

# Common Knowledge of Abstract Groups

Merlin Humml, Lutz Schröder

Friedrich-Alexander-Universität Erlangen-Nürnberg, Erlangen, Germany  
{merlin.humml, lutz.schroeder}@fau.de

## Abstract

Epistemic logics typically talk about knowledge of individual agents or groups of explicitly listed agents. Often, however, one wishes to express knowledge of groups of agents specified by a given property, as in ‘it is common knowledge among economists’. We introduce such a logic of common knowledge, which we term *abstract-group epistemic logic (AGEL)*. That is, AGEL features a common knowledge operator for groups of agents given by concepts in a separate agent logic that we keep generic, with one possible agent logic being *ALC*. We show that AGEL is EXPTIME-complete, with the lower bound established by reduction from standard group epistemic logic, and the upper bound by a satisfiability-preserving embedding into the full  $\mu$ -calculus. Further main results include a finite model property (not enjoyed by the full  $\mu$ -calculus) and a complete axiomatization.

## Introduction

Epistemic (modal) logic is concerned with the individual and collective knowledge of agents. One of the most important modalities for collective knowledge is *common knowledge*: A fact  $\phi$  is common knowledge in a given group of agents if everyone in the group knows  $\phi$ , and everyone knows that everyone knows  $\phi$ , etc. In the present work, our focus of attention is on the involved notion of group of agents. The most basic variant of the common knowledge operator, typically written  $C$ , refers to *all* agents in a predetermined finite set  $Ag$  that forms a parameter of the logic as a whole (Fagin et al. 1995). In a more fine-grained variant,  $C$  can be annotated with an explicitly given subset of the set of agents: For  $A \subseteq Ag$ ,  $C_A\phi$  says that  $\phi$  is common knowledge among the agents in  $A$ . For instance, if Alice and Bob are legitimate participants in a communication protocol and  $\phi$  is a fact concerning a shared key, then  $\phi$  would ideally be common knowledge of Alice and Bob but not of a malicious third party Charlie – i.e.  $C_{\{Alice, Bob\}}\phi$  would hold but  $C_{\{Alice, Bob, Charlie\}}\phi$  would not.

Listing agents in a group explicitly is appropriate in well-controlled settings such as the above, where the participants in the epistemic situation are fixed and previously known. In other application contexts, however, this may not always be the case, in particular in statements found in real-world

argumentation. Consider, for example, the sentence ‘Doctors agree that smoking is bad for your health.’ We take this sentence (maybe debatably) as making a statement about common knowledge of all doctors. Encoding this claim as a formula of the form  $C_A\phi$  where  $A$  is a finite set explicitly enumerating all doctors is clearly neither feasible nor even semantically desirable, as the statement is presumably meant to hold without regard to exactly how many, and which, doctors are practising in the world at the moment. Rather, one would want  $A$  to be given by the defining property of *being a doctor*.

In the present paper, we introduce an epistemic logic that allows precisely this: *abstract-group epistemic logic (AGEL)* features a common knowledge operator for groups of agents described by concepts in a dedicated *agent logic*. We keep the technical development generic in the choice of agent logic, subject to some technical requirements on the agent logic that are satisfied, for instance, by the description logic *ALC*; so we can describe groups of agents such as ‘doctors and pharmacists’ or ‘parents of teenagers’. We note that we treat the agent logic as rigid, i.e. there is no uncertainty about membership in the groups it describes. In other words, group descriptions are *de re* rather than *de dicto*. We further illustrate the logic on a variant of the muddy children puzzle where the number of participants is unspecified and potentially large. Our main results on AGEL are EXPTIME-completeness of the satisfiability problem; a bounded (specifically, doubly exponential) model property; and a complete axiomatization. Technically, we establish the lower complexity bound by a satisfiability-preserving translation of standard group epistemic logic (Fagin et al. 1995) into AGEL, and the upper bound by a satisfiability-preserving translation of AGEL into the full  $\mu$ -calculus (i.e. *ALC* with inverse roles and fixpoint operators (Vardi 1998)). Use of the full  $\mu$ -calculus avoids the exponential blow-up that would be incurred by a more naive reverse translation of AGEL into group epistemic logic. However, the full  $\mu$ -calculus does not have the finite model property (Vardi 1998; Streett 1982). Instead, we show the bounded model property and completeness by means of a filtered model construction that uses ideas from the finite model construction for propositional dynamic logic (Fischer and Ladner 1979; Blackburn, de Rijke, and Venema 2001), in particular transitive closure of (small) canonical pseudo-

models.

Complete proofs can be found in the full version (Humml and Schröder 2022).

**Related Work** There is a line of research on indexing knowledge modalities with *names* that designate groups of agents (Grove and Halpern 1993) (Fagin et al. 1995, Chapter 6). We refer to such groups as *named groups*; they are similar to the atoms of our agent logic but are *non-rigid*, i.e. their interpretation depends on the current world, in an approach that is focused on the analysis of knowledge about the identity of agents. Although common knowledge of such name-defined groups has been mentioned early on (Grove and Halpern 1993), results have largely focused on operators of the type ‘every agent / some agent with name  $n$  knows’ (recall that generally, ‘everyone knows  $\phi$ ’ differs from  $\phi$  being common knowledge in that it need not imply that everyone knows that everyone knows  $\phi$ , etc.). Recently, Bílková, Christoff, and Roy (2021) have shown completeness and the finite model property (but no complexity bound) for common knowledge of name-defined groups. (A different form of common knowledge for non-rigid groups has been considered earlier, without considering axiomatization (Moses and Tuttle 1988).)

Extending the descriptive means for groups of agents has been considered already by Grove and Halpern (1993), who give an axiomatization (but no complexity bound) of a logic that features ‘everyone knows’ and ‘someone knows’ operators for propositional combinations of names. Additional expressive means are provided by subsequent first-order extensions of the logic that allow quantifying over agent names (Grove 1995; Naumov and Tao 2019) in the style of term modal logic (Fitting, Thalmann, and Voronkov 2001). Quantifying over agents provides a straightforward way of encoding, e.g., the ‘everybody knows operator’; for instance, ‘every doctor knows  $\phi$ ’ would be expressed as  $\forall x. \text{doctor}(x) \rightarrow K_x \phi$  if  $K_x$  denotes the usual single-agent knowledge modality ‘ $x$  knows’. Common knowledge, on the other hand, involves transitive closure and as such is not first-order expressible, hence not easily accommodated in such frameworks. Also, decidability results in a first-order setting will, of course, require additional restrictions (e.g. it has recently been shown that the two-variable fragment of term-modal logic is decidable, with complexity between NEXPTIME and 2EXPSpace (Padmanabha and Ramanujam 2019)).

