the high level work and boosting performance. Our Weighted Focal method improves performance by finding a sweet spot in the middle. We introduce a hyper-parameter  $r$  in the low-level focal search which controls trading off low level work (finding paths) and high level work (resolving conflicts), allowing us to find low-level plans faster with a little more high level work than current suboptimal CBS methods.

planner and a high-level constraint tree (CT) to resolve conflicts. Several methods speed up CBS by reducing the CT size by explicitly pruning branches, selectively expanding branches, adding sets of constraints, detecting symmetries, and improving high-level heuristics (Boyerski et al. 2015, 2021; Li et al. 2019, 2020a, 2021).

Enhanced CBS (ECBS) (Barer et al. 2014) introduced the first bounded-suboptimal version of CBS, utilizing a focal search on the high level as well as another focal search planner on the low level that minimizes path conflicts with other agents and therefore decreases the CT size. ECBS specifically mentions how modifying the low level planner to use a

weighted cost-to-go heuristic returns paths with many conflicts, leading to a larger CT tree and proved “ineffective in [their] experiments” as direct motivation for reducing the path conflicts instead. Explicit Estimation CBS (EECBS) (Li, Ruml, and Koenig 2021) replaces ECBS’s high level focal search with Explicit Estimation Search (Thayer and Ruml 2011) but keeps the same low level focal search. Continuous-time CBS (CCBS) (Andreychuk et al. 2019) incorporates Safe Interval Path Planning (SIPP) (Phillips and Likhachev 2011) to speed up the low level search by reasoning about waits but also does not employ a weighted heuristic. To the authors’ best knowledge, no prior work has effectively used a weighted cost-to-go heuristic in any manner in the CBS framework, with the prevailing norm that doing so would lead to more conflicts, reduce performance, or remove bounded sub-optimality.

Our initial insight is that we can use the weighted cost-to-go heuristic along with the conflict heuristic. We introduce the first bounded sub-optimal CBS methods that incorporate a weighted cost-to-go heuristic with the conflict heuristic within CBS’s single agent planner. We find that performance (and large performance gains) is controlled by the relative weight of the cost-to-go and conflict heuristic which allows us to trade off low-level and high-level work, see Figure 1. This runs counter to the experience of researchers familiar with single agent planning where we would expect the weighted cost-to-go heuristic to improve performance by finding paths faster. In order of importance, our contributions are:

1. Providing insight on the *conflict* heuristic’s impact on performance. We combine a weighted cost-to-go heuristic with a weighted conflict heuristic ratio in the focal queue and demonstrate that the relative weight, not the weighted cost-to-go heuristic, dictates performance. We provide additional analysis and show how this behaviour is novel in respect to existing CBS and single-agent planning intuition.
2. Incorporating the weighted cost-to-go heuristic in the open queue, and studying how the path lower bounds interact with certain CBS improvements.
3. Reducing PP to a particular step of suboptimal CBS and showing that our methods are the natural generalization.

## Incorporating Weighted Cost-to-go Heuristic

MAPF problems have many variants; we focus on the typical MAPF problem which is defined by a graph, a set of agents with corresponding start-goal vertices, and the objective of finding collision free paths that minimizes the sum of their travel times. Like related work, the graph is defined by a discretized environment with discretized timesteps, where at each timestep agents can wait in their current vertex or move to adjacent vertices. Our final path must avoid vertex conflicts (two agents at the same vertex) and edge conflicts (two agents using the same edge in different directions).

CBS is a foundational algorithm for several modern MAPF solvers which uses a high-level conflict solver and a low-level path planner to find a set of conflict free minimum cost paths. In a broader perspective, the CBS frame-

work employs a high-level constraint tree that searches over conflicts in a best-first manner, and a low level A\* search that searches individual paths while respecting constraints. CBS’s space-time A\* low level planner utilizes a precomputed cost-to-go heuristic that measures the optimal distance to goal ignoring conflicts. Bounded sub-optimal CBS methods (e.g. ECBS, EECBS) modify the single agent planner to a focal search that computes  $w_{so}$  sub-optimal path that minimizes the number of conflicts with other agents (which reduces future constraints in the CT). The low level planner must also return a lower bound on the optimal solution cost which is required for certain CBS improvements, specifically prioritized conflicts and symmetry reasoning, see Li, Ruml, and Koenig (2021) for full justification. The low level focal search has two queues; OPEN which searches over optimal paths (paths sorted by cost) and maintains an optimality bound, and FOCAL which prioritizes  $w_{so}$  sub-optimal paths with fewer conflicts (paths sorted by conflicts). We specifically discuss our method in relation to EECBS as it was shown to outperform ECBS and other MAPF planners, but our method is directly usable in ECBS and any other bounded sub-optimal CBS planner using a low level focal planner (see Table 4).

