

# From Monopoly to Competition: Optimal Contests Prevail

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## Abstract

We study competition among contests in a general model that allows for an arbitrary and heterogeneous space of contest design and symmetric contestants. The goal of the contest designers is to maximize the contestants' sum of efforts. Our main result shows that optimal contests in the monopolistic setting (i.e., those that maximize the sum of efforts in a model with a single contest) form an equilibrium in the model with competition among contests. Under a very natural assumption these contests are in fact dominant, and the equilibria that they form are unique. Moreover, equilibria with the optimal contests are Pareto-optimal even in cases where other equilibria emerge. In many natural cases, they also maximize the social welfare.

## 1 Introduction

Many important economic and social interactions may be viewed as contests. The designer aims to maximize her abstract utility (e.g. workers' productivity, sales competitions, innovative ideas for new projects, useful information from contestants) by forming a contest, and contestants exert effort in hopes of winning a prize. The design of optimal contests is by now well understood in the monopolistic (single-contest) setting. In particular, in many cases, a winner-takes-all contest is optimal in terms of maximizing either the sum of contestants' efforts or the single maximal effort (e.g. (Barut and Kovenock 1998; Kalra and Shi 2001; Moldovanu and Sela 2001; Terwiesch and Xu 2008; Chawla, Hartline, and Sivan 2019)).

While most of the existing literature on contest design focuses on a monopolistic contest with an exogenously given set of contestants, in reality, many times, there are multiple contests on a market and these contests must compete to attract contestants, which induces a participation vs. effort trade-off. Although the optimal contest in the single-contest setting induces maximal effort exertion *after* contestants choose to participate in their contests, contestants might at the same time be discouraged from choosing the more demanding contests. To attract contestants, it seems helpful to design lucrative and easy contests that leave a large fraction of the total surplus to contestants. Thus, these two aspects appear to be contradicting.

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Previous literature has already started to acknowledge this issue with few models that formally studied it (e.g., (Azmat and Möller 2009; Stouras, Erat, and Lichtendahl 2020; Deng et al. 2022)). In particular, (Azmat and Möller 2009) conclude that, despite the competitive environment, contest designers should still choose effort-maximizing contests since the effort aspect dominates the participation aspect. However, it is not clear how robust this conclusion really is, since these previous papers analyze models that are restricted in two main aspects. First, they assume that all contests have the same total prize to offer.<sup>1</sup> Second, and perhaps even more important, they restrict the choice of a contest and assume that designers choose a multiple-prize contest where contestants' winning probabilities for each prize are determined by a Tullock success function that is exogenous and identical for all contest designers. Generalizing the model of (Azmat and Möller 2009), our paper provides a more general framework and analysis of competition among multiple contests, showing that effort still dominates participation even allowing asymmetric contest designers and general contest design space.

### 1.1 Overview of Results and Techniques

At a high level, our competition model is composed of three phases. In the first phase, contest designers choose their contests (and commit to them) from a class of contests available to them which could be any arbitrary class of contests. In the second step, after seeing the contests chosen by designers, each contestant chooses (possibly in a random way) one contest to participate in. Finally, in each contest, contestants invest effort by playing a symmetric Nash equilibrium (which previous literature has shown to exist, see details in Section 2). Designers aim to maximize the sum of efforts exerted in their own contests and contestants aim to maximize the reward they receive minus their effort.

We identify two properties of contests and show as our first main result that if every contest designer chooses a contest that satisfies these two properties then we are at an equilibrium. The first property, which we term Monotonically Decreasing Utility (MDU), simply says that a con-

<sup>1</sup>Previous analysis substantially relies on this assumption. It also formally assumes exactly two competing contests, although this aspect may be more easily generalized.

testant's symmetric-equilibrium utility in the single contest game decreases as the number of contestants increases. The second property, which we term Maximal Rent Dissipation (MRD), is defined with respect to the space of possible contests  $\mathcal{S}_i$  that contest designer  $i$  has. A specific contest  $C_i \in \mathcal{S}_i$  has maximal rent dissipation if, for any other contest  $C'_i \in \mathcal{S}_i$  that could be a possible choice for designer  $i$ , and for any number of contestants,  $k$ , the contestant's symmetric-equilibrium utility in the single contest  $C_i$  when there are  $k$  contestants is not larger than the contestant's equilibrium utility in the single contest  $C'_i$  when there are  $k$  contestants. Thus,  $C_i$  minimizes the contestants' utilities and therefore maximizes the utility of the contest designer, among all contests available to designer  $i$ . In this sense,  $C_i$  is optimal for the designer in the single contest game.