The above logics and ours employ ternary transition relations relating agents to pairs of worlds. Beyond epistemic logic, ternary relations appear, e.g., in the logic of Pierce algebras (de Rijke 1993), in arrow logic (Venema 1996), and in Routley-Meyer-style semantics of *relevance logic* (Mares 2020). While such modalities are formally similar to *everyone-knows* modalities for abstract groups, the frame conditions imposed on models and, consequently, the attached meta-theory are quite different from ours.

## Abstract-Group Epistemic Logic

We proceed to introduce the syntax and semantics of *abstract-group epistemic logic* (AGEL). Details and proofs

are often omitted or only sketched; a full version can be found on arXiv <https://arxiv.org/abs/2211.16284>.

**Syntax** We parametrize the logic over the choice of an *agent logic*  $\mathcal{L}_{\text{Ag}}$  that serves to specify groups of agents. We will discuss assumptions on the semantics of  $\mathcal{L}_{\text{Ag}}$  later in this section. Syntactically, we require only that  $\mathcal{L}_{\text{Ag}}$  has a formula syntax where formulae are expressions in some grammar, in particular giving rise to a standard notion of *subformula*, and includes propositional atoms from a set  $\text{At}_{\text{Ag}}$ , referred to as *agent atoms*, as well as the full set of Boolean connectives. Properties of  $\mathcal{L}_{\text{Ag}}$  needed in our main results will be named explicitly in the respective theorems. One choice for  $\mathcal{L}_{\text{Ag}}$  that satisfies all requisite properties is the standard description logic  $\mathcal{ALC}$  (Baader et al. 2003).

We further assume a set  $\text{At}_W$  of *world atoms*. The set of (*world*) *formulae*  $\phi, \chi, \dots$  of AGEL is then defined by the grammar

$$\phi, \chi := \perp \mid p \mid \neg\phi \mid \phi \wedge \chi \mid C_\psi\phi \quad (p \in \text{At}_W, \psi \in \mathcal{L}_{\text{Ag}}).$$

That is, we include propositional atoms and Boolean connectives; further Boolean connectives  $\vee, \rightarrow, \leftrightarrow$  are defined as usual. The key feature is then the common knowledge operator  $C_\psi$  for groups of agents defined by an agent formula  $\psi$ ; a formula  $C_\psi\phi$  is read ‘ $\phi$  is common knowledge among agents satisfying  $\psi$ ’, using the term *common knowledge* in the sense recalled in the introduction.

**Example 1.** We may encode the statement ‘parents of teenagers know that education is pointless’ as the AGEL formula  $C_{\exists \text{hasChild. Teenager}} \text{ep}$ , using  $\mathcal{ALC}$  as the agent logic and understanding the world atom  $\text{ep}$  as ‘education is pointless’. The even more frustrating fact that parents know that their offspring know this as well would be captured by the formula  $C_{\exists \text{hasChild. Teenager}} C_{\text{Teenager}} \text{ep}$ .

**Semantics** We assume that the agent logic comes with a notion of *agent model*, and that every agent model  $\mathcal{A}$  is equipped with an underlying set  $\text{Ag}$  of *agents* and a satisfaction relation  $\models_{\mathcal{A}} \subseteq \text{Ag} \times \mathcal{L}_{\text{Ag}}$ ; we write  $\mathcal{A}, a \models \psi$  for  $(a, \psi) \in \models_{\mathcal{A}}$ , and  $\llbracket \phi \rrbracket_{\mathcal{A}} = \{a \in \text{Ag} \mid \mathcal{A}, a \models \phi\}$ . We require that  $\mathcal{L}_{\text{Ag}}$  *conservatively extends classical propositional logic*. By this we mean more specifically that  $\mathcal{L}_{\text{Ag}}$  does not impose restrictions on valuations of agent atoms, i.e. given a set  $\text{Ag}$  and a valuation  $V_{\text{Ag}}: \text{At}_{\text{Ag}} \rightarrow \mathcal{P}(\text{Ag})$ , there always exists an agent model  $\mathcal{A}$  with underlying set  $\text{Ag}$  such that  $\mathcal{A}, a \models q$  iff  $a \in V_{\text{Ag}}(q)$ , for  $a \in \text{Ag}$  and  $q \in \text{At}_{\text{Ag}}$ .

Then, an (AGEL) *model*  $\mathcal{M} = (X, \mathcal{A}, V_W, \sim)$  consists of a set  $X$  of *worlds*, an agent model  $\mathcal{A}$ , a *world valuation*  $V_W: \text{At}_W \rightarrow \mathcal{P}(X)$  interpreting the world atoms, and a family  $\sim$  of *indistinguishability relations*  $\sim_a \subseteq X \times X$  indexed over agents  $a \in \text{Ag}$ . We require that each  $\sim_a$  is an equivalence relation (in keeping with the usual view that epistemic indistinguishability relations should be equivalence relations); see however Remark 3. For a set  $A \subseteq \text{Ag}$  of agents, we write  $\sim_A = (\bigcup_{a \in A} \sim_a)^*$ <sup>1</sup> where  $(-)^*$  denotes reflexive-transitive closure (note that  $\sim_A$  is symmetric,

<sup>1</sup>This should not be confused with similar notation used to denote the intersection of the  $\sim_a$  in work on epistemic logic with quantification over agents (Naumov and Tao 2019)

hence an equivalence). We define satisfaction  $\mathcal{M}, x \models \psi$  ( $x$  satisfies  $\psi$ ) of a formula  $\psi$  at a world  $x$  recursively by

$$\begin{aligned} \mathcal{M}, x \not\models \perp & \quad \mathcal{M}, x \models p \text{ iff } x \in V_W(p) \\ \mathcal{M}, x \models \neg\phi & \text{ iff } \mathcal{M}, x \not\models \phi \\ \mathcal{M}, x \models \phi \wedge \chi & \text{ iff } \mathcal{M}, x \models \phi \text{ and } \mathcal{M}, x \models \chi \\ \mathcal{M}, x \models C_\psi\phi & \text{ iff whenever } x \sim_{[\psi]_{\mathcal{A}}} y, \text{ then } \mathcal{M}, y \models \phi; \end{aligned}$$

that is,  $C_\psi$  is the standard box modality for  $\sim_{[\psi]_{\mathcal{A}}}$ . When  $\mathcal{M}, x \models \phi$ , then we also say that  $(\mathcal{M}, x)$  is a *model* of  $\phi$  (and we will use this phrasing in general, also for other logics). The formula  $\phi$  is *satisfiable* if there is a model of  $\phi$ , and *valid* (notation:  $\models \phi$ ) if  $\mathcal{M}, x \models \phi$  for all  $\mathcal{M}, x$ . We write  $\llbracket \phi \rrbracket_{\mathcal{M}} = \{x \in X \mid \mathcal{M}, x \models \phi\}$ .

