

Spearman Rank Correlation Screening for Ultrahigh-Dimensional Censored Data

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Abstract

Herein, we propose a Spearman rank correlation based screening procedure for ultrahigh-dimensional data with censored response cases. The proposed method is model-free without specifying any regression forms of predictors or response variables and is robust under the unknown monotone transformations of these response variables and predictors. The sure-screening and rank-consistency properties are established under some mild regularity conditions. Simulation studies demonstrate that the new screening method performs well in the presence of a heavy-tailed distribution, strongly dependent predictors or outliers, and offers superior performance over the existing nonparametric screening procedures. In particular, the new screening method works well when a response variable is observed under a high censoring rate. An illustrative example is provided.

Introduction

Ultrahigh-dimensional covariates are often encountered in many fields of study, e.g., mechanical systems; genetic engineering (Zhou et al. 2021), and biomedical engineering. Under the “larger p smaller n ” data framework, numerous penalized variable selection approaches have been developed for high-dimensional Cox model (Zhang and Lu 2007; Zou 2008), additive hazard model (Chen and Cai 2018; Leng and Ma 2007; Martinussen and Scheike 2009; Lin and Lv 2013), linear regression model (Huang, Horowitz, and Ma 2008; Wang et al. 2008) and unconditional moment model (Tang, Yan, and Zhao 2018). However, the aforementioned variable selection methods may not perform well because of the simultaneous challenges of computational expediency, statistical accuracy, sophisticated algorithm techniques and strong model assumptions (Fan, Samworth, and Wu 2009; Tang, Yan, and Zhao 2018). Therefore, developing new approaches to deal with ultrahigh-dimensional censored data is necessary and urgent.

Emerging feature screening approaches have recently been developed for a complete response with ultrahigh-dimensional covariates. The existing model-based feature-screening methods include sure independence screening

(SIS) for linear regression (Fan and Lv 2008), a maximum-marginal-likelihood (MML) approach for generalized linear models (Fan and Song 2010), a nonparametric feature screening method for missing response (Li et al. 2020), a nonparametric independence screening (NIS) for additive models (Fan, Feng, and Song 2011), partially linear models with missing responses (Tang, Xia, and Yan 2019), and marginal empirical-likelihood screening (ELS) for linear regression (Chang, Tang, and Wu 2013). Model-free feature-screening methods have been developed to overcome the limitation variable above models. These include sure independent ranking and screening (SIRS) (Zhu et al. 2011), rank-correlation screening (RCS) (Li et al. 2012), distance correlation screening (DCS) (Li, Zhong, and Zhu 2012; Chen, Chen, and Liu 2019), category-adaptive variable screening (Xie et al. 2020), quantile-adaptive screening (QAS) (He, Wang, and Hong 2013), fused Kolmogorov filter screening (FKFS) (Mai and Zou 2015), conditional quantile screening (CQS) (Wu and Yin 2015), and the fused mean-variance (FMV) filter (Yan et al. 2018). Rank correlation can characterize the nonlinear relationship between two variables.

However, the aforementioned screening procedures proposed a complete response failure in performing well for the censored model because it cannot be estimated reliably. In particular, the performance will be dramatically decreased under a high censoring rate or in the existence of outliers in the predictors. Furthermore, several emerging feature-screening procedures have been proposed for censored responses. The existing model-based screening methods mainly focus on the Cox model. These include the lasso-penalization approach for prescreening (Tibshirani 1997), the standardized-marginal-maximum partial-likelihood estimators (Zhao and Li 2012), and the marginal-sure-independence-screening procedure (Fan and Song 2010). However, model-free screening methods with a censored response, such as the censored quantile-adaptive screening procedure (He, Wang, and Hong 2013), the censored-rank-correlation screening (RCS_{cen}) procedure with inverse probability-of-censoring weighted as Kendall’s τ (Song et al. 2014), the conditional-quantile screening (CQS_{cen}) procedure for a covariate-independent censoring method (Wu and Yin 2015), the adjusted-distance-correlation screening procedure (DCS_{cen}) (Chen, Chen, and

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Wang 2018), and the censored sure independent ranking and screening methods (SIRS_{cen}) (Zhou and Zhu 2017), may be more robust under model misspecification. Recent research on the screening issue of ultrahigh-dimensional censored data has also included the following works (Liu and Chen 2018; Lin, Liu, and Hao 2018; Liu, Zhang, and Zhao 2018; Zhang, Liu, and Wu 2017; Zhang et al. 2018). Additionally, a feature screening procedure is developed through a marginal Buckley–James index (Yan et al. 2021); a multiple-imputation sure independence screening (MI-SIS) procedure is proposed to distinguish between the active and inactive predictors (Xie, Yan, and Tang 2021).

The main contributions of this article include the followings:

- This is the attempt using imputation technic to adjust the feature screening method in high-dimensional censored data, called Spearman rank correlation screening (SRCS_{cen}).
- It is model-free due to it being invariant under monotonic transformations of the response and robust in the presence of monotonic transformations of predictor variables.
- The feature screening performance still behaves under a high censoring rate.