**Main Results** Our first main result (Theorem 3.1) is that MRD contests form an equilibrium of the contest competition game, and that, in the case where designers can choose only MDU contests, MRD contests are weakly dominant. In the latter case, MRD contests are the only possible contests that emerge in equilibria (Theorem 3.3). If designers can choose non-MDU contests, non-MRD contests may emerge as additional equilibria (Example 3.2) on top of the equilibria that MRD contests form. However, we show that even when non-MRD contests form equilibria, choosing MRD contests is a Pareto-optimal outcome for contest designers (Theorem 3.7), maintaining the attractiveness of MRD contests to contest designers. In summary, effort indeed dominates participation in the aforementioned trade-off for the competing designers. We additionally show that MRD contests are welfare optimal in many natural cases (although not always, see Section 5). These conclusions hold regardless of the number of designers, the rewards they have, and the classes of contests they can choose from subject to the assumptions detailed below.

**Important Modeling Assumptions** Our analysis relies on the fact that, in our model, the social welfare generated by any contest is fixed and independent of effort: as long as  $k \geq 1$  contestants participate in a contest, the social welfare is simply the reward offered by the designer (the efforts cancel out in the social welfare summation because they appear in a plus sign for the designer and in a minus sign for the contestants). This is an implication of assuming a linear and symmetric cost of effort, and of assuming that the reward has the same value for all contestants. We also assume that the reward is fully allocated to the contestants. In particular, if only one contestant shows up, that contestant receives the full reward. The latter two assumptions are natural in many types of contests, e.g., in R&D contests, in crowd-sourcing websites, in rewarding athletes, musicians, actors, for their performance, etc. The assumption that the reward is valued the same way by all contestants may be less natural, though, in incomplete-information auction-like settings where the reward is a non-monetary object that may be valued differently by different contestants/bidders. This is a key reason why our results may not necessarily carry over to an incomplete-information setting.

**Intuition** To see why the above assumptions imply our first main result that MRD contests are at equilibrium in our contest competition game, consider the following intuition. Suppose that the designer switches from some MRD contest  $C_i$  to a non-MRD (i.e., less surplus-extracting) contest  $C'_i$ , to attract more contestants (we indeed prove that there are no contests that attract *less* contestants than MRD contests). Consider the contribution of one additional contestant to the designer's utility, i.e., consider designer  $i$ 's marginal utility of increasing the symmetric equilibrium probability  $p_i$  that a contestant participates in contest  $i$ . Each additional contestant generates social welfare equal to the prize awarded by this contest when no other contestants participate; however, in this case, the designer's utility is zero in both  $C_i$  and  $C'_i$  since in this case, the contestant receives the full reward without exerting any effort. So, when no other contestants participate, the contestant contributes 0 to the designer's utility in both  $C_i$  and  $C'_i$ . On the other hand, when other contestants are participating, this added contestant does not contribute to the social welfare and we show that the sum of contestants' utilities in  $C'_i$  is higher than that in  $C_i$ , thus strictly lowering the designer's utility. There are two contradicting effects here –  $C'_i$  leaves more surplus to contestants thus increasing their utility but on the other hand, attracts more contestants hence decreasing the utility of each contestant – and it turns out that this is true only when the other designers' contests are MDU. We conclude that as long as  $p_i > 0$ , the designer has an incentive to decrease  $p_i$  as much as possible; this is achieved by an MRD contest.