We record a fixpoint characterization of  $C_\psi$ :

**Lemma 2.** *The set  $\llbracket C_\psi\phi \rrbracket_{\mathcal{M}}$  is the greatest fixed point of the function  $F: \mathcal{P}(X) \rightarrow \mathcal{P}(X)$  given by  $F(U)$  being the set of worlds  $x$  such that  $\mathcal{M}, x \models \phi$  and whenever  $\mathcal{A}, a \models \psi$  and  $x \sim_a y$ , then  $y \in U$ .*

**Remark 3.** Since the modality  $C_\psi$  takes reflexive-transitive closures, it is in fact immaterial whether the indistinguishability relations  $\sim_a$  are individually reflexive and transitive; we impose the corresponding requirement mainly to ease notation and discussion.

**Remark 4.** In logical settings with non-rigid (i.e. world-dependent) agent models (e.g. (Grove and Halpern 1993; Grove 1995; Naumov and Tao 2019)), one can accommodate uncertainty about the identity and properties of agents, and moreover specify agent groups by their knowledge, as in ‘people who know  $\phi$  also know  $\psi$ ’. As indicated in the introduction, AGEL, which assumes the agent logic to be rigid, trades this mode of expression for a computationally and axiomatically tractable treatment of common knowledge. One may still envisage extending AGEL with an operator  $I$ , with  $I\phi$  read as ‘knows about the truth or falsity of  $\phi$ ’. That is, an agent satisfies  $I\phi$  if in every world, she knows either  $\phi$  or  $\neg\phi$ . For instance, if  $\phi$  represents the publicly contested fact whether or not transfer negotiations are under way concerning a football player  $P$ , then  $I\phi$  would hold for the relevant officials of the involved clubs and maybe for  $P$ . However, harnessing such an operator technically is likely to be challenging. To name one potential obstacle, the first-order translation of  $I\phi$  would presumably need to involve three variables (i.e. unlike the translation of most modal logics would not end up in the two-variable fragment): one for the agent  $x$ , and two for worlds that are indistinguishable for  $x$ , and then required to agree on  $\phi$  (see the appendix).

**Remark 5.** Since AGEL is effectively a fixpoint logic, it is expected that compactness fails. Indeed, given atomic agent concepts  $A, B$  and a world atom  $p$ , the set consisting of the formula  $\neg C_{A \vee B} p$  and all formulae of the form  $C_{D_1} C_{D_2} \dots C_{D_n} p$ , for  $n \geq 0$  and  $D_1, \dots, D_n \in \{A, B\}$ , is unsatisfiable but all its finite subsets are satisfiable. It follows that there is no finitary proof system for AGEL that is *strongly* complete, i.e. makes all unsatisfiable sets of formulae inconsistent. We later give a proof system that is *weakly* complete, i.e. derives all valid formulae.

## Complexity

We show next that the satisfiability problem of AGEL is EXPTIME-complete.

### Lower Bound:

#### Reduction from Group Epistemic Logic

We prove EXPTIME-hardness by a satisfiability-preserving encoding of standard group epistemic logic (GEL) (with common knowledge), which is known to be EXPTIME-hard (Fagin et al. 1995). (To facilitate the subsequent discussion, we reduce from a slightly more expressive logic than strictly necessary for the hardness proof.) We briefly recall the syntax and semantics of GEL: The logic is parametrized over a finite set  $Ag$  of agents and a set  $\text{At}_W$  of (world) atoms. The set of *formulae*  $\phi, \psi, \dots$  of GEL is then given by the grammar

$$\phi, \psi ::= \perp \mid p \mid \neg\phi \mid \phi \wedge \psi \mid C_G\phi$$

where  $p \in \text{At}_W$  and  $\emptyset \neq G \subseteq Ag$ , with  $C_G\phi$  read ‘ $\phi$  is common knowledge among the agents in  $G$ ’. (Knowledge operators  $K_a$  for individual agents  $a$  are included as common knowledge operators  $C_{\{a\}}$ .) As indicated in the introduction, the difference with AGEL is that in GEL, groups  $G$  of agents need to be given as enumerated finite subsets of a known fixed set of named agents. Models  $\mathcal{M} = (X, V_W, \sim)$  consist of a set  $X$  of worlds, a (world) valuation  $V_W: \text{At}_W \rightarrow \mathcal{P}(X)$ , and a family  $\sim$  of indistinguishability relations  $\sim_a \subseteq X \times X$ , indexed over agents  $a \in Ag$  and again required to be equivalence relations. For  $G \subseteq Ag$ , we write  $\sim_G := (\bigcup_{a \in G} \sim_a)^*$ . Then, satisfaction  $\mathcal{M}, x \models \phi$  of a formula  $\phi$  at a world  $x$  is defined recursively by the expected clauses for atoms and propositional connectives, and

$$\mathcal{M}, x \models C_G\phi \text{ iff whenever } x \sim_G y, \text{ then } \mathcal{M}, y \models \phi.$$

The encoding  $q$  of GEL into AGEL is given as follows. We introduce a fresh agent atom  $p_a$  for each  $a \in Ag$ . For a GEL formula  $\phi$ ,  $q(\phi)$  is then defined recursively by

$$q(C_G\phi) := C_{\bigvee_{a \in G} p_a} q(\phi)$$

and commutation with all other constructs (i.e.  $q(\neg\phi) = \neg q(\phi)$  etc.). Using the running assumption that  $\mathcal{L}_{\text{Ag}}$  conservatively extends classical propositional logic, we obtain

**Theorem 6** (Lower complexity bound). *The satisfiability problem for AGEL is EXPTIME-hard.*

*Proof sketch.* We need to show that  $q$  is indeed satisfiability-preserving. A model  $(\mathcal{M}, x)$  of a GEL formula  $\phi$  over the set  $Ag$  of agents, with  $\mathcal{M} = (X, V_W, \sim)$ , is transformed into a model  $(\mathcal{M}', x)$  of the AGEL formula  $q(\phi)$ , with  $\mathcal{M}' = (X, \mathcal{A}, V_W, \sim')$ , by taking  $Ag$  to be the underlying set of  $\mathcal{A}$ , and  $V_{\text{Ag}}(p_a) = \{a\}$  for  $a \in Ag$ ; this uses the running assumption that  $\mathcal{L}_{\text{Ag}}$  conservatively extends classical propositional logic.