Screening Procedures

Let Y be a continuous response with a support \mathbb{R}_y^1 , and \mathbf{X} be a vector of continuous covariates with a support \mathbb{R}_x^p . Define $F_k(x) = \text{pr}(X_k \leq x)$, $F_{Y|X_k}(y|x) = \text{pr}(Y \leq y | X_k = x)$, $F_k(x, y) = \text{pr}(X_k \leq x, Y \leq y)$ ($k = 1, \dots, p$), and $F(y) = \text{pr}(Y \leq y)$. To investigate the relationship between X_k and Y , Fan and Lv (Fan and Lv 2008) utilized absolute marginal Pearson correlation $E(X_k Y) - E(X_k)E(Y)$ to rank the linear correlation between X_k and Y . Zhu et al. (Zhu et al. 2011) proposed a marginal screening utility based on $E\{X_k F_{Y|X_k}(Y | X_k)\}$, while Li et al. (Li et al. 2012) developed the robust rank correlation screening method based on $E\{F_k(X_k, Y)\} - E\{F_k(X_k)\}E\{F(Y)\}$. Motivated by these marginal screening utilities, we consider investigating the correlation between the distributions $F_k(x)$ and $F(y)$, because such correlations include the linear and nonlinear relationship between X_k and Y . Therefore, we propose the following index for the k th covariate,

$$\omega_k = E\{F_k(X_k)F(Y)\} - E\{F_k(X_k)\}E\{F(Y)\} = \iint \{F_k(x)F(y)\}dF_k(x, y) - \frac{1}{4}, \quad (1)$$

which can be used to measure the dependence between X_k and Y . Then, ω_k serves as the population quantity of our proposed marginal-utility measure for ranking the potential correlations between predictors and responses. ω_k has the remarkable property of being 0 if X_k and Y are statistically independent. This motivates us to utilize it for feature-screening to characterize linear and nonlinear relationships between responses and covariates.

Suppose that one observes the right-censored data $(\mathbf{X}, Y^*, \delta) = \{\mathbf{X}, Y \wedge C, I(Y \leq C)\}$, where Y is a cen-

sored response of interest, C represents a censoring variable, and $\mathbf{X} = (X_1, \dots, X_p)^T$. Let $\{\mathbf{X}_i, Y_i^*, \delta_i : i = 1, \dots, n\}$ comprise independent copies of $(\mathbf{X}, Y^*, \delta)$. If we know the distributions $F_k(x)$ and $F(y)$, the moment estimator of $E\{F_k(x)F(y)\}$ in (1) is $1/n \sum_{i=1}^n \{F_k(X_{ki})F(Y_i)\}$. Since Y_i cannot be completely observed due to censoring, we replace $F(Y_i)$ with its conditional expectation given the observed response Y_i^* , censoring indicator δ_i and predictor X_{ki} . For this, let $\mathcal{F}(Y_i^*, \delta_i, X_{ki}) = E\{F(Y_i) | Y_i^*, \delta_i, X_{ki}\}$. Note that

$$\begin{aligned} & \mathcal{F}(Y_i^*, \delta_i, X_{ki}) \\ &= \delta_i F(Y_i^*) + (1 - \delta_i) E\{F(Y_i) | Y_i > Y_i^*, Y_i^*, X_{ki}\} \\ &= \delta_i F(Y_i^*) + (1 - \delta_i) \frac{E\{F(Y_i)I(Y_i > Y_i^*) | X_{ki}\}}{1 - F(Y_i^* | X_{ki})} \\ &= \delta_i F(Y_i^*) + (1 - \delta_i) \frac{\int_{Y_i^*}^{\infty} F(y)dF(y | X_{ki})}{1 - F(Y_i^* | X_{ki})}, \end{aligned}$$

where the equality is derived from the formula of conditional expectation given an event, $F(y|x)$ is the Y distribution given the fixed value x . Then we conclude that

Proposition 1 *The unbiased moment estimator of $E\{F_k(x)F(y)\}$ is $1/n \sum_{i=1}^n \{F_k(X_{ki})\mathcal{F}(Y_i^*, \delta_i, X_{ki})\}$. i.e., $E\{F_k(X_{ki})\mathcal{F}(Y_i^*, \delta_i, X_{ki})\} = E\{F_k(x)F(y)\}$.*

However, the distributions of $F_k(x)$, $F(y)$ and $F(y|x)$ are usually unknown in practice. $F_k(x)$ can be estimated by its empirical distribution $\hat{F}_k(x) = 1/n \sum_{i=1}^n I(X_{ki} \leq x)$ with $I(\cdot)$ being the indicator function. We can also turn to employ imputation technic that

$$F(y) = EI(Y \leq y) = E\{\delta I(Y \leq y) + (1 - \delta) \frac{\int_{Y_i^*}^{\infty} I(Y \leq y)dF(Y | X_k)}{1 - F(Y_i^* | X_k)}\}$$

to obtain the estimator

$$\begin{aligned} \hat{F}_n(y) &= 1/n \sum_{i=1}^n \{\delta_i I(Y_i \leq y) + \\ & (1 - \delta_i) \frac{\int_{Y_i^*}^{\infty} I(Y \leq y)d\hat{F}_n(Y | X_{ki})}{1 - \hat{F}_n(Y_i^* | X_{ki})}\}, \end{aligned}$$

where $\hat{F}_n(y|x)$ is the estimator of conditional distribution $F(y|x)$ and given by $\hat{F}_n(y|x) = \sum_{i=1}^n \frac{\delta_i B_{ni}(x)}{\hat{G}(Y_i^* | x)} I(Y_i^* \leq y)$, where $\hat{G}(y|x)$ is the local Kaplan-Meier estimator (He et al. 2014) of $G(y|x) = \text{pr}(C \geq y|x)$. More specifically,

$$\hat{G}(y|x) = \prod_{i=1}^n \left\{ 1 - \frac{B_{ni}(x)}{\sum_{j=1}^n I(Y_j^* \geq Y_i^*) B_{nj}(x)} \right\}^{I(Y_i^* \leq y, \delta_i = 0)}, \quad (2)$$

where $B_{nj}(x) = K(\frac{x - X_{kj}}{h}) / \{\sum_{i=1}^n K(\frac{x - X_{ki}}{h})\}$ ($j = 1, \dots, n$), are the Nadaraya-Watson weights, h is the bandwidth and $K(\cdot)$ is a density function. Therefore, the empirical version of $\mathcal{F}(Y_i^*, \delta_i, X_{ki})$ is

$$\hat{\mathcal{F}}_n(Y_i^*, \delta_i, X_{ki}) = \delta_i \hat{F}_n(Y_i^*) + (1 - \delta_i) \frac{\int_{Y_i^*}^{\infty} \hat{F}_n(y)d\hat{F}_n(y | X_{ki})}{1 - \hat{F}_n(Y_i^* | X_{ki})}. \quad (3)$$