**Applications** We apply our general framework to a competition among Tullock contests with varying prize structures. As corollaries, we obtain: (1) Choosing winner-takes-all contests, i.e., giving the entire reward to the winner, is an equilibrium for designers who can only adjust prize structures but not their observability of effort and/or quality, i.e., the Tullock parameter  $\tau$  is exogenous (Corollary 4.3). (2) Choosing all-pay auctions is an equilibrium for designers who can only adjust observability but must choose winner-takes-all; in fact, we show that choosing any  $\tau \geq 2$  is an equilibrium (Corollary 4.2). (3) Choosing the winner-take-all contest with the largest possible observability is an equilibrium for designers that can adjust both prize structures and observability (Corollary 4.4).

## 1.2 Related Literature

In the full version we review three strands of literature. One strand considers the optimality of contests in a monopolistic (single contest) setting, in terms of revenue for the designer, agent participation, etc. The other two, more recent strands, consider equilibrium outcomes when multiple contest (or more generally, mechanism) designers compete over agents' participation and effort. Our result is interesting in the way it ties together these domains: We show that effort-maximizing contests (those that were identified in the first literature strand) are in an equilibrium in our general model, that belongs to and follows the second strand. The takeaway message to a contest designer is that in case she is interested to maximize the sum of contestants' efforts, introduc-

ing competition does not change her basic goal of maximizing over effort extraction, given a fixed set of contestants.

(Körpeoğlu, Korpeoglu, and Hafalir 2017) consider an incomplete information contest model where contestants can participate in multiple contests, and contest designers use winner-takes-all contests while strategically choosing rewards to maximize the maximal submission quality minus reward. They show that, in several cases, contest designers benefit from contestants' participation in multiple contests.

## 2 Model and Preliminaries

All missing proofs appear in the full version.

### 2.1 A Single-Contest Game

A contest designer designs a contest among several contestants to maximize the sum of efforts exerted by the contestants in return for some reward to be divided among them according to some winning rule determined by the designer.

Formally, a contest  $C$  is composed of a reward (or a total prize)  $R$  and contest success functions (CSF)  $f^k : \mathbb{R}_{\geq 0}^k \rightarrow [0, 1]^k$  for each number of contestants  $k > 0$ . Contestants exert efforts  $(e_1, \dots, e_k) \in \mathbb{R}_{\geq 0}^k$  to compete for the reward. Each contestant  $i$  receives a fraction  $f_i^k(e_1, \dots, e_k)$  of the reward, where  $f_i^k(e_1, \dots, e_k)$  is the  $i$ -th coordinate of the vector  $f^k(e_1, \dots, e_k)$ . We allow general functions  $f_i^k(\cdot)$  and only require that  $\sum_{i=1}^k f_i^k(e_1, \dots, e_k) \leq 1$ . This captures a wide variety of contest success functions. For example, in a single-prize (or winner-takes-all) setting,  $f_i^k(e_1, \dots, e_k)$  is the probability that contestant  $i$  wins the entire prize; Section 4.2 and Footnote 6 discuss how multi-prize settings are captured by our model. The utility of a contestant is the reward she gets minus the effort she exerts:  $f_i^k(e_1, \dots, e_k)R - e_i$ . Altogether, for a given number of contestants  $k$ , this defines a complete-information game for the contestants. Later on, we also consider the utility of the contest designer which we define as the sum of efforts  $\sum_{i=1}^k e_i$ . A designer's utility is 0 when  $k = 0$ .

**Definition 2.1.** A contest is anonymous if its contest success functions  $f^k : \mathbb{R}_{\geq 0}^k \rightarrow [0, 1]^k$  satisfy, for any  $k > 0$ , for any  $(e_1, \dots, e_k) \in \mathbb{R}_{\geq 0}^k$  and any permutation  $\pi$  of  $(1, \dots, k)$ ,

$$f^k(e_{\pi(1)}, \dots, e_{\pi(k)}) = \left( f_{\pi(1)}^k(e_1, \dots, e_k), \dots, f_{\pi(k)}^k(e_1, \dots, e_k) \right).$$

**Definition 2.2.** A contest fully allocates the reward if its CSF  $f^k : \mathbb{R}_{\geq 0}^k \rightarrow [0, 1]^k$  satisfy,  $\forall k > 0, \forall (e_1, \dots, e_k) \in \mathbb{R}_{\geq 0}^k$ ,  $\sum_{i=1}^k f_i^k(e_1, \dots, e_k) = 1$ .