Conversely, a model  $(\mathcal{M}, x)$  of  $q(\phi)$ , with  $\mathcal{M} = (X, \mathcal{A}, V_W, \sim)$ , is transformed into a model  $(\mathcal{M}', x)$  of  $\phi$ , with  $\mathcal{M}' = (X, V_W, \sim')$ , by taking  $\sim'_a = \sim_{V_{\text{Ag}}(p_a)}$  (using notation like in the formal semantics of AGEL).  $\square$

**Remark 7.** Depending on additional restrictions on the agent logic, it will sometimes be possible to give also a fairly straightforward satisfiability-preserving translation in the reverse direction, from AGEL to GEL. For instance, if the agent logic is just classical propositional logic, then we can proceed as follows: Let  $\phi$  be an AGEL formula, and let  $A \subseteq \text{At}_{\text{Ag}}$  be the set of agent atoms mentioned in  $\phi$ . Let  $Ag$  be the (finite) set of truth valuations  $\kappa: A \rightarrow 2$  in the set  $2 = \{\perp, \top\}$  of Boolean truth values, and write  $\kappa(\psi)$  for the truth value of a propositional formula  $\psi$  over  $A$  under  $\kappa$ ; then a translation of  $\phi$  into a satisfiability-equivalent GEL formula  $s(\phi)$  is defined recursively by

$$s(C_\psi\chi) = C_{\{\kappa \in Ag \mid \kappa(\psi) = \top\}}s(\chi)$$

and commutation with all other constructs. However, even in this basic case, such a translation will be of limited use as it has exponential blowup (the set  $\{\kappa \in Ag \mid \kappa(\psi) = \top\}$  can be exponentially large). For more expressive agent logics, e.g. whenever the agent logic extends  $\mathcal{ALC}$ , one has (series of) formulae that are satisfiable only over exponentially large agent models: Given agent formulae  $\psi_n$  that are of polynomial size in  $n$  but satisfiable only over agent models of exponential size in  $n$  (in  $\mathcal{ALC}$ , such  $\psi_n$  exist), the AGEL formulae  $p \wedge C_{\psi_n}\neg p$  are satisfiable only over models whose agent model components are of exponential size in  $n$ . Indeed, from a purely computational point of view (and for suitably restricted agent logics), one may see AGEL as a way of dealing with exponentially many agents without incurring doubly exponential computational cost. We realize this by an encoding into a different target logic, discussed after the next example.

**Example 8** (Lots of muddy children). The classical *muddy children* puzzle with  $k$  many children can be seen as  $k$  many agents  $A_i$  communicating according to a fixed protocol to gain common knowledge of a length- $k$  bitstring where each agent  $A_i$  can see all bits except the one at index  $i$  (Pavlovic 2021, Section 4.3). Specifically, it is commonly known initially that at least one bit is set, and the protocol then proceeds in rounds in which the agents announce whether they have learned their missing bit. The full modelling of the puzzle thus requires a dynamic epistemic logic with common knowledge and public announcements (Baltag, Moss, and Solecki 1998; Lutz 2006). Here, we concentrate on modelling, in an extended setting, how knowledge is gained in individual rounds of the protocol, which does not require public announcements; we generalize the textbook treatment by Huth and Ryan (2004).

We consider a variant of the puzzle that can essentially be seen as a product of  $n$  copies of the original puzzle. We then have an  $n \times k$ -matrix of bits, and each agent has an *invisibility type* consisting of one *invisibility index* per row determining the bit she cannot see in that row (in the original puzzle, there is only one row, and the invisibility type is the identity of the agent). We require that every bit of the matrix is seen by at least one agent. We do not otherwise restrict which invisibility types are realized; also, a given invisibility type may be realized by more than one agent. Note that there are exponentially many (viz,  $k^n$ ) invisibility

types; the point of these considerations being that the number of (distinguishable) agents is a) not fixed, and b) potentially large. We introduce propositional atoms  $p_{(j,i)}$  for  $1 \leq j \leq n$  and  $1 \leq i \leq k$  indicating whether the bit at position  $(j, i)$  in the matrix is *set* (1). We use agent atoms  $h_{j,i}$  to describe agents who cannot see the value of the bit at position  $(j, i)$  of the matrix, in an agent logic that extends propositional logic with a propositional background theory, in this case hardwiring the above description of the scenario (every agent sees all bits except one per row, and every bit is seen by some agent). The common knowledge resulting from the visibility conditions is hence

$$C_\top(\bigwedge_{i,j}(p_{j,i} \rightarrow C_{\neg h_{j,i}}p_{j,i}) \wedge (\neg p_{j,i} \rightarrow C_{\neg h_{j,i}}\neg p_{j,i})).$$

We write  $\alpha_j^{\leq x}$  for the (purely propositional) formula stating that at most  $x$  bits are set in row  $j$ ; that is,  $\alpha_j^{\leq x}$  is the disjunction of all conjunctions of the form  $\bigwedge_{i \in H} p_{i,j} \wedge \bigwedge_{i \in \{1, \dots, k\} \setminus H} \neg p_{i,j}$  where  $H \subseteq \{1, \dots, k\}$  and  $|H| \leq x$ . (This formula is of exponential size in the number  $k$  of columns, but note that this happens already in the original muddy children puzzle, i.e. in the case  $n = 1$ .) The initial knowledge available to the agents before the first round is that at least one bit is set in each row:

$$C_\top(\bigwedge_j \neg \alpha_j^{\leq 0}).$$

In each communication round the agents then choose one row, and communicate whether or not they know the value of the bit they cannot see in that row. Assuming all agents do not know the value of their respective bit, this establishes common knowledge about everyone's uncertainty:

$$C_\top(\bigwedge_{i,j} \neg C_{h_{j,i}}p_{j,i} \wedge \neg C_{h_{j,i}}\neg p_{j,i}).$$