We propose an adjusted Spearman rank correlation screening utility of ω_k for a censored response (SRCS_{cen}) given by

$$\widehat{\omega}_k = 1/n \sum_{i=1}^n [\widehat{F}_k(X_{ki}) \widehat{F}_n(Y_i^*, \delta_i, X_{ki})] - 1/4, \quad (4)$$

which is invariant to under any strictly increasing transformation of the response. Therefore, we choose $\widehat{\omega}_k$ as a marginal utility to measure the importance of X_k for response Y in the presence of censoring. The corresponding screening set is defined as

$$\widehat{\mathcal{A}} = \{k : |\widehat{\omega}_k| \geq cn^{-\tau}, 1 \leq k \leq p\}, \quad (5)$$

where c and τ are pre-determined thresholding values defined by Condition (C1) below. We note that if all δ_i 's are equal to 1 (i.e., all observed responses are complete), $\widehat{\omega}_k$ in (4) can be rewritten as $\widehat{\omega}_k = 1/n^3 \sum_{j=1}^n R_{kj} Q_j - 1/4$, where $R_{kj} = \sum_{i=1}^n I(X_{ki} \leq X_{kj})$ and $Q_j = \sum_{i=1}^n I(Y_i \leq Y_j)$ denote the rank of X_{kj} in all observations of \mathbf{X}_k and the rank of Y_j in all observations of \mathbf{Y} , respectively. By setting $\bar{R}_k = \frac{1}{n} \sum_{j=1}^n R_{kj}$ and $\bar{Q} = \frac{1}{n} \sum_{j=1}^n Q_j$, we can express Spearman's rank-correlation coefficient as

$$\rho_{kn} = \frac{\sum_{j=1}^n (R_{kj} - \bar{R}_k)(Q_j - \bar{Q})}{\sqrt{\sum_{j=1}^n (R_{kj} - \bar{R}_k)^2 \sum_{j=1}^n (Q_j - \bar{Q})^2}} = 12 \left\{ \frac{1}{n(n^2-1)} \sum_{j=1}^n R_{kj} Q_j - \frac{1}{4} \frac{n+1}{n-1} \right\}, \quad (6)$$

where the detailed derivative process of (6) is shown in the Appendix. Obviously, the form of ρ_{kn} is analogous to that of $\widehat{\omega}_k$ and converges to $12\omega_k$. Therefore, we call this screening procedure Spearman rank correlation screening with a screening utility $\widehat{\omega}_k$ (SRCS_{cen}) in (4). The Spearman correlation is a nonparametric measure of the statistical dependence between two variables. Unlike Pearson correlation, it assesses how well the relationship between two variables can be described using a monotonic function. This property allows us to discover the nonlinear relationship between the response and predictor values. Therefore, it can be directly used to deal with semiparametric models such as those of transformation regression models and single-indices models with monotonic constraints on the link function.

Theoretical Properties

In this section, we investigate the sure-screening, rank-consistency and false-discovery controlling properties of the proposed screening procedures. Without specifying any regression model of Y and $\mathbf{X} = (X_1, \dots, X_p)^\top$, where $p \gg n$ (with n being the sample size), we define the active predictor subset as

$$\mathcal{D} = \{k : F(y | \mathbf{X}) \text{ functionally depends on } X_k \text{ for some } y, k = 1, \dots, p\},$$

where $F(y | \mathbf{X}) = \text{pr}(Y \leq y | \mathbf{X})$. Then, the sparsity assumption states that $p \gg |\mathcal{D}|$. Our goal is to recover the active set \mathcal{D} as precisely as possible. To this end, we apply

the screening procedure depicted in Section 2 for each pair (X_k, Y) as a marginal utility to measure the importance of X_k for the response Y . We require the following conditions.

(C1) There exist positive constants $c > 0$ and $0 \leq \tau < 1/2$ such that $\min_{k \in \mathcal{D}} |\omega_k| > 2cn^{-\tau}$.

(C2) $\min_{k \in \mathcal{D}} |\omega_k| - \max_{k \notin \mathcal{D}} |\omega_k| \geq c_1 n^{-\kappa}$, where c_1 and κ are some positive constants.

(C3) Y and C are independent given covariates X_k ($k = 1, \dots, p$). $G(y | x)$ has uniformly bounded first derivative and bounded (uniformly in y) second-order partial derivatives with respect to x . Furthermore, $\inf_x \text{pr}(y \leq Y_i \leq C_i | x) \geq \lambda_1 > 0$ for some positive constant λ_1 and any $y \in [0, b_H]$, and $y_1 \leq \sup\{y : G(y | x) > 0\} \leq y_2$ uniformly in x for some positive constants y_1 and y_2 ; The kernel function $K(\cdot)$ is a probability density function such that it is bounded and has compact support.

Condition (C1) allows the minimum true signal to be on the order of $n^{-\tau}$, degenerating to zero as the sample size increases. Furthermore, this condition can be relaxed by assuming $c = O(n^{-\psi})$ with $0 < \psi < 2\tau$ (Cui, Li, and Zhong 2015). Under the relaxed condition, the sure-screening property in Theorem 1 still holds, but the convergence rate becomes relatively slower. Condition (C2) ensures that the screening utility can separate informative and non-informative predictors well at the population level, and it is much weaker than the partial-orthogonality condition, i.e., $|\omega_k| \neq 0$ for $k \in \mathcal{D}$ and $|\omega_k| = 0$ for $k \notin \mathcal{D}$. Condition (C3) is commonly used in the survival analysis literature to ensure that the Kaplan-Meier estimator and its reciprocal function are well behaved (He, Wang, and Hong 2013).