Our main idea is to incorporate a  $w_h$  weighted cost-to-go heuristic in the single agent planner in two ways: one in OPEN independent of the conflict heuristic and the other in FOCAL along with the conflict heuristic. Algorithm 1 shows cases EECBS’s general low level search pseudocode, with W-EECBS changes underlined. Ties in FOCAL are broken by  $f_{open}$ . The user’s suboptimality hyper-parameter  $w_{so}$  is assumed to be fixed and outside our optimization.

**Intuition:** Our intuition is that by incorporating the cost-to-go weight with the conflict heuristic, we should be able to plan faster while also minimizing conflicts. This intuition is informed by single-agent planning where weighting the cost-to-go heuristic speeds up search. We therefore think that increasing  $w_h$  alongside using the conflict heuristic in FOCAL will result in the low-level planner finding similar quality paths faster without many additional conflicts, resulting in speed ups. However, we find that increasing  $w_h$  does *not* help performance significantly. Instead we find that modulating the conflict heuristic’s relative weight in FOCAL substantially improves overall performance.

## Weighted Open Variant (WO-EECBS)

OPEN’s priority function is weighted by  $w_h$ , while FOCAL remains unchanged, prioritized by the number of conflicts. To maintain our overall suboptimality bound, the focal bound  $w_f$  is scaled to  $w_{so}/w_h$  which constrains  $w_h \in [1, w_{so}]$  as we need  $w_f \geq 1$ . Since the f-values in OPEN are now weighted by  $w_h$ , we obtain a lower bound on the optimal path cost by scaling the minimum f-value in OPEN,  $F_{best}$ , to  $F_{best}/w_h$ . Note that  $w_h = 1$  trivially results in regular EECBS.

Prioritized conflicts (PC) is technique which attempts to speed up the CT search by categorizes which conflicts should be chosen by the CT tree to resolve, and is applied when an agent is on its shortest path (determined when agent’s lowerbound path cost equals the found path cost)

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**Algorithm 1: Suboptimal CBS low level focal search planner**

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**Input:**  $n_{start}$ , atGoal(), Paths  $P_I$  of other agents**Output:** Lower bound  $LB$  on optimal path cost, Path from  $n_{start}$  with sub-optimality  $\leq w_{so}$  (i.e. cost  $\leq w_{so} * LB$ )

```
1: Set $W_f()$ 
2: OPEN = FOCAL =  $\{n_{start}\}$ ,  $LB = F_{best} = 0$ 
3: while FOCAL  $\neq \emptyset$  do
4:    $n \leftarrow$  FOCAL.pop()
5:   OPEN.remove( $n$ )
6:    $LB \leftarrow$  max( $LB$ , UpdateLowerBound())
7:   if atGoal( $n$ ) then
8:     return  $LB$ , Solution backtracking from  $n$ 
9:   for  $n' \in$  succ( $n$ ) do
10:     $g \leftarrow n.g + cost(n, n')$ 
11:     $h \leftarrow$  getCostToGoHeuristic( $n'$ )
12:     $n'.F_{open} \leftarrow f_{open}(g, h)$ 
13:    OPEN.insert( $n'$ )
14:     $c \leftarrow$  getNumConflictsFromPaths( $n'$ ,  $P_I$ )
15:     $n'.F_{focal} \leftarrow f_{focal}(g, h, c)$ 
▷ Update FOCAL
16:    $F_{best} \leftarrow \min_{k \in OPEN} k.F_{open}$ 
17:   for all  $n' \in OPEN, n' \notin FOCAL$  do
18:     if  $n'.F_{open} \leq w_f * F_{best}$  then
19:       FOCAL.insert( $n'$ )
20: return NaN, No solution

21: procedure  $f_{open}(g, h)$ :
22:   return  $g + h$ 
23: procedure  $f_{focal}(g, h, c)$ :
24:   return  $c$ 
25: procedure Set $W_f()$ :
26:    $w_f \leftarrow w_{so}$ 
27: procedure UpdateLowerBound():
28:   return  $F_{best}$ 
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(Čáp et al. 2015; Li, Ruml, and Koenig 2021). Symmetry reasoning (SR) techniques add groups of constraints to speed up the CT search and can only be applied when agents are on their shortest paths as well (Li et al. 2021; Li, Ruml, and Koenig 2021). One side effect of WO-EECBS is that the naively computed lower bound is usually substantially lower than the optimal path cost even though the path may not have been very sub-optimal. Several papers have discussed this pessimistic lower bound in weighted A\* single agent search (Thayer and Ruml 2008; Holte et al. 2019). This pessimistic lower bound should then theoretically reduce the amount of prioritized conflicts (PC) and symmetry reasoning (SR) applied. We therefore *a posteriori* compute a better lower bound using Holte et al. (2019) and test if this increases the usage of PC and SR, and boosts performance.

### Weighted Focal Variant (WF-EECBS)

We keep OPEN unweighted and instead incorporate the weighted heuristic in FOCAL along with the inadmissible conflict heuristic. This requires us to balance the importance

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**Algorithm 2: Weighted Open modifications**

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**Parameters:** Cost-to-go heuristic weight  $w_h$ 