Of course the order of rounds is irrelevant here, and the state of the protocol can hence simply be represented by a tuple  $(x_1, \dots, x_n)$  where each  $x_j$  counts how many communication rounds have taken place for row  $j$  (counting communication of the initial knowledge that at least one bit is set in each row). The key invariant of the protocol is that if these counters reach  $(x_1, \dots, x_n)$  without anyone having learned new bits, then this results in the accumulated common knowledge

$$C_\top(\bigwedge_j \neg \alpha_j^{\leq x_j}).$$

This clearly holds in the beginning of the game due to the initial knowledge. Then in state  $(x_1, \dots, x_n)$ , after querying row  $j$ , the common knowledge increases according to the inference

$$\begin{aligned} \Gamma, C_\top(\neg \alpha_j^{\leq x_j}), C_\top(\bigwedge_i \neg C_{h_{j,i}}p_{j,i} \wedge \neg C_{h_{j,i}}\neg p_{j,i}) \\ \vdash C_\top(\neg \alpha_j^{\leq x_j+1}). \end{aligned}$$

where  $\Gamma$  represents the visibility axioms and  $\vdash$  denotes local (i.e. per-world) consequence.. The formal proof is similar to the textbook proof for the original puzzle (Huth and Ryan 2004), and sketched as follows:

- Assume  $\alpha_j^{\leq x_j+1}$ .

- From the accumulated knowledge of the previous rounds, the agents already know that more than  $x_j$  bits are set in row  $j$ . From the assumption, they can conclude that exactly  $x_j + 1$  bits are set.
- Given that nobody knew whether their respective missing bit in row  $j$  is set, the agents can conclude that more than  $x_j + 1$  bits are set in row  $j$ , contradicting the assumption. (Otherwise at least one agent whose bit is set would see only  $x_j$  many set bits and could hence have deduced that her missing bit is set.)

Similar reasoning is used to conclude missing bits once enough communication rounds have been performed.

### Upper Bound: Encoding into the Full $\mu$ -Calculus

We establish the EXPTIME upper bound on satisfiability checking by a satisfiability-preserving translation of AGEL into the  $\mu$ -calculus with converse, also known as the *full  $\mu$ -calculus*, whose satisfiability problem is in EXPTIME (Vardi 1998). We emphasize that the full  $\mu$ -calculus does not have the finite model property (Vardi 1998; Streett 1982); we therefore establish a bounded model property separately in the next section.

In the translation, we use fixpoints to take transitive-reflexive closures, and inverse roles to close under symmetry. The main idea is then to view the family  $\sim$  of per-agent indistinguishability relations  $\sim_a$  featuring in the definition of AGEL models as a ternary relation on a single domain, and to encode this ternary relation as a binary relation between worlds and (agent, world)-pairs.

In fact, the single-variable fragment of the full  $\mu$ -calculus suffices for the translation; we briefly recall its syntax and semantics. The syntax is parametrized over sets AP and Prog of *atomic propositions* and *atomic programs*, respectively. A *program* is either an atomic program or a *converse program*  $\alpha^-$  of  $\alpha \in \text{Prog}$ . Moreover, we fix a single *fixpoint variable*  $z$ . Then, the set of *formulae*  $\phi, \psi, \dots$  is given by the grammar

$$\phi, \psi ::= \perp \mid p \mid z \mid \neg\phi \mid \phi \wedge \psi \mid [\alpha]\phi \mid \nu z. \phi$$

where  $p \in \text{AP}$  and  $\alpha$  is a program. The box operator  $[\alpha]$  is read ‘for all  $\alpha$ -successors’. The  $\nu z$  operator takes greatest fixpoints, and binds  $z$ ; that is, an occurrence of  $z$  is *free* if it lies outside the scope of any  $\nu z$ . Application of  $\nu z$  is restricted to formulae  $\phi$  in which every free occurrence of  $z$  is *positive*, i.e. lies under an even number of negations  $\neg$ . Further propositional connectives  $\top, \vee, \rightarrow, \leftrightarrow$  are defined as usual. Moreover, we define diamond operators as  $\langle \alpha \rangle \phi := \neg[\alpha]\neg\phi$ . (Also, one can define least fixpoints  $\mu z. \phi$ .)

The semantics is defined over *models*  $\mathcal{M} = (X, R, V)$  that consist of a domain  $X$ , an assignment  $R$  of a transition relation  $R(\alpha) \subseteq X \times X$  to every atomic program  $\alpha \in \text{Prog}$ ; and a valuation  $V: \text{AP} \rightarrow \mathcal{P}(X)$  of the atomic propositions. The interpretation of converse programs is defined by  $R(\alpha^-) = \{(e, d) \mid (d, e) \in R(\alpha)\}$ . The semantics of a formula  $\phi$  is then given as a function  $\llbracket \phi \rrbracket_{\mathcal{M}}: \mathcal{P}(X) \rightarrow \mathcal{P}(X)$  whose argument serves as the extension of  $z$ , inductively defined by the expected clauses for Boolean operators,

$$\llbracket p \rrbracket_{\mathcal{M}} = V(p), \text{ and}$$

$$\llbracket z \rrbracket_{\mathcal{M}}(U) = U$$

$$\llbracket [\alpha]\phi \rrbracket_{\mathcal{M}}(U) = \{w \in X \mid \forall v \in X. (w, v) \in R(\alpha) \rightarrow v \in \llbracket \phi \rrbracket_{\mathcal{M}}(U)\}$$

$$\llbracket \nu z. \phi \rrbracket_{\mathcal{M}}(U) = \bigcup \{Z \subseteq X \mid Z \subseteq \llbracket \phi \rrbracket_{\mathcal{M}}(Z)\}.$$

One shows by induction that when  $\nu z. \phi$  is well-formed, then  $\llbracket \phi \rrbracket_{\mathcal{M}}$  is monotone w.r.t. set inclusion, so  $\llbracket \nu z. \phi \rrbracket_{\mathcal{M}}(U)$  as defined above is, by the Knaster-Tarski fixpoint theorem, the greatest fixpoint of  $\llbracket \phi \rrbracket_{\mathcal{M}}$ .