Theorem 1 (i) (Sure-Screening Property) *If Condition (C3) and other conditions in Lemmas hold, there exists a positive constant b depending on c , such that*

$$P(\max_{1 \leq k \leq p} |\widehat{\omega}_k - \omega_k| \geq cn^{-\tau}) \leq O[p(n+1) \exp(-bn^{1-2\tau})];$$

and under Conditions (C1) and (C3),

$$\text{pr}(\mathcal{D} \subset \widehat{\mathcal{A}}) \geq 1 - O[|\mathcal{D}|(n+1) \exp(-bn^{1-2\tau})].$$

(ii) (Rank-Consistency) *If Conditions (C2), (C3), and the additional condition $\log(p) = o(n^{1-2\kappa})$ with $\kappa < 1/2$ hold, then*

$$\liminf_{n \rightarrow \infty} (\min_{k \in \mathcal{D}} |\widehat{\omega}_k| - \max_{k \notin \mathcal{D}} |\widehat{\omega}_k|) > 0.$$

Theorem 1 gives the sure-screening and rank-consistency properties of our proposed screening procedure. The conditions for the sure-screening property are milder, because we do not require the regression function of Y onto \mathbf{X} to be linear and there are few requirements on the moments of covariates. Since ω_k inherits the robustness of a distribution function, our proposed screening utility is robust against heavy-tailed distributions of predictors and the presence of potential outliers. The exponential tail probability bound of the sure-screening property is less than or equal to that of Wu and Yin (Wu and Yin 2015) and the equality holds only if the quantile q in CQScen(q) is adopted as 0.5. We find that our method can handle the NP-dimensionality $\log(p) = O(n^\zeta)$,

where $\zeta < 1 - 2\tau$ with $0 \leq \tau < 1/2$, which depends on the minimum true signal strength. In this case, we have

$$\begin{aligned} & \text{pr}\left(\max_{1 \leq k \leq p} |\hat{\omega}_k - \omega_k| \leq cn^{-\tau}\right) \geq \\ & 1 - O\left[\Lambda \exp\{-\Lambda n^{1-2\tau} + \log(n+1)\}\right], \end{aligned} \quad (7)$$

where Λ is some constant. The rank-consistency property implies that the values of $|\hat{\omega}_k|$ of active predictors can be ranked ahead inactive ones with high probability. Thus we can separate the active and inactive predictors by taking an ideal threshold value following Mai and Zou (Mai and Zou 2015).

Theorem 2 (Controlling false discovery) *If Condition (C3) holds, there exists a positive constant b' depending on c ,*

$$\text{pr}\{|\hat{\mathcal{A}}| \leq n^\tau \sum_k |\omega_k|/c\} \geq 1 - O(n)p \exp(-b'n^{1-2\tau}).$$

Theorem 2 implies that the model obtained after screening is of polynomial size with high probability. Although we investigate the theoretical conclusion on controlling the false-positive rate in Theorem 2, the result is conservative for screening purposes because the lower false-positive rate may lead to larger false-negative error. Then an alternative strategy to specify $\hat{\mathcal{A}}$ practically is to use $\hat{\mathcal{A}} = \{k : |\hat{\omega}_k| \text{ is among the } d_n\text{th largest}\}$. We note that the sure-screening property of the SRCS_{cen} filter does not involve d_n or the censoring rate explicitly, leading to tremendous practical convenience for our choice of d_n , because we can utilize a reasonably large d_n to guarantee a high probability of the hold of the sure-screening property. As Mai and Zou (Mai and Zou 2015) suggested we can use $d_n = a[n/\log(n)]$, where a is some constant whose value may reflect the researchers' prior knowledge of the number of susceptible predictors, or the budget limitations (Song et al. 2014). Therefore, its choice is flexible and a more conservative choice could be $d_n \leq n$, so that a follow-up regression analysis could be performed in a $p < n$ scenario.

Simulation Studies

Simulation studies were conducted to evaluate the performance of the proposed new feature-screening procedure and to compare it with existing screening methods. We consider six model-free methods, including rank correlation screening (RCS) (Li et al. 2012), censored rank independence screening (RCS_{cen}) (Song et al. 2014), conditional quantile screening with complete and censored responses (CQS(q), CQS_{cen}(q), where q represents the quantile used) (Wu and Yin 2015), respectively, our proposed SRCS_{cen} in (4), and the SRCS defined in (6). Naturally, we want to know whether we can simply use the screening procedures developed for a complete response in the presence of censoring. To answer this question, we design a naive screening procedure for RCS, CQS(q) and SRCS based on datasets $\{(\mathbf{X}_i^\top, Y_i^*) : i = 1, \dots, n\}$. First, we focus on checking the model-based behaviors of screening procedures combined with some shrinkage methods. Then we aim to show the robust model-free performance of the proposed nonparametric screening procedure.

Linear Model

In the linear model, we mainly investigate how to decide the true model size. Considering that some truly unimportant covariates are also retained in the screening stage, we next perform the penalized method to further remove these covariates, and specify the eventual true model size.

Example 1 (Linear model). *In this simulation study, we suppose that censored response variable Y takes the following linear model*

$$Y_i = X_i^\top \beta + \epsilon_i,$$

where $X_i = (x_{i1}, \dots, x_{ip})$ and (x_{i1}, \dots, x_{i5}) are derived from $\text{Unif}(0,1)$, (x_{i6}, \dots, x_{ip}) are assumed to be generated from a multivariate normal distribution with zero mean and covariance matrix $\Phi = (d_{jl})$ with $d_{jl} = 0.7^{|j-l|}$, and the censored variable C_i was generated from $C_i = X_i^\top \theta + \epsilon_i$, ϵ_i and ϵ_i were assumed to follow the standard normal distribution $\mathcal{N}(1, 1)$ and $\mathcal{N}(0, 1)$, respectively. We set the sample size to $n = 200$ and the number of regressors to $p = 3,000$. The true coefficients $\beta = (\beta_1, \dots, \beta_p)^\top = (3, 1.5, 0, 0, 2, \mathbf{0}_{p-5})$, $\theta = (\theta_0, 0, 0, -4, -4, \mathbf{0}_{p-4})^\top$, and θ_0 were chosen to achieve censoring ratios of 45% and 65%.