**Example 2.3.** A Tullock contest (or, more accurately, a single-prize Tullock contest) parameterized by  $\tau \in [0, +\infty]$  has the following contest success function:

$$f_i^k(e_1, \dots, e_k) = \begin{cases} \frac{e_i^\tau}{\sum_{j=1}^k e_j^\tau} & \text{if } e_j > 0 \text{ for some } j \in \{1, \dots, k\} \\ \frac{1}{k} & \text{otherwise} \end{cases}$$

When  $\tau = +\infty$  the contest is an "All Pay Auction (APA)" where the contestant with the highest effort wins with certainty (to maintain anonymity, if several contestants exert

the highest effort, they all win with equal probability). A Tullock contest is anonymous and it fully allocates the reward.

The Tullock contest model captures a noisy mapping of effort to output via the randomness parameter  $\tau$ . As  $\tau$  increases the mapping becomes less noisy (more accurate). For example, suppose a contestant who exerts effort  $e_i$  produces random output  $Y_i = \tau \log e_i + Z_i$  where  $Z_i$  follows some noise distribution. Efforts are unobservable and the reward is allocated to the contestant with maximal  $Y_i$ . In this setting,  $f_i^k(e_1, \dots, e_k)$  is the expected fraction of reward received by contestant  $i$ , as a function of contestants' unobserved efforts. When the noise distribution is a Gumbel distribution,  $f$  becomes a Tullock function (Fu and Wu 2019). Our model therefore captures some of the aspects underlying the issue of unobservable efforts / a noisy mapping.

**Definition 2.4.** Let  $\mathcal{C}_R$  be the set of all anonymous contests with reward  $R$  that fully allocate the reward and have a symmetric Nash equilibrium among  $k$  contestants  $\forall k > 0$ .

For example, (Alcalde and Dahm 2010) and (Baye, Kovenock, and De Vries 1996) show that Tullock contests with parameters  $\tau \in [0, \infty)$  and  $\tau = \infty$  admit a symmetric Nash equilibrium (NE); thus,  $\mathcal{C}_R$  contains all Tullock contests with reward  $R$ . Other examples of contests that admit symmetric NE are given in, e.g., the seminal works of (Hirschleifer 1989; Nti 1997), a survey by (Corchón 2007), as well as later works such as (Amegashie 2012).

We assume throughout that all contestants in the same contest (with  $k$  players) play a symmetric NE of that contest. Formally, for every  $C \in \mathcal{C}_R$  we fix a (mixed strategy) symmetric NE, i.e.,  $e_1, \dots, e_i, \dots, e_k$  are i.i.d. random variables that follow a distribution  $F$  defined by a mixed strategy NE. Since  $C$  is anonymous, in the symmetric NE all contestants get an equal expected fraction of the reward and hence their expected utilities are identical. We denote their identical expected utility by  $\gamma_C(k) = \mathbb{E}_{e_1, \dots, e_k \sim F} [f_i^k(e_1, \dots, e_k)R - e_i]$ . Moreover, since  $C \in \mathcal{C}_R$  fully allocates the reward, we must have  $\mathbb{E}_{e_1, \dots, e_k \sim F} [f_i^k(e_1, \dots, e_k)] = \frac{1}{k}$  and hence

$$\gamma_C(k) = \frac{R}{k} - \mathbb{E}_{e_i \sim F} [e_i]. \quad (1)$$

If  $k = 1$ , the single contestant does not exert any effort, hence  $\gamma_C(1) = R$ . We also have  $\gamma_C(k) \geq 0 \forall k > 0$  since a contestant can choose to exert zero effort and guarantee non-negative utility. Moreover, since  $e_i \geq 0$ ,  $\gamma_C(k) \leq \frac{R}{k}$ . We use  $\gamma_C(k)$  to express the utility of a designer that uses a contest  $C \in \mathcal{C}_R$  with  $k \geq 1$  contestants by rearranging (1):

$$\mathbb{E}_{e_1, \dots, e_k \sim F} \left[ \sum_{i=1}^k e_i \right] = k \mathbb{E}_{e_i \sim F} [e_i] = R - k\gamma_C(k). \quad (2)$$

We assume that the utility of a contest designer is the expected sum of efforts, even if this is non-observable. This fits settings like workplace contests that aim to improve workers' productivity, when workers' productivity is linear. In an additive noise model, the expected sum of efforts is equal to the expected sum of qualities of submissions since the expected noise is usually assumed to be zero.