In detail, the translation is then defined as follows. We assume for the translation that the agent logic is given as a fragment of the full  $\mu$ -calculus. For clarity, we write  $u$  for the syntactic embedding; we assume that each  $u(\psi)$  is *closed*, i.e. does not contain a free occurrence of  $z$ . As indicated previously, a typical choice would be  $\mathcal{ALC}$  (modulo the usual slight syntactic shifts), but for purposes of the complexity analysis, the agent logic could in fact be the full  $\mu$ -calculus itself (whereas the axiomatization introduced in the next section needs assumptions on the agent logic that are not satisfied by the full  $\mu$ -calculus, such as the finite model property).

We use three fresh atomic programs  $\pi_1, \pi_2$ , *edge*. The intention is that *edge* relates worlds to (agent, world)-pairs, out of which  $\pi_1$  extracts the agent component, and  $\pi_2$  the world component. Speaking more precisely, we do not insist that  $\pi_1, \pi_2$  are functional, so when  $(x, r) \in R(\text{edge})$  in a model, then  $r$  may in fact represent several (agent, world) pairs, namely all pairs  $(a, y)$  such that  $(r, a) \in R(\pi_1)$  and  $(r, y) \in R(\pi_2)$  (and indeed it may happen that  $r$  represents no pair at all). As per Remark 3, we do not need to worry at this point about the fact that the induced indistinguishability relations are not forced to be equivalence relations. As an intermediate step in the translation, we introduce forward and backward binary ‘next-step’ modalities by abbreviation:

$$\psi \rightsquigarrow \phi := [\text{edge}](\langle \pi_1 \rangle \psi \rightarrow \langle \pi_2 \rangle \phi),$$

is understood as ‘all worlds that are reached by a single forward indistinguishability step for an agent satisfying  $\psi$  satisfy  $\phi$ ’, and

$$\phi \leftarrow \psi := [\pi_2^-](\langle \pi_1 \rangle \psi \rightarrow [\text{edge}^-]\phi)$$

describes the opposite direction, ‘all worlds that are reached by a single backward indistinguishability step for an agent satisfying  $\psi$  satisfy  $\phi$ ’. Using these two abbreviations to model a step along the symmetrization of the indistinguishability relation, we can encode the common knowledge modality, modelling transitive-reflexive closure via a greatest fixpoint as indicated above: We define the translation  $t$  of a AGEL formula  $\phi$  into a formula  $t(\phi)$  in the full  $\mu$ -calculus recursively by

$$t(C_\psi \phi) = \nu z. t(\phi) \wedge (u(\psi) \rightsquigarrow z) \wedge (z \leftarrow u(\psi))$$

and commutation with all other constructs. (Since no recursive calls of  $t$  are made on  $\psi$ , its duplication does not lead to exponential blowup; recall that by the current assumption, the translation  $u$  of agent formulae does essentially nothing.) We thus obtain the following result.

**Theorem 9** (Upper complexity bound). *If the agent logic is a fragment of the full  $\mu$ -calculus, then the satisfiability problem of AGEL is in EXPTIME.*

*Proof sketch.* We need to show that the translation  $t$  is really satisfiability-equivalent. From a model  $\mathcal{M}, x_0$  of an AGEL formula  $\phi$ , with  $\mathcal{M} = (X, \mathcal{A}, V_W, \sim)$ , we construct a model  $(\mathcal{M}', x_0)$  of  $t(\phi)$ , with  $\mathcal{M}' = (X', R, V)$ , as follows. Following the intention of the translation  $t(\phi)$  as indicated above, we take  $X'$  to be the union of the set  $X$  of worlds, the set  $Ag$  of agents (the underlying set of  $\mathcal{A}$ ), and the set  $Ag \times X$  of (agent, world) pairs, assuming w.l.o.g. that these sets are disjoint. We interpret the syntactic material (atomic programs and atomic propositions) mentioned in agent formulae in  $\phi$  over  $Ag \subseteq X'$  like in the given agent model  $\mathcal{A}$ , exploiting that the agent logic is a fragment of the full  $\mu$ -calculus. Similarly, we interpret world atoms on  $X$  in the same way as in  $\mathcal{M}$ . Finally, we interpret  $\pi_1, \pi_2$  as the expected projections  $R(\pi_1) = \{((a, x), a) \mid (a, x) \in Ag \times X\}$ ,  $R(\pi_2) = \{((a, x), x) \mid (a, x) \in Ag \times X\}$ , and edge as

$$R(\text{edge}) = \{(x, (a, y)) \mid x \sim_a y \text{ in } \mathcal{M}\}.$$

It is not hard to see that  $(\mathcal{M}', x)$  is really a model of  $\phi$ .

In the converse direction, given a model  $(\mathcal{M}, x_0)$  of the  $\mu$ -calculus formula  $t(\phi)$ , with  $\mathcal{M} = (X, R, V)$ , we construct a model  $(\mathcal{M}', x_0)$  of  $\phi$ , with  $\mathcal{M}' = (X, \mathcal{A}, V_W, \sim)$ , as follows. We use the original set  $X$  of worlds also as the underlying set  $Ag$  of the agent model  $\mathcal{A}$ , whose structure we then obtain by just suitably restricting  $\mathcal{M}$ . Similarly, we take  $V_W$  to be the restriction of  $V$  to world atoms, i.e. we interpret world atoms in  $\mathcal{M}'$  like in  $\mathcal{M}$ . Finally, as indicated above, we define the indistinguishability relation  $\sim_a$  for  $a \in Ag = X$  as the symmetric closure of the relation

$$\{(x, y) \mid \exists(x, e) \in R(\text{edge}), \\ (e, a) \in R(\pi_1), (e, y) \in R(\pi_2)\}.$$

Again, it is not hard to check that  $(\mathcal{M}, x)$  is really a model of  $\phi$ .  $\square$

In combination with Theorem 6, we thus have

**Corollary 10** (Complexity of AGEL). *If the agent logic is a fragment of the full  $\mu$ -calculus, then the satisfiability problem of AGEL is EXPTIME-complete.*

**Remark 11.** Indeed the above encoding implies that one can raise the expressiveness of the agent logic to extensions of the full  $\mu$ -calculus that remain decidable in EXPTIME. One candidate is the *full hybrid  $\mu$ -calculus* (Sattler and Vardi 2001), which extends the full  $\mu$ -calculus with *nominals*, i.e. propositional atoms denoting single objects. This opens the possibility of combining explicitly named agents in the standard sense with abstract groups of agents, as in the formula

$$C_{\text{John} \vee \exists \text{ hasFriend. John}} \text{ Pub\_on\_Wednesdays},$$

which says that John and his friends know that their regular pub night is on Wednesdays.