```
1: procedure  $f_{open}(g, h)$ :
2:   return  $g + w_h * h$ 
3: procedure Set $W_f()$ :
4:    $w_f \leftarrow w_{so}/w_h$ 
5: procedure GetLowerBound(): ▷ Naive
6:   return  $F_{best}/w_h$ 
7: procedure GetLowerBound(): ▷ Improved
8:    $g_{min} \leftarrow \min_{n \in OPEN} n.g$ 
9:   return  $(F_{best} + (w_h - 1) * g_{min})/w_h$ 
```

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**Algorithm 3: Weighted Focal modifications**

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**Parameters:**  $w_h$ , Relative conflict weight  $r$ 

```
1: procedure  $f_{focal}(g, h, c)$ :
2:   return  $g + w_h * (h + r * c)$ 
```

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of these competing heuristics in FOCAL’s priority function  $g + w_h * (h + r * c)$  with  $w_h \geq 1, r \geq 0$ . Manipulating  $r$  changes the relative importance of finding a solution fast (lower  $r$ ) vs avoiding conflicts (higher  $r$ ). Note that  $w_h = 1$  and  $r \rightarrow \infty$  results in regular EECBS (preferring paths with lowest conflicts). Due to the use of FOCAL,  $w_h$  (and  $r$ ) can be arbitrarily large and is not bounded by  $w_{so}$  unlike WO-EECBS. In our experiments WF-EECBS significantly outperforms WO-EECBS and EECBS, therefore Weighted EECBS (W-EECBS) refers to this weighted focal version.

**Lemma 1.** *WO-EECBS and WF-EECBS are both  $w_{so}$  sub-optimal.*

*Proof.* EECBS’s overall optimality is split between the high-level CT sub-optimal search and the low-level sub-optimal search. Since the high-level search is unchanged and identical to EECBS, we just need to prove that WO-EECBS and WF-EECBS have the same low-level sub-optimality  $w_{so}$  as EECBS.

In WO-EECBS: FOCAL returns a node at most  $w_f$  sub-optimal compared to OPEN which is weighted by  $w_h$ . Our overall optimality is then  $w_f * w_h = w_{so}/w_h * w_h = w_{so}$ .

In WF-EECBS: FOCAL’s sub-optimality is fixed regardless of  $f_{focal}$ , and OPEN is optimal, so our overall optimality is trivially  $w_f = w_{so}$ .  $\square$

### Relating CBS, Prioritized Planning, and W-EECBS

CBS-based algorithms and PP are usually treated as distinct categories of MAPF search based methods. Ma et al. (2019) introduces priorities in CBS as a distinction to regular CBS and Li et al. (2022) employs a modified PP planner that return paths with least conflicts, but neither attempt to relate PP and CBS.

Here we prove that PP is actually equivalent to the first step of generating the initial agent paths in the root CT node in EECBS (and other bounded sub-optimal CBS planners like ECBS) with an infinite sub-optimality. With  $w_{so} = \infty$  in EECBS, all states in OPEN in the single agent planner are inserted into FOCAL, and therefore expansions are

sorted first by their number of conflicts, and then the path f-value. In the root CT node, agents will try to avoid all previous agents and search over all conflict=0 paths, then conflict=1 after exhausting all conflict=0 paths, then conflict=2, etc. This first step is identical to PP; EECBS with  $w_{so} = \infty$  differs only in its ability to continue planning over conflicts while PP fails in that scenario. To the authors’ knowledge, this is the first time there has been an explicit relation between sub-optimal CBS and PP. WO-EECBS and WF-EECBS are the two generalized methods combining the weighted low-level planner commonly used in PP with EECBS’s conflict resolution mechanism.