In this simulated dataset, a total of 200 simulation replications were conducted. Let $\hat{\beta}_{(k)} = (\hat{\beta}_{1(k)}, \dots, \hat{\beta}_{p(k)})^\top$ be the estimator realized in the k th simulation replication. Then, the model selected by $\hat{\beta}_{(k)}$ is given by $\hat{\mathcal{M}}_{(k)} = \{j : |\hat{\beta}_{j(k)}| > 0\}$ and the corresponding average model size (AMS) is $\frac{1}{200} \sum_{k=1}^{200} |\hat{\mathcal{M}}_{(k)}|$, where $|\hat{\mathcal{M}}_{(k)}|$ denotes the cardinality of $\hat{\mathcal{M}}_{(k)}$. Let $\mathcal{D}^* = \{j : \beta_j \neq 0\}$ denote the index of the true model; we evaluate the coverage probability (CP) as $\frac{1}{200} \sum_{k=1}^{200} I(\mathcal{D}^* \subseteq \hat{\mathcal{M}}_{(k)})$, which measures how likely it is for all relevant variables to be discovered by one particular method. To characterize the capability of this method in producing sparse solutions, we define the percentage of correct zeros (PCZ; %) as $\frac{100\%}{p-|\mathcal{D}^*|} \left\{ \frac{1}{200} \sum_{k=1}^{200} \sum_{j=1}^p I(\hat{\beta}_{j(k)} = 0) I(\beta_j = 0) \right\}$ and percentage of incorrect zeros (PIZ; %) as $\frac{100\%}{|\mathcal{D}^*|} \left\{ \frac{1}{200} \sum_{k=1}^{200} \sum_{j=1}^p I(\hat{\beta}_{j(k)} = 0) I(\beta_j \neq 0) \right\}$. If all zero coefficients are correctly identified for all inactive predictors and no sparse solution is mistakenly estimated for all relevant variables, the true model is perfectly identified, meaning that giving larger PCZ and smaller PIZ imply a good model-fitting procedure. We utilize the residual-mean-square (RMS) error to examine the performance of the estimated true coefficients, i.e., $\text{RMS} = \frac{1}{200} \sum_{k=1}^{200} \sum_{j \in \mathcal{D}^*} \sqrt{\frac{1}{|\mathcal{D}^*|} (\hat{\beta}_{j(k)} - \beta_{0j})^2}$. We choose the bandwidth $h = \sigma_{X_k} n^\alpha$, $\alpha = -1/5$, where σ_{X_k} is the estimated standard deviation of X_k in the sample and adopt standard normal distribution as the kernel density function $K(\cdot)$.

To prepare candidate models for fitting, we ordered the regressors via the suggested SRCS_{cen} and the above mentioned screening methods, i.e., RCS_{cen}, CQS_{cen}(0.5), SRCS, CQS and RCS. We separated the first $\lceil n/\log(n) \rceil \approx 38$ active predictors using each screening method, and then

considered conducting variable selection and parameter estimation procedures using a combining Buckley-James-type least square objective function and the corresponding penalized functions, i.e.,

$$\ell_p(\boldsymbol{\beta}; \lambda) = \frac{1}{2} \left\| \mathbf{Y} \left(\boldsymbol{\beta}, \widehat{F}_\beta \right) - \mathbf{X} \boldsymbol{\beta} \right\|^2 - \frac{1}{2n} \left[\sum_{i=1}^n \left\{ Y_i \left(\boldsymbol{\beta}, \widehat{F}_\beta \right) - X_i^\top \boldsymbol{\beta} \right\}^2 \right] + \sum_{j=1}^p p_\lambda(|\beta_j|), \quad (8)$$

where $\mathbf{Y}(\boldsymbol{\beta}, F) = (Y_1(\boldsymbol{\beta}, F), \dots, Y_n(\boldsymbol{\beta}, F))^\top$, and $Y_i(\boldsymbol{\beta}, F) = E(Y_i | X_i, Y_i^*, \delta_i) = \delta_i Y_i^* + (1 - \delta_i) \left\{ X_i^\top \boldsymbol{\beta} + \frac{\int_{Y_i^* - X_i^\top \boldsymbol{\beta}}^{\infty} t dF_\beta(t)}{1 - F_\beta(Y_i^* - X_i^\top \boldsymbol{\beta})} \right\}$. Here \widehat{F}_β is the Kaplan-Meier estimator of F_β given by $\widehat{F}_\beta(t) = 1 - \prod_{i: v_i(\boldsymbol{\beta}) \leq t} \left\{ 1 - \frac{1}{G_n(\boldsymbol{\beta}, v_i(\boldsymbol{\beta}))} \right\}^{\delta_i}$, and $v_i(\boldsymbol{\beta}) = \min(\zeta_i(\boldsymbol{\beta}), \epsilon_i(\boldsymbol{\beta}))$, $\epsilon_i(\boldsymbol{\beta}) = Y_i - X_i^\top \boldsymbol{\beta}$, $\zeta_i(\boldsymbol{\beta}) = C_i - X_i^\top \boldsymbol{\beta}$, $i = 1, \dots, n$, $G_n(\boldsymbol{\beta}, u) = \sum_{i=1}^n I(v_i(\boldsymbol{\beta}) \geq u)$. Next we generate the plug-in response $Y_i(\boldsymbol{\beta}, \widehat{F}_\beta)$. $p_\lambda(\cdot)$ are some sparsity-inducing penalties and here we consider SCAD, MCP and LASSO to obtain penalized estimator; the optimal value of λ can be selected via the following EBIC criterion (Chen and Chen 2008; Wang 2009),