## 2.2 A Contest Competition Game

We study a game where multiple contest designers compete by choosing their contest success functions. Contestants observe these contests and choose in which one to participate.

**Definition 2.5.** A complete-information contest competition game is denoted by  $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ , where  $m \geq 2$  is the number of contest designers,  $n \geq 1$  is the number of contestants,  $R_i > 0$  is the reward of contest  $i$ , and  $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$ . The game has two phases:

1. **Designers choose contests.** Each designer  $i$  chooses a contest  $C_i \in \mathcal{S}_i$  simultaneously. Contestants observe the chosen contests  $(C_1, \dots, C_m)$ .
2. **Contestants play a normal-form game of choosing in which contest to participate.** A pure strategy of each contestant in this game is to choose one contest. Importantly, contestants may play a mixed strategy: each contestant  $\ell$  participates in each contest  $C_i$  ( $i = 1, \dots, m$ ) with some probability  $p_{\ell i}$ ,  $\sum_{i=1}^m p_{\ell i} = 1$ . Let the vector of probabilities chosen by contestant  $\ell$  be  $\mathbf{p}_\ell = (p_{\ell 1}, \dots, p_{\ell m})$ .

After Nature assigns contestants to contests, utilities are as follows. If  $k \geq 1$  contestants participate in contest  $C_i$ , each of them gains utility  $\gamma_{C_i}(k)$  and designer  $i$  gains utility  $R_i - k\gamma_{C_i}(k)$ . If  $k = 0$ , the designer's utility is 0.

Contestants must participate in some contest, i.e., they cannot decide not to participate. This assumption is innocuous since, as remarked above, in equilibrium contestants utilities are always non-negative. An important element of our model is the space  $\mathcal{S}_i$  of all possible contests a designer can *strategically* choose. We treat this as an abstract set of anonymous contests that fully allocate the reward and admit a symmetric Nash equilibrium among  $k$  contestants for any  $k > 0$ . An important application is Tullock contests. When a contestant decides on a level of effort to exert, she knows the total number of contestants  $k$  that participate in the same contest. In practice, contestants observe this number in physical contests (like sports contests) or if the designer reveals this information. (Myerson and Wärneryd 2006) show that contest designers have an incentive to do so because the expected aggregate effort in a contest with a commonly known number of contestants is in general higher; (Lim and Matros 2009) show that for Tullock contests with binomial participation distribution, the designer always reveals the number  $k$ ; (Ryvkin and Drugov 2020) characterizes that this is true in general for contests with concave marginal costs.

In the second phase, each contestant has a finite number  $m$  of possible actions and the game is symmetric, hence there must exist at least one symmetric mixed NE (Nash 1951). We assume throughout that contestants play this symmetric equilibrium, i.e., we only consider equilibria in which the probability vector of every contestant is the same ( $\mathbf{p}_1 = \mathbf{p}_2 = \dots = \mathbf{p}_n$ ). The full version discusses the case where the contestants choose an asymmetric equilibrium. As argued by (Burdett, Shi, and Wright 2001), for example, a symmetric equilibrium requires no coordination among contestants and is more robust and natural than an asymmetric equilibrium. Formally, we denote by  $\mathbf{p}(C_1, \dots, C_m) \in \mathbb{R}^m$  the probability vector chosen by the contestants at their

symmetric equilibrium when the designers choose contests  $(C_1, \dots, C_m)$  in the first phase.<sup>2</sup>

Assume that designers choose contests  $\mathbf{C} = (C_1, \dots, C_m)$  and contestants choose symmetric participation probabilities  $(p_1, \dots, p_m) = \mathbf{p}(\mathbf{C})$ . For a contestant who participates in  $C_i$ , the number of other contestants in  $C_i$  follows the binomial distribution  $\text{Bin}(n-1, p_i)$ . Thus, the expected utility of a contestant participating in  $C_i$ , denoted by  $\beta(C_i, p_i)$ , is:

$$\beta(C_i, p_i) = \mathbb{E}_{k \sim \text{Bin}(n-1, p_i)} [\gamma_{C_i}(k+1)] = \sum_{k=0}^{n-1} \binom{n-1}{k} p_i^k (1-p_i)^{n-1-k} \gamma_{C_i}(k+1). \quad (3)$$

Let  $\text{Supp}(\mathbf{C}) = \{i : p_i(\mathbf{C}) > 0\}$ .