## Completeness and Bounded Models

We axiomatize AGEL in Hilbert style using the following system  $C5$  of axioms and rules:

$$\begin{array}{ll} (T) & C_\psi \phi \rightarrow \phi \\ (\perp) & \phi \rightarrow C_\perp \phi \\ (K) & C_\psi(\phi \rightarrow \gamma) \rightarrow (C_\psi \phi \rightarrow C_\psi \gamma) \\ (4) & C_\psi \phi \rightarrow C_\psi C_\psi \phi \\ (5) & \neg C_\psi \phi \rightarrow C_\psi \neg C_\psi \phi \\ (Ind) & C_{\psi \vee \chi}(\phi \rightarrow (C_\psi \phi \wedge C_\chi \phi)) \rightarrow (\phi \rightarrow C_{\psi \vee \chi} \phi) \\ (Nec) & \frac{\phi}{C_\psi \phi} \quad (AM) \quad \frac{\gamma \rightarrow \psi}{C_\psi \phi \rightarrow C_\gamma \phi} \end{array}$$

Recall that by our running assumptions, the agent logic  $\mathcal{L}_{Ag}$  is closed under all propositional connectives. We write  $C5 \vdash \phi$  if an AGEL formula  $\phi$  is derivable in this system;  $\phi$  is *consistent* if  $C5 \not\vdash \neg \phi$ . For a finite set  $\Gamma$  of formulae, we write  $\widehat{\Gamma}$  for the conjunction of all formulae in  $\Gamma$ , and we say that  $\Gamma$  is *consistent* if  $\widehat{\Gamma}$  is consistent. The system includes the usual  $S5$  axioms for common knowledge, reflecting normality (axiom (K)), reflexivity (axiom (T)), transitivity (axiom (4)), and Euclideanity (axiom (5)), as well as the usual necessitation rule (Nec). As usual, the (K) axiom implies commutation of  $C_\psi$  with conjunction, and hence, together with the necessitation rule (Nec), monotonicity and replacement of equivalents for  $C_\psi$ . Specific properties of AGEL are reflected in the axiom ( $\perp$ ), which, together with (T), says that  $C_\perp \phi$  holds almost vacuously, in the sense that it does not claim any agent to know anything but requires  $\phi$  to be true in the current world; in the rule (AM), which says that  $C_\psi \phi$  is antimonotone in  $\psi$ , as requiring fewer agents to know  $\phi$  is a weaker claim; and, centrally, in the induction axiom (Ind), which captures the fact that the indistinguishability relation  $\sim_{A \cup B}$  for a union  $A \cup B$  of two sets  $A, B$  of agents is the reflexive-transitive closure of the union of the indistinguishability relations  $\sim_A, \sim_B$  for the original sets. The rule (AM) implies replacement of equivalents in the index of  $C_\psi$  (from equivalence of  $\psi$  and  $\psi'$ , derive equivalence of  $C_\psi \phi$  and  $C_{\psi'} \phi$ ). Via (AM), the system depends on reasoning in the agent logic, which we will assume to be completely axiomatized.

We note first that the induction axiom generalizes to multiple disjuncts:

**Lemma 12.** *Every formula of the form*

$$C_{\bigvee_{i=1}^n \psi_i}(\phi \rightarrow (\bigwedge_{i=1}^n C_{\psi_i} \phi)) \rightarrow (\phi \rightarrow C_{\bigvee_{i=1}^n \psi_i} \phi)$$

(for  $n \geq 0$ ) is derivable in  $C5$ .

We explicitly record soundness of the system:

**Theorem 13** (Soundness). *If  $C5 \vdash \phi$  then  $\models \phi$ .*

We show completeness via a finite canonical model construction that is related to the standard treatment of propositional dynamic logic (PDL) in that it needs to close canonical models under transitivity (Fischer and Ladner 1979; Blackburn, de Rijke, and Venema 2001). This construction requires some restrictions on the agent logic:

**Definition 14.** We say that the agent logic  $\mathcal{L}_{\text{Ag}}$  has the *filtered model property* if, for each finite set  $\Sigma_{\text{Ag}}$  of agent formulae that is closed under subformulae, there is an agent model  $\mathcal{A}(\Sigma_{\text{Ag}})$ , with underlying set denoted by  $\text{Ag}(\Sigma_{\text{Ag}})$ , such that on the one hand any two distinct agents in  $\text{Ag}(\Sigma_{\text{Ag}})$  are distinguished by a formula in  $\Sigma_{\text{Ag}}$  (in particular  $\text{Ag}(\Sigma_{\text{Ag}})$  is finite, namely at most exponential in  $|\Sigma_{\text{Ag}}|$ ), and on the other hand for every satisfiable subset  $\Gamma \subseteq \Sigma_{\text{Ag}}$ , there is an agent in  $\text{Ag}(\Sigma_{\text{Ag}})$  that satisfies all formulae in  $\Gamma$ . Since the agent logic is closed under propositional connectives, we then have, for each agent  $a \in \text{Ag}(\Sigma_{\text{Ag}})$ , a *characteristic agent formula*  $\hat{a}$  (a propositional combination of  $\Sigma_{\text{Ag}}$ -formulae) such that  $\llbracket \hat{a} \rrbracket_{\mathcal{A}(\Sigma_{\text{Ag}})} = \{a\}$ . We put

$$\text{Clo}_{\text{Ag}}(\Sigma_{\text{Ag}}) = \Sigma_{\text{Ag}} \cup \{\hat{a} \mid a \in \text{Ag}(\Sigma_{\text{Ag}})\}.$$

**Example 15.** As indicated in the introduction,  $\mathcal{ALC}$  has the filtered model property (and is completely axiomatized), and in fact one can go beyond  $\mathcal{ALC}$  to some degree. In particular, the extension of  $\mathcal{ALC}$  with *nominals*,  $\mathcal{ALCO}$ , still has the filtered model property. (See also comments in Remark 11.)

We fix from now on a consistent AGEL formula  $\rho_0$ . We base our canonical model construction on a suitable notion of closure.

**Definition 16** (Normalized negation). We let  $\dot{\neg}\phi = \chi$  if  $\phi$  has the form  $\phi = \neg\chi$ , and  $\dot{\neg}\phi = \neg\phi$  otherwise.