## Experimental Results

We test our methods with different numbers of agents, in increments of 50, on 8 diverse maps from Stern et al. (2019) and report the mean values across 5 seeds. Table 1 shows the diversity of the maps; plots contain 4 maps in the same order sorted by decreasing free states (with the full plots showing all 8 in the appendix).

Map name	Max # agents	Raw Area	# free states
Paris_1_256	1000	256x256	47240
den520d	1000	256x257	28178
ht_chantry	1000	162x141	7461
den312d	1000	65x81	2445
empty-48-48	1000	48x48	2304
empty-32-32	500	32x32	1024
random-32-32-10	450	32x32	922
random-32-32-20	400	32x32	819

Table 1: Map statistics — We show the maximum number of agents, height by width raw area, and number of free states on each of the maps we use in our evaluations. Figures are sorted in this same order horizontally (top left subplot will be the largest map, bottom right will be the smallest), to showcase the relationship of performance with map size.

We use  $w_{so} = 2$  and a timeout of 300 seconds in all our experiments unless otherwise specified. In all figures, if a method failed (timed out on all 5 seeds) on a particle number of agents on a map, we do not report larger number of agents. The speed up  $S_{method} = T_{baseline}/T_{method}$  is reported to normalize differences in hardware, where the baseline is the unweighted method (ECBS or EECBS) based on context. In all tables, speeds up are computed only on instances where the baseline did not timeout.

### Weighted Open

Overall, performance with the weighted anchor variant varies based on the map; it provides large speed ups (10+) in 2, medium (1-5) in 3, and hurts (0-1) in 3. Figure 2 shows that improving the lower bound on the usage of CBS improvements does lead to higher utilization. Contrary to our expectations, Table 2 and Figure 3 reveal this causes worse performance even though this computation has negligible overhead.

The relative performance of  $w_h$  in WO-EECBS fits our intuition with larger  $w_h$  helping to a certain extent and then

Method		Speed up		% instances faster than Baseline	# solved
$w_h$	LB+	Max	Median		
1	False	1	1	N/A, is baseline	98
1.2	False	3	1.4	69%	102
1.2	True	3	1.2	61%	<b>103</b>
1.5	False	<b>37</b>	<b>2.7</b>	70%	101
1.5	True	36	2.4	<b>72%</b>	<b>103</b>
2	False	32	0.66	39%	67
2	True	28	0.31	14%	49

Table 2: WO-EECBS Results — We report the max and median speed up across all 8 maps, as well as the number of instances solved and better than the baseline. We see that  $w_h = 1.5$  produces the best speed up and that in general improving the lower bound (LB+ set to True) decreases performance.

Method		Speed up		% faster than Baseline	# solved
$r$	$w_h$	Max	Median		
$\infty$	1	1	1	-	98
2.5	8	84	5.5	77%	106
2.5	16	112	6.8	79%	104
5	4	<b>131</b>	9.5	<b>92%</b>	<b>113</b>
5	8	127	9.4	89%	109
5	16	113	<b>11</b>	88%	109
10	4	109	5.1	90%	111
10	8	91	7.2	89%	110

Table 3: WF-EECBS Results — Comparing against Table 2 we see that WF-EECBS greatly outperforms WO-EECBS and the baseline in the majority of instances. The first row describes the EECBS baseline in WF-EECBS parameters.