$$\text{EBIC}(\mathcal{M}) = \log n^{-1} \mathbf{Y}^\top(\boldsymbol{\beta}_{\mathcal{M}}, \widehat{F}_{\boldsymbol{\beta}_{\mathcal{M}}}) [I_n - \mathbf{X}_{\mathcal{M}}(\mathbf{X}_{\mathcal{M}}^\top \mathbf{X}_{\mathcal{M}})^{-1} \mathbf{X}_{\mathcal{M}}^\top] \mathbf{Y}(\boldsymbol{\beta}_{\mathcal{M}}, \widehat{F}_{\boldsymbol{\beta}_{\mathcal{M}}}) + n^{-1} |\mathcal{M}| (\log n + 2 \log p),$$

where $\boldsymbol{\beta}_{\mathcal{M}}$ and $\mathbf{X}_{\mathcal{M}}$ are respectively, the sub-parameter coefficients and sub-design matrix corresponding to \mathcal{M} . To calculate all slopes, we specify a proper initial value and adopt a Buckley-James iterative procedure (Miller and Halpern 1982) to update the the regression estimators until input of some convergence criterion.

Table 1 presents the simulated results for Example 1. We note that (i) traditional screening methods, (i.e., SRCS, CQS, and RCS), fail to recover the true model combined with the penalized procedure, because according to Table 1, they show lower coverage probability and their estimated average model size approximates zero under the censored case; (ii) the SRCS_{cen}+penalized method shows the best performance in terms of the highest coverage probability; (iii) although all methods show higher percentages of correct zeros, the SRCS_{cen}+penalized method shows a lower percentage of incorrect zeros, this result is attributed to the wrongly selected predictors in the first screening step; (iv) unlike the estimated result of RCS_{cen}, The coverage probability obtained by SRCS_{cen} still approximates one under a 65% censoring rate, implying that our proposed method remains robust, whereas RCS_{cen} is dramatically reduced under a 65% censoring rate; (v) the SRCS_{cen}-concave penalties method is superior in recovering the true model, because according to Table 1, the estimated average model size under this method approximates the true model size, i.e., 3.

Other Models

To measure the model-free performances of all screening methods, we consider measurements including (i) the selected model size, i.e., the average number of active vari-

ables contained in the top m selected variables. We consider $m = 4, 10$, (i.e., m_4 and m_{10}) for Example 2 and $m = 3, 10$, for Examples 3, because m_3 and m_{10} have corresponding numbers of true active predictors given by 4, 3 respectively; (ii) the coverage probability (\mathcal{P}_{all}) of the the top $\lceil n/\log(n) \rceil$ estimated variables covering the true ones; (iii) the median minimum model size, i.e., the minimum number of predictors needed to keep all the active predictors; (iv) the interquartile range (IQR) of the minimum model size needed to include all active predictors. Here we adopt the range of the 0.8 and 0.2-quantiles. To show the robustness of our proposed nonparametric screening procedures, we consider the following three models. For each setting, we consider $n = 200$, $p = 3000$ and 200 replicates.

Example 2 (*Linear transformation model*). We adopt the model setting of (Song et al. 2014) and generate failure time T_i from the class of linear transformation models

$$H(T_i) = \mathbf{X}_i^\top \boldsymbol{\beta} + \epsilon_i, \quad i = 1, \dots, n,$$

where $Y_i = H(T_i)$ and $H(t) = \log 0.5(e^{2t} - 1)$, $\boldsymbol{\beta} = (-1, -0.9, \mathbf{0}_4^\top, 0.8, 1, \mathbf{0}_{p-8}^\top)$, $\mathbf{X}_i \sim N(0, \Sigma)$ and $\Sigma = (\sigma_{kj})_{1000 \times 1000}$ with $\sigma_{kk} = 1$ and $\sigma_{kj} = 0.5^{|k-j|}$. Although Y_i is unobserved for an unknown function $H(\cdot)$, our method is still applicable to this model setting for its invariance under any strictly increasing transformation. The model is flexible, because if we adopted three error distributions, including the standard normal distribution and the standard logistic distribution or standard extreme-value distribution, this model would correspond to a normal transformation model, a proportional-odds model, or a proportional-hazards model, respectively. The censoring time was generated from $\text{Unif}(0, \kappa)$ or $\text{Unif}(0, \kappa \exp(|X_1 - X_2|))$ for random and non-random censoring, where κ was chosen to achieve censoring rates of 45% and 65%. To study the robustness of our method, we contaminated the predictors by adding outliers to them using the strategy of (Song et al. 2014).

Example 3 (*Additive model*). We consider the additive model of the form: $Y = \sum_{j=1}^p g_j(X_j) + \epsilon$. Set $g_1(X_1) = 4X_1$, $g_2(X_2) = 2 \tan(\pi X_2/2)$, $g_3(X_3) = 5X_3^2$. Assume that the X_j were generated from $\text{Unif}(0, 1)$ independently and that $\epsilon \sim N(0, 1)$ is independent of \mathbf{X} . The censoring variable was generated from a 2-component normal-mixture distribution plus a constant, i.e., $\kappa_1 N(-5, 2) + \kappa_2 N(5, 1) + \kappa_3$ for censoring rates of 45% and 65%, respectively.