**Claim 2.1** (Equilibrium condition). *Suppose that designers choose contests  $\mathbf{C} = (C_1, \dots, C_m)$  in the first phase of the game and contestants participate in contests with probabilities  $(p_1, \dots, p_m) = \mathbf{p}(\mathbf{C})$  in equilibrium. Then,*

- If  $i \in \text{Supp}(\mathbf{C})$ ,  $\beta(C_i, p_i) \geq \beta(C_j, p_j) \forall j = 1, \dots, m$ .
- Thus, if  $i, j \in \text{Supp}(\mathbf{C})$ , then  $\beta(C_i, p_i) = \beta(C_j, p_j)$ .

## 2.3 Equilibrium among Contest Designers

We use  $\mathbf{C} = (C_i, \mathbf{C}_{-i}) = (C_1, \dots, C_m)$  to denote the contests (strategies) chosen by all designers, where  $\mathbf{C}_{-i}$  denotes the contests chosen by designers other than  $i$ . Let  $u_i(C_i, \mathbf{C}_{-i})$  be the expected utility of contest designer  $i$  given that contestants use  $\mathbf{p}(C_i, \mathbf{C}_{-i})$ . Formally, by (2) the utility of the designer of contest  $C_i$  equals  $R_i - k\gamma_{C_i}(k)$  when there are  $k \geq 1$  contestants. Since each contestant participates in  $C_i$  independently with probability  $p_i = p_i(C_i, \mathbf{C}_{-i})$ , the total number  $k$  of contestants in  $C_i$  follows the binomial distribution  $\text{Bin}(n, p_i)$ , and hence the designer's expected utility equals

$$u_i(C_i, \mathbf{C}_{-i}) = \mathbb{E}_{k \sim \text{Bin}(n, p_i)} [(R_i - k\gamma_{C_i}(k)) \cdot \mathbb{1}_{k \geq 1}]. \quad (4)$$

Since a contest is a constant-sum game where the overall utility of all players (i.e., the welfare) equals the total reward  $R_i$  whenever there is at least one contestant, designer  $i$ 's expected utility  $u_i(C_i, \mathbf{C}_{-i})$  is equal to the expected welfare  $R_i[1 - (1 - p_i)^n]$  minus the sum of contestants' expected utilities obtained from contest  $i$ ,  $np_i\beta(C_i, p_i)$ . Formally,

**Claim 2.2.**

$$u_i(C_i, \mathbf{C}_{-i}) = R_i [1 - (1 - p_i)^n] - np_i\beta(C_i, p_i). \quad (5)$$

We analyze the following solution concepts:

**Definition 2.6.** Given some  $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ ,

- A contest  $C_i \in \mathcal{S}_i$  is (weakly) dominant if  $\forall C'_1 \in \mathcal{S}_1, \dots, C'_m \in \mathcal{S}_m$ ,  $u_i(C_i, \mathbf{C}_{-i}) \geq u_i(C'_i, \mathbf{C}'_{-i})$ .
- A tuple of contests  $(C_1, \dots, C_m)$ , where  $C_i \in \mathcal{S}_i$  for all  $i$ , is a contestant-symmetric subgame-perfect equilibrium (contestant-symmetric SPE) if  $u_i(C_i, \mathbf{C}_{-i}) \geq u_i(C'_i, \mathbf{C}_{-i}), \forall C'_i \in \mathcal{S}_i, \forall i$ .

For simplicity and also for practical purposes, we do not consider the case where designers play mixed strategies.