hurting due to the interplay with the focal queue. Concretely, WO-EECBS with a “saturated” anchor weight of 2 provides lower speed-ups as the focal queue in that instance has  $w_f = w_{so}/w_{so} = 1$  and thus flexibility to reduce the number of collisions. The next section demonstrates that this variant is dominated by the weighted focal variant.

### Weighted Focal

Table 3 demonstrates that WF-EECBS’s speed up is consistently higher than the baseline and WO-EECBS. Overall WF-EECBS helps on 7 out of 8 maps, providing large speed ups (10+) on three and massive speed ups (50-100+) on two. Weighted EECBS (W-EECBS) therefore refers to this weighted focal version.

Figure 5 show a surprising and important relationship between the cost-to-go weight  $w_h$  and relative conflict weight  $r$  in WF-EECBS; the performance is dominated by  $r$  rather than  $w_h$ , with optimal values  $r \in [2, 16]$ . The relative weight  $r$  explicitly dictates the tradeoff between searching longer to avoid a future conflict or planning shorter and incurring the conflict which will need to be resolved by the constraint tree afterwards. Regular EECBS lacks this flexibility and with  $r \rightarrow \infty$  will prioritize planning longer to avoid conflicts. Table 3 shows that increasing  $w_h$  with the same  $r$  usually but not necessarily increases median speed up.

We check how incorporating the weighted heuristic gen-

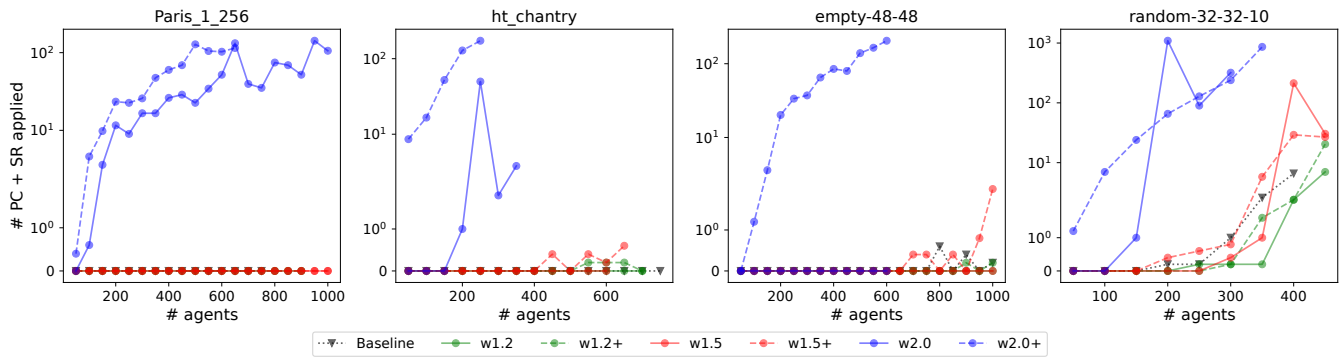


Figure 2: Viewing the effect of improving the lowerbound on the WO-EECBS — The “+” label denotes using an improved lowerbound; improving the lowerbound leads to a significant higher usage of CBS improvements with the y-axis denoting the average number (across 5 seeds) of cardinal conflicts and symmetry reasoning applied for each problem instance. Without the improved lower bound, WO-EECBS is usually unable to use these CBS improvements. Methods terminate on a map once they fail all 5 seeds on a certain number of agents or they reach the maximum number of agents in a scene, fractional values are due to the averaging over 5 seeds.

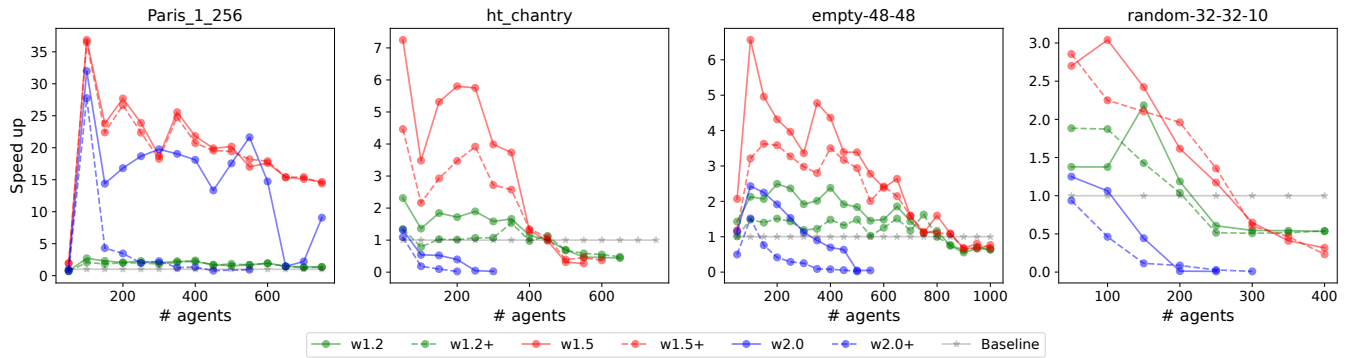


Figure 3: WO-EECBS full results — The “+” label denotes using an improved lowerbound. A medium weighted value of  $w=1.5$  performs the best on both maps. However, the maximum speed up peaks at around 35 in the Paris scenario and it struggles for larger number of agents (e.g.  $> 400$  in ht\_chantry) and in all the harder scenarios (e.g. both random maps, empty-32-32, den312d in appendix). Additionally the improved lowerbound actually decreases performance contrary to our expectations.

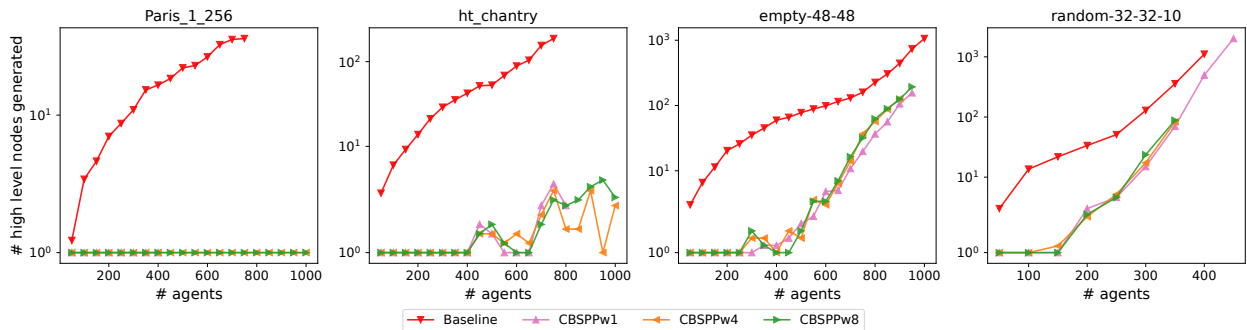


Figure 4: PP, CBS, W-EECBS equivalence — We see that running “CBSPP” (WO-EECBS with a very large suboptimality factor simulating  $w_{so} \leftarrow \infty, w_h = w$ , or equivalently WF-EECBS with  $w_{so}, r \leftarrow \infty, w_h = w$ ) is equivalent to running  $w$  weighted prioritized planning with CBS’s conflict resolution capability. This is highlighted by the number of high level nodes sticking to one with low numbers of agents (identical to PP) as opposed to the baseline with several high level nodes, and then increasing only after conflicts are forced. Observe how the larger maps (top row) are able to be solved in only one high level node (i.e. no conflict resolution required), but smaller maps require reasoning over conflicts. Note the differing log y-axis values across different graphs. Fractional values due to averaging across 5 seeds.