The simulation result for Example 2 is reported in Table 2 shows that the performance of the proposed method is comparable to or better than other screening methods under random censoring; we note that (i) SRCS_{cen} performs better than RCS_{cen}, because according to Table 2, SRCS_{cen} shows much higher coverage probability of the top $\lceil n/\log(n) \rceil$ estimated variables covering the true ones, a larger average number of active variables contained in the top m_4 and m_{10} , and smaller 0.8 and 0.2-quantiles ranges of the minimum model size. (ii) SRCS_{cen} generates the same median minimum model size as CQScen(0.5) in Table 2, but SRCS_{cen}

Screening method	Selection method	CR=45%					CR=65%				
		CP(%)	PCZ(%)	PIZ(%)	RMS	AMS	CP(%)	PCZ(%)	PIZ(%)	RMS	AMS
SRCS _{cen}	SCAD	91.50	99.13	2.50	0.49	3.21	86.50	98.57	5.67	0.82	2.42
	MCP	91.50	99.16	2.50	0.49	3.24	86.50	98.53	5.67	0.84	2.43
	LASSO	91.50	99.59	2.50	0.86	3.01	86.00	98.99	2.74	1.49	2.23
CQS(0.5) _{cen}	SCAD	63.50	98.87	14.65	0.66	1.72	60.00	98.42	15.22	0.91	1.21
	MCP	63.50	98.99	14.65	0.66	1.72	60.00	98.43	15.22	0.91	1.20
	LASSO	63.50	99.47	15.33	0.99	1.61	58.50	99.39	14.02	1.55	1.18
RCS _{cen}	SCAD	81.50	99.11	11.27	0.68	1.91	11.00	98.78	45.07	1.35	0.11
	MCP	81.50	99.30	11.27	0.68	1.92	11.00	98.79	45.07	1.35	0.11
	LASSO	81.50	99.79	11.27	1.09	0.64	10.00	99.69	43.21	1.68	0.00
SRCS	SCAD	65.50	99.10	15.20	0.94	0.72	22.50	98.62	33.43	1.16	0.25
	MCP	65.50	99.23	15.20	0.95	0.72	22.50	98.64	33.43	1.16	0.25
	LASSO	65.50	99.48	15.20	1.23	0.02	20.50	99.34	39.54	1.69	0.00
CQS(0.5)	SCAD	31.00	98.95	33.43	0.95	0.12	4.50	98.62	59.08	1.78	0.00
	MCP	31.00	99.15	33.43	0.95	0.12	4.50	98.69	59.08	1.78	0.00
	LASSO	31.00	99.84	35.62	1.56	0.00	4.00	99.92	61.27	1.99	0.00
RCS	SCAD	54.50	98.86	23.50	0.85	0.92	14.00	98.55	40.22	1.36	0.06
	MCP	54.50	98.83	23.50	0.85	0.93	14.00	98.65	40.22	1.36	0.06
	LASSO	54.00	99.55	26.52	1.44	0.02	13.50	99.26	43.27	1.98	0.00

Table 1: Simulation results by different screening procedures and variable selection methods in Example 1.

Error	Method	CR=45%					CR=65%				
		m_4	m_{10}	\mathcal{P}_{all}	Median	IQR	m_4	m_{10}	\mathcal{P}_{all}	Median	IQR
Normal	SRCS _{cen}	3.78	4.00	1	4	0	3.35	3.59	0.92	3.90	0.25
	CQS(0.5) _{cen}	3.72	3.90	0.98	4	0	3.25	3.41	0.86	3.75	1.50
	RCS _{cen}	3.64	3.81	0.92	4	1.68	0.72	1.44	0.00	1000	10.2
	SRCS	2.64	3.20	0.63	12.5	72.4	0.31	0.56	0.00	653.5	141.2
	CQS(0.5)	2.35	2.97	0.67	20	50.8	0.22	0.44	0.00	458	381.2
	RCS	3.20	3.55	0.71	5	23.4	0.53	0.79	0.00	452	240.5
Logistic	SRCS _{cen}	3.75	3.79	1	4	0.15	3.51	3.65	0.97	3.90	3.05
	CQS(0.5) _{cen}	3.52	3.64	0.95	3.90	3.42	3.58	3.60	0.88	3.75	8.25
	RCS _{cen}	2.85	3.20	0.70	10.5	59.4	0.81	1.94	0	1000	0
	SRCS	1.55	2.20	0.19	103.5	413.4	0.19	0.75	0	664.5	571.2
	CQS(0.5)	11isted	2.00	0.16	119.5	403.2	0.09	0.59	0	538	679.8
	RCS	2.29	2.62	0.40	68.5	133	0.21	1.06	0	478	632
Weibull	SRCS _{cen}	3.87	3.95	1	4	0.15	3.77	3.90	0.98	4	1.08
	CQS(0.5) _{cen}	3.68	3.80	0.98	4	1.98	3.57	3.75	0.90	4	5.24
	RCS _{cen}	3.45	3.47	0.85	4	9.2	2.21	2.32	0.17	197.5	788.2
	SRCS	2.09	2.27	0.37	93	253.4	1.31	1.39	0.15	344	515.2
	CQS(0.5)	1.65	2.38	0.35	91	273.4	1.19	1.76	0.11	215.5	413
	RCS	2.72	2.72	0.68	82.5	133.2	1.79	2.58	0.29	145.5	234.4

Table 2: Simulation results by different screening methods for Example 2 with random censoring.

Method	CR=45%					CR=65%				
	m_3	m_{10}	\mathcal{P}_{all}	Median	IQR	m_3	m_{10}	\mathcal{P}_{all}	Median	IQR
SRCS _{cen}	2.51	2.82	0.97	3	2	2.36	2.74	0.90	3	5.02
CQS(0.5) _{cen}	2.31	2.62	0.90	4	10.05	2.05	2.33	0.83	7	15.2
RCS _{cen}	1.49	1.89	0.41	67	318	0.52	0.72	0.07	435	455.6
SRCS	2.03	2.17	0.82	4.5	18.4	1.03	1.69	0.41	587	790.6
CQS(0.5)	1.42	2.01	0.64	57.5	92.6	0.55	1.05	0.18	352.5	303.2
RCS	1.88	2.22	0.79	6	47.4	1.09	1.35	0.38	182	539.4

Table 3: Simulation results of different screening methods for Example 3.

outperforms CQScen(0.5), because SRCS_{cen} always shows smaller 0.8- and 0.2-quantile ranges of minimum model size and larger coverage probabilities. For simulated results of the additive model in Example 3, one can see that our proposed method SRCS_{cen} still performs best among all screening methods because according to Table 3, SRCS_{cen} shows a higher coverage probability, larger selected model size and smaller interquantile range than other screening procedures and RCS_{cen} still cannot work under a censoring ratio of 65%, because the coverage probability approximates or equals zero in Table 3. The screening results of Example 3 again proves that our proposed nonparametric screening method is a robust model-free procedure.