<sup>2</sup>All our results hold for *all* symmetric equilibria. In addition, Lemma 2.8 shows that the symmetric equilibrium is unique if a certain condition (satisfied, e.g., by all Tullock contests) holds.

## 2.4 Additional Important Properties of Contests

Our results use the following three properties of contests:

**Definition 2.7.** A contest  $C_i \in \mathcal{C}_{R_i}$  has monotonically decreasing utility (MDU) if  $\gamma_{C_i}(1) \geq \gamma_{C_i}(2) \geq \dots \geq \gamma_{C_i}(n)$ . In words, a contestant's symmetric NE expected utility is decreasing as the number of contestants increases.

A contest  $C_i \in \mathcal{S}_i \subseteq \mathcal{C}_{R_i}$  has maximal rent dissipation (MRD) in  $\mathcal{S}_i$  if  $\forall C'_i \in \mathcal{S}_i$  and  $\forall k = 1, \dots, n$ ,  $\gamma_{C_i}(k) \leq \gamma_{C'_i}(k)$ . Let  $\text{MRD}(\mathcal{S}_i) \subseteq \mathcal{S}_i$  denote the set of all MRD contests in  $\mathcal{S}_i$ . In words, MRD contests maximize the designer's utility regardless of the number of contestants (equivalently, minimize contestants' symmetric NE expected utility).

A contest  $C_i \in \mathcal{C}_{R_i}$  has full rent dissipation if  $\gamma_{C_i}(1) = R_i$  and  $\gamma_{C_i}(k) = 0$  for  $k = 2, \dots, n$ .

Lemma 4.1 shows that all Tullock contests satisfy MDU; This is not unique to Tullock contests, e.g., Proposition 2 of (Nti 1997) shows a large class of contests that satisfy MDU.

A full-rent-dissipation contest  $C_i$  satisfies both MDU and MRD in any set  $\mathcal{S}_i$  that contains it. (Baye, Kovenock, and De Vries 1996) show that APA has full rent dissipation. In fact, as a corollary of (Ewerhart 2017) we observe in Section 4.1 that every Tullock contest with  $\tau \geq 2$  has full rent dissipation. MDU also implies:

**Lemma 2.8.** For MDU contests  $\mathcal{C} = (C_1, \dots, C_m)$ , the contestants' symmetric equilibrium  $\mathbf{p}(\mathcal{C})$  is unique.

## 3 Main Results: Equilibria in Contest Competition Games

Our first main result shows that choosing MRD and MDU contests form a subgame perfect equilibrium of the CCG game. Moreover, MRD contests are dominant if the set of all possible contests contains only MDU contests:

**Theorem 3.1.**

1. Fix any CCG( $m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m$ ) where each  $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$  contains a maximal rent dissipation contest that has monotonically decreasing utility, denoted by  $T_i \in \text{MRD}(\mathcal{S}_i)$ . Then,  $(T_1, \dots, T_m)$  is a contestant-symmetric SPE.
2. Moreover, if each  $\mathcal{S}_i$  contains only MDU contests,  $T_i$  is dominant for all  $i$ .
3. Corollary of 1: For any CCG( $m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m$ ) where each  $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$  contains a full rent dissipation contest (e.g., APA) denoted by  $F_i$ ,  $(F_1, \dots, F_m)$  is a contestant-symmetric SPE.

*Proof sketch.* Fix a contest designer,  $i$ . Suppose each of the  $n$  contestants participates in  $i$ 's contest with some probability  $p_i$  (assuming a symmetric participation equilibrium). By Claim 2.2,  $i$ 's expected utility is

$$u_i(C_i, \mathcal{C}_{-i}) = R_i [1 - (1 - p_i)^n] - np_i \beta(C_i, p_i), \quad (6)$$

where we recall that  $R_i [1 - (1 - p_i)^n]$  is the expected welfare generated in contest  $C_i$  and  $\beta(C_i, p_i)$  is each contestant's expected utility conditioning on participating in  $C_i$ . Now, suppose that contest designer  $i$  switches to a contest  $C'_i$  that requires less effort from the contestants (namely, leaving more utility to the contestants) and hence increases the

participation probability to  $p'_i = p_i + \Delta p$ . The welfare term  $R_i [1 - (1 - p_i)^n]$  is increased by

$$\Delta p \cdot \frac{\partial}{\partial p_i} R_i [1 - (1 - p_i)^n] = n \Delta p R_i (1 - p_i)^{n-1}. \quad (7)$$