Method		Speedup		% faster than Baseline	# solved vs Baseline
-CBS	$w_{so}$	Max	Median		
E	1.01	1.7	0.45	23%	30/42
EE	1.01	1.8	0.71	32%	36/46
E	1.1	9.9	2.6	75%	71/76
EE	1.1	10	2.3	73%	68/75
E	1.2	16	3.7	80%	83/77
EE	1.2	22	3.1	80%	79/77
E	1.5	35	3.5	83%	95/73
EE	1.5	47	3.7	78%	94/73
E	2	88	5.5	85%	108/77
EE	2	88	5.5	85%	105/77
E	4	137	7.8	91%	110/66
EE	4	137	8.5	92%	110/67
E	8	164	6.9	91%	112/69
EE	8	164	9.6	91%	111/68

Table 4: Generalizing weighting FOCAL to different suboptimal CBS methods and suboptimality. — We compare the effect of weighting FOCAL on both ECBS and EECBS across different suboptimality. We use  $r = 5, h = 8$  and a timeout of 60 seconds across all experiments, and report statistics as in Table 2. The last column shows the number of instances solved (numerator) vs the baseline (denominator). We see that incorporating the weighted FOCAL hurts at a very low suboptimality  $w_{so} = 1.01$ , but then produces large benefits for  $w_{so} \geq 1.5$ . Additionally, we see very similar speedups across different methods at the same suboptimality, demonstrating how our method’s benefits are generalizable across different suboptimal CBS methods.

eralizes across different methods and suboptimality. Table 4 shows the effect of using a weighted focal with  $r = 5, h = 8$  across different suboptimality on ECBS and EECBS. These hyper-parameters were chosen solely based on Table 3 (on EECBS with  $w_{so} = 2$ ) and were intentionally not optimized for this experiment. We employ a timeout of 60 seconds and compare against the corresponding unweighted baseline. We observe that our weighted focal hurts at a very low suboptimality  $w_{so} = 1.01$  but then steadily results in larger performance boosts as  $w_{so}$  increases. In particular, for large suboptimality  $w_{so} \geq 1.5$ , we see that our weighted methods start to solve significantly more instances than the baseline. For  $w_{so} \geq 2$ , we also get large and consistent speed up benefits ( $> 80\%$  faster than baseline, median speed up  $> 5$ ) and with  $w_{so} = 4, 8$  solve almost double the number of instances as their unweighted baseline. It is important to observe that our method produces very similar speeds up in both ECBS and EECBS. This highlights how our technique identically speeds up the low level planner regardless of the high level search, demonstrating how our technique is readily generalizable to future suboptimal CBS methods.

### Understanding Weighted Suboptimal CBS

Figure 5 surprised us as it contradicted our initial intuition from single agent planning that the cost-to-go weight  $w_h$  would have a direct effect on performance, with larger  $w_h$  generally causing larger speed ups. Instead, the relative conflict weight  $r$  primarily dictates performance, with increas-

Method		Total # Low Level Nodes	# CT Nodes + bypasses	# LL per low level call	Speed up
$r$	$w_h$				