In summary, the proposed new screening method is more robust than existing ones and provides superior performance. In addition, when a response is censored at a high rate such as 65%, our screening procedure still works well. To intuitively expose the mechanism of robustness under a high censoring ratio, we assume that (X_j, Y) follows a bivariate normal distribution with zero mean vector and correlation $\rho_j = \text{Cor}(X_j, Y)$; thus, X_j and Y have standard normal distributions. We generated a censored variable C from $\mathcal{N}(\eta_0, 0.3^2)$ and chose different values of η_0 to achieve four censoring ratios (CRs) of 30%, 50%, 70% and 90%. Thus, the values of screening statistics are the functions of ρ_j and CR. Figure 1 (see Supplemental Material) presents the change of the screening utilities of RCScen, CQScen and SRCS_{cen} against ρ_j under different CR's. And it shows that (i) the three screening utilities are strictly increasing functions of $|\rho_j|$ under relatively low censoring ratios; (ii) RCScen is invalid for screening active predictors under higher censoring ratios; and (iii) the statistical value of SRCS_{cen} increases more quickly with increasing $|\rho_j|$.

An Application

In this section, we applied our proposed screening procedure with a censored response to the mantle cell lymphoma microarray data, available from <http://llmpp.nih.gov/MCL/>. The data consist of the survival times of 92 patients and the gene-expression measurements of 8,810 genes for each patient. However, we only considered 6,312 genes for each patient after deleting 2,498 that appeared to be missing. During the follow-up, 64 patients died of mantle-cell lymphoma and the other 28 were censored, causing a censoring rate of 36%. Our goal is to identify genes with great influence on patient survival risk. Under a given model size $\lceil 92/\log(92) \rceil = 20$, we ranked the most influential of the 6312 genes using the proposed screening procedure in Section 2 and two other methods developed by Song et al. (Song et al. 2014) and Wu and Yin (Wu and Yin 2015). We summarized the top 20 selected genes in Table 4 (see Supplemental Material) and found 12 genes commonly selected by the three screening methods. Their unique identifications are 28990, 30157, 27095, 34771, 28346, 28872, 34790, 30334, 25234, 31420, 17326 and 17123. Wu and Yin (Wu and Yin 2015) have confirmed that these genes may be strongly associated with patients' survival risk. In addition, Gene 28990, i.e., cell division cycle 2, G1 to S and G2 to M, was ranked first by our proposed method and its importance has also been

identified by Huang and Ma (Huang and Ma 2010) and Wu and Yin (Wu and Yin 2015). Among the top 20 genes, the numbers commonly selected by SRCS_{cen} and CQScen(0.5) are 14 with 16 being commonly selected by SRCS_{cen} and RCS_{cen}, and 12 by CQScen(0.5) and RCS_{cen}. Therefore, our approach shows the highest total number of common genes with other two methods, i.e., 30.

Next, we examine how a variable-screening procedure helps to predict the conditional mean of censored response variables. To this end, we regard the logarithm of patients' survival time as response Y and 6,312 genes as covariates $\mathbf{X} = (X_1, \dots, X_{6312})^\top$. All covariates were standardized before applying all methods. Then, we consider the linear model

$$Y_i = \mathbf{X}_i^\top \boldsymbol{\beta} + \epsilon_i, i = 1, \dots, 92,$$

we require the following two steps to be implemented for model-fitting, i.e., the *Screening+penalized methods* procedure. The first step is to order the set of 6312 regressors and adopt the top $\lceil \frac{92}{\log(92)} \rceil = 20$ predictors for the six screening index magnitudes (i.e., SRCS, CQS(0.5), RCS, SRCS_{cen}, CQScen(0.5), and RCS_{cen}). In the second step we consider combining the Buckley-James-type least-squares objective function and the corresponding penalized functions (i.e., SCAD, MCP, Lasso), i.e., Equation (8), to obtain a penalized estimator $\hat{\boldsymbol{\beta}}$; then, we obtain a conditional mean estimator $\hat{\mu}_i$. To evaluate the prediction performance of various methods, we adopt bootstrap strategy and let $B = \{i : \text{observation } i \text{ is resampled}\}$ as the index set of observations resampled and \mathcal{N}_i denotes the resampled number of i . We used the average squared prediction errors (ASPE)

$$\text{ASPE} = \frac{1}{\sum_{i \in B} \mathcal{N}_i \delta_i} \sum_{i \in B} \mathcal{N}_i \delta_i (Y_i - \hat{\mu}_i)^2.$$

Figures 2 (see Supplemental Material) presents boxplots of the ASPEs after 500 replications. And it shows that our proposed SRCS_{cen} method together with any shrinkage procedure offers a better performance than other methods in terms of the smallest median of 500 ASPE values among the three censored screening methods.

Conclusion

In this article, to address the new challenges from ultrahigh-dimensional data in robustness and the presence of censoring, especially under a high censoring rate, we have proposed a new model-free screening procedure to improve screening efficiency, and established its sure-screening and rank-consistency properties under very weak regularity conditions. The proposed screening approach is invariant under the monotone transformation and is robust under the presence of heavy-tailed distributions, outliers, and dependent covariates. In particular, the new screening method still works well when a response variable is observed under a high censoring rate. The superior performance of the proposed screening approach over the existing methods has been demonstrated through simulation studies and real data analysis.

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