A contestant's conditional utility in contest  $i$  is increased, because as contestants participate in *other* contests  $C_j$  ( $j \neq i$ ) with *less* probabilities, the utility a contestant obtains from  $C_j$  is *increased* because  $C_j$  has MDU by assumption; since contestants are indifferent between contests  $i$  and  $j$ , their utility obtained from contest  $i$  must be increased as well. Suppose it is increased to  $\beta(C'_i, p'_i) = \beta(C_i, p_i) + \Delta \beta$ . Then, the utility term  $np_i \beta(C_j, p_i)$  is increased by

$$\begin{aligned} & np'_i \beta(C'_i, p'_i) - np_i \beta(C_i, p_i) \\ &= n(p_i + \Delta p)(\beta(C_i, p_i) + \Delta \beta) - np_i \beta(C_i, p_i) \\ &= n \Delta p \beta(C_i, p_i) + np_i \Delta \beta + n \Delta p \Delta \beta \\ &> n \Delta p \beta(C_i, p_i) \geq n \Delta p R_i (1 - p_i)^{n-1}, \end{aligned}$$

where the last inequality is because a contestant obtains utility  $R_i$  when no other contestants participate in  $C_i$ , which happens with probability  $(1 - p_i)^{n-1}$ . This increase outweighs the increase of the welfare term (7), so the designer's utility is decreased.  $\square$

Theorem 3.1 gives a sufficient condition for the existence of a specific type of contestant-symmetric subgame-perfect equilibria, namely that the  $\mathcal{S}_i$ 's contain MDU and MRD contests. Monotonically Decreasing Utilities rule out some design instruments, for example, caps on efforts (one could imagine a design in which the cap is decreasing in the number of contestants). We emphasize that, in our model, the  $\mathcal{S}_i$ 's can contain other arbitrary types of contests, including non-MDU contests; the point is that if the  $\mathcal{S}_i$ 's contain MDU and MRD contests then these contests form contestant-symmetric subgame-perfect equilibria. Nevertheless, by introducing caps on efforts (thus violating MDU), the following example shows that other types of equilibria may also exist and that MDU and MRD contests are not dominant. See the full version for additional examples and a more detailed discussion.

**Example 3.2.** Let  $m = 2, n = 6, R_1 = R_2 = 1$ , both  $\mathcal{S}_1$  and  $\mathcal{S}_2$  consist of two contests: APA, and a contest  $C$  that gives the reward for free when the number of contestants is  $k = 5, 6$  and runs APA otherwise. Thus,  $\gamma_C = (1, 0, 0, 0, 1/5, 1/6)$ , which is not MDU. The full version shows that  $(C, C)$  is a contestant-symmetric SPE and that APA is not a best-response to  $C$  (and therefore not dominant). By Theorem 3.1,  $(\text{APA}, \text{APA})$  is still a contestant-symmetric SPE of this game. Furthermore, by Theorem 3.7, both designers strictly prefer  $(\text{APA}, \text{APA})$  over  $(C, C)$ .

When the sets  $\mathcal{S}_i$  contain only MDU contests, the contestant-symmetric subgame-perfect equilibria that Theorem 3.1 describes are the only possible equilibria:

**Theorem 3.3.** Fix any CCG( $m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m$ ) where each  $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$  only contains MDU contests. Assume  $\text{MRD}(\mathcal{S}_i) \neq \emptyset$  for each  $i$ . Pick  $T_i \in \text{MRD}(\mathcal{S}_i)$ , and let

## 4 Applications

### 4.1 Competition among Tullock Contests

We first apply our results to the special case where the sets  $\mathcal{S}_i$  are arbitrary subsets of Tullock contests, i.e., the parameter  $\tau$  becomes a strategic choice (as suggested in e.g. (Michaels 1988; Nitzan 1994; Wang 2010)). Some of the previous literature views  $\tau$  as an exogenous parameter representing how accurately the designer can observe the ranking of