$\infty$	—	1,046,159	20.2	6,145	1
16	4	223,000	22.8	1,294	3.9
8	4	135,196	37.8	702	6.1
4	4	51,000	69.4	228	9.9
2	4	56,224	312	121	1.8

Table 5: Comparing low and high level statistics — WF-EECBS expands less total low level nodes by generating more CT nodes significantly faster (fewer low level nodes per low level call) than EECBS (top row). Although we show one example ( $w_{so} = 2$ , den312d with 150 agents), we observe the relative conflict weight  $r$  controls this balance throughout different  $w_{so}, w_h$ , and scenarios.

ing  $w_h$  only marginally increasing performance given  $r$ . Table 5 shows how  $r$  directly controls the balance between low level and high level work in WF-EECBS. Decreasing  $r$  from  $\infty$  (which is what EECBS implicitly has) shifts the work load from the low level to the high level search. Across all 8 maps, we generally observe that each WF-EECBS’s low level planner call does 5-150x fewer low level expansions than EECBS while WF-EECBS’s CT search does 2-20x more work than EECBS, resulting in a net reduction of 5-100x less total low-level nodes expanded. The increased numbers of CT nodes does add high level overhead, resulting in a sweet spot in the middle. This leads us to believe that current Suboptimal CBS methods place too large a burden on the low level search; each CT node requires too many expansions to find a suboptimal path with minimal conflicts. Our WF method is able to “balance” the low level and high level work better, and improves performance by expanding more CT nodes significantly faster than their unweighted counterpart. Figure 1 illustrates this effect of using our Weighted Focal method compared to CBS and current Suboptimal CBS methods.

The role of  $r$  might be obvious in retrospect but the impact leads to a novel insight; minimizing conflicts as a separate mechanism loosely connected to path length (as currently done in FOCAL) can lead to the low-level planner doing significantly too much work. We believe this finding has direct relevance for future Suboptimal CBS based methods, and even non-CBS MAPF methods like MAPF-LNS2 (Li et al. 2022) whose low level planner minimizing unweighted conflicts will likely suffer similarly. We predict that using an explicit trade off with  $r$  will better balance low-level and high-level work and improve performance.

### Relating CBS, Prioritized Planning, and W-EECBS

We run WO-EECBS with a very large sub-optimality value ( $w_{so} = 10000$ ) and different anchor weights to see how this mimics running weighted prioritized planning. We denote these as “CBSPP” with their specific weights to emphasize the relation. Figure 4 verifies that the number of generated CT nodes stays at 1 for low levels of agents until conflicts become unavoidable. The instances with CT nodes greater than 1 also demonstrate how CBSPP’s ability to replan using CBS’s conflict resolution increases success rate compared to prioritized planning which would fail in those instances. At

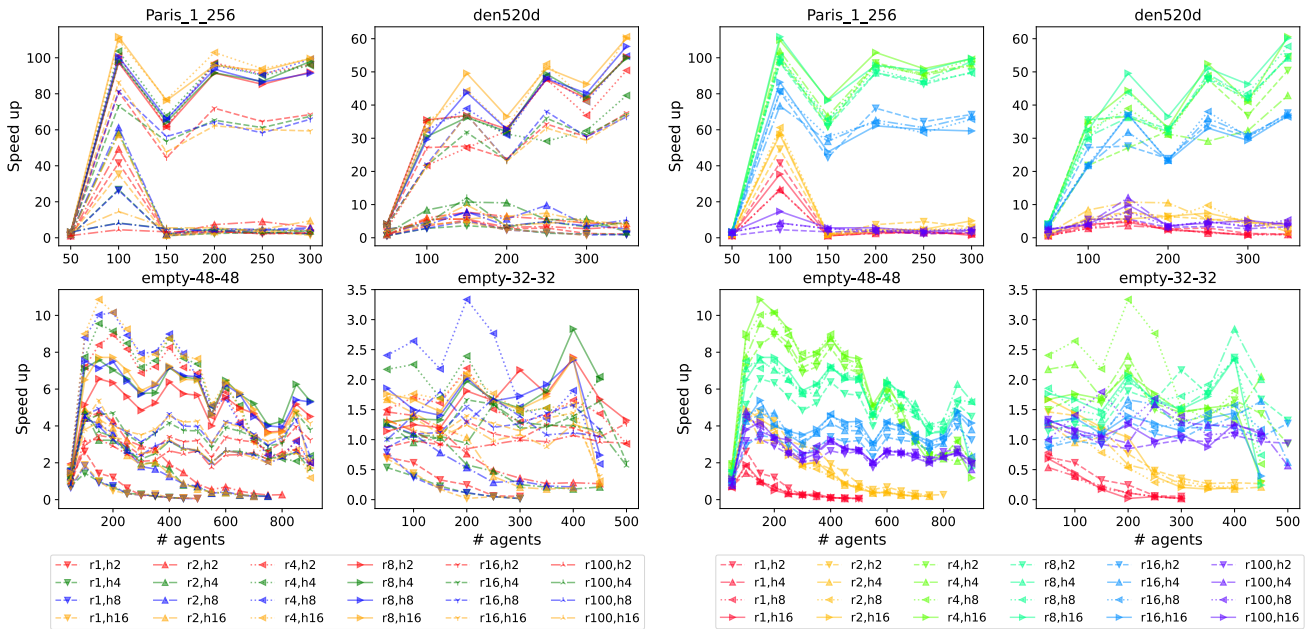