**Definition 17** (Closure). Let  $\Sigma_{\text{Ag}}$  be the closure of the set of agent formulae occurring in  $\rho_0$  under taking subformulae. Then the *closure*  $\Sigma$  of  $\rho_0$  is the least set of world formulae containing  $\rho_0$  that is closed under (world) subformulae and normalized negation, and moreover satisfies

$$\text{if } C_\chi\phi \in \Sigma \text{ and } \psi \in \text{Clo}_{\text{Ag}}(\Sigma_{\text{Ag}}), \text{ then } C_\psi\phi \in \Sigma$$

(with  $\text{Clo}_{\text{Ag}}(\Sigma_{\text{Ag}})$  as per Definition 14).

We next construct a weak form of model of  $\rho_0$  that assigns indistinguishability relations to sets of agents without regard to their definition via reflexive-transitive closure; this will be rectified in a subsequent step.

**Definition 18** (Pseudo-model). An AGEL *pseudo-model*  $\mathcal{M}^p = (X, \mathcal{A}, V_W, \sim^p)$  consists of a set  $X$  of worlds, an agent model  $\mathcal{A}$  with underlying set  $\text{Ag}$  of agents, a valuation  $V_W: \text{At}_W \rightarrow \mathcal{P}(X)$  of the world atoms, and an equivalence relation  $\sim_A^p$  on  $X$  for each subset  $A \subset \text{Ag}$ . The *semantics* of AGEL over pseudo-models is defined like over models, except that the interpretation of  $C_\psi$  uses the relation  $\sim_{\llbracket \psi \rrbracket_{\mathcal{A}}}^p$  in place of  $\sim_{\llbracket \psi \rrbracket_{\mathcal{A}}}$ .

We construct a *canonical pseudo-model*  $\mathcal{C}_\Sigma^p = (X_\Sigma, \mathcal{A}(\Sigma_{\text{Ag}}), V_W, \sim^p)$  by taking  $X_\Sigma$  to consist of the maximal consistent subsets of  $\Sigma$ ;  $\mathcal{A}(\Sigma_{\text{Ag}})$  as per Definition 14;  $V_W(p) = \{\Gamma \in X_\Sigma \mid p \in \Gamma\}$ ; and

$$\Gamma \sim_{\llbracket \psi \rrbracket_{\mathcal{A}}}^p \Delta \leftrightarrow \hat{\Gamma} \wedge P_\psi \hat{\Delta} \text{ consistent}$$

where by  $P_\psi$  we denote the dual of  $C_\psi$ , i.e.  $P_\psi\phi := \neg C_\psi \neg\psi$ . Well-definedness is guaranteed by the properties of  $\mathcal{A}(\Sigma_{\text{Ag}})$ ; indeed we shall often write  $\sim_{\llbracket \psi \rrbracket_{\mathcal{A}}}^p$  in place of  $\sim_{\llbracket \psi \rrbracket_{\mathcal{A}}}^p$  for readability, similarly for  $\sim$ . We note that  $\Sigma$  inherits exponential size from  $\text{Clo}_{\text{Ag}}(\Sigma_{\text{Ag}})$ , so  $X_\Sigma$  is of doubly exponential size in the size of  $\rho_0$ .

**Lemma 19.** The relations  $\sim_{\llbracket \psi \rrbracket_{\mathcal{A}}}^p$  are reflexive and symmetric.

The point of using the canonical pseudo-model is that it allows for a straightforward proof of the usual existence lemma:

**Lemma 20** (Existence Lemma). Let  $\Gamma \in X_\Sigma$  be a world in the canonical pseudo-model  $\mathcal{C}_\Sigma^p$  such that  $\neg C_\psi\phi \in \Gamma$ . Then there exists  $\Delta \in X_\Sigma$  such that  $\Gamma \sim_{\llbracket \psi \rrbracket_{\mathcal{A}}}^p \Delta$  and  $\neg\phi \in \Delta$

Leveraging characteristic agent formulae, we can derive a proper AGEL-model, the *canonical model*  $\mathcal{C}_\Sigma$ , from the canonical pseudo-model  $\mathcal{C}_\Sigma^p$  by taking

$$\sim_a = \sim_a^p,$$

where we exploit that by Remark 3, we do not need to care whether  $\sim_a$  is transitive. The key point is then that the existence lemma survives this transition thanks to the following fact, which hinges on the induction axiom (*Ind*):

**Lemma 21.** For all formulae  $\psi \in \text{Clo}_{\text{Ag}}(\Sigma_{\text{Ag}})$ ,  $\sim_{\llbracket \psi \rrbracket_{\mathcal{A}}}^p \subseteq \sim_{\llbracket \psi \rrbracket_{\mathcal{A}}}$

It is then straightforward to establish the expected truth lemma, making use of the fact that  $\text{Clo}_{\text{Ag}}(\Sigma_{\text{Ag}})$  contains all requisite characteristic agent-formulae:

**Lemma 22** (Truth Lemma). Let  $\Gamma$  be a world in the canonical model  $\mathcal{C}_\Sigma$ . Then for all  $\phi \in \Sigma$ ,

$$\phi \in \Gamma \leftrightarrow \mathcal{C}_\Sigma, \Gamma \models \phi.$$

Completeness and the bounded model property are then immediate:

**Corollary 23** (Completeness over finite models). Suppose that  $\mathcal{L}_{\text{Ag}}$  has the filtered model property and is completely axiomatized. Then  $\mathcal{C}5$ , together with the axiomatization of  $\mathcal{L}_{\text{Ag}}$ , is weakly complete, i.e. every valid AGEL formula is derivable. Moreover, AGEL has the bounded model property: if a formula  $\rho_0$  is satisfiable, then  $\rho_0$  has a finite model with at most doubly exponentially many worlds in the size of  $\rho_0$ .

## Conclusions

We have introduced *abstract-group epistemic logic* (AGEL), a logic for reasoning about the common knowledge of groups of agents that are described abstractly via defining properties. We have established EXPTIME-completeness, a bounded model property, and (necessarily weak) completeness of a natural axiomatization. The EXPTIME upper bound holds in spite of the fact that the expected encoding into standard group epistemic logic (with a common knowledge operator for enumerated groups of agents) incurs exponential blowup, and relies instead on a satisfiability-preserving translation into the  $\mu$ -calculus with converse. Key directions for future research concern in particular extensions of the logic by a distributed knowledge operator; by dynamic epistemic modalities such as public announcements; by expressive means for describing groups of agents via their individual knowledge; and by allowing non-rigid agent names and agent atoms to capture knowledge about agents.

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