Figure 5: WF-EECBS  $w_h, r$  Analysis — Left half: Lines with the same color have the same  $w_h$  values, each  $w_h$  color has 6 different line/marker styles corresponding to different  $r$  values. We expect performance to be primarily driven by  $w_h$ , with methods with same  $w_h$  values performing similarly and larger  $w_h$  resulting in larger speed ups. However we see that methods with the same  $w_h$  value (e.g. all the blue lines) are wildly scattered.

Right half: Lines with the same color have the same  $r$  values. We see a striking grouping effect across runs with same  $r$  values but different  $w_h$  values, showing how performance is tightly linked to  $r$  and *not*  $w_h$  across different  $r$  and  $w_h$  values across all the maps. Additionally, values too low  $r = 1, 2$  (red, yellow) and too high  $r = 16, 100$  (blue, purple) perform worse than  $r = 4, 8$  (lime green, turquoise), implying some optimal region of  $r \in [2, 16]$ .

the expense of additional engineering complexity, we recommend that practitioners using PP should instead use W-EECBS with a large suboptimality as they get the same prioritized planning behavior in the root node along with the natural robustness and completeness of CBS.

## Future Work and Conclusion

We see several avenues to build upon our work. Our work keeps  $r$  and  $w_h$  fixed in MAPF instances; adaptively changing  $r$  and  $w_h$  during a single MAPF search, or predicting a fixed optimal  $r$  and  $w_h$  could increase performance and robustness across different maps. Our insights about the cost-to-go heuristic’s weak role and the relative conflict heuristic’s significant role can be incorporated into non-CBS based MAPF methods which employ either heuristic. Understanding WO-EECBS improved bound’s negative performance effect would also be interesting investigative work.

Our experiments provide compelling evidence for MAPF practitioners to use Weighted EECBS and more broadly incorporate relative conflict weights along with cost-to-go heuristics. We first introduce WO-EECBS which incorporating the weighted cost-to-go in the open queue, and analyze the effect of improving the lower bound on utilizing prioritized conflicts and symmetry reasoning. We then introduce WF-EECBS by modifying the focal priority to include a weighted cost-to-go and relative weighted conflict heuris-

tic, and show significant speeds up compared to EECBS. We demonstrate how these speeds ups change across different hyper-parameters  $w_{so}, w_h, r$  and different scenario (map sizes, numbers of agents). We provide novel insight that the cost-to-go weight  $w_h$  does not primarily impact performance as expected, but that instead the relative weight  $r$  dictates performance by trading off low-level and high-level work effectively. We show that our weighted focal technique results in similar speed ups regardless of the high level search, illustrating how our technique is readily generalizable to other suboptimal CBS methods. Finally, we show that PP is actually just one specific step in suboptimal CBS with an infinite sub-optimality, and show Weighted Suboptimal CBS is the natural generalization of the two.

Overall, our proposed methods bear no additional overhead and are directly usable in other suboptimal CBS planners. More broadly, we hope this work inspires future MAPF work to incorporate the conflict heuristic with more nuance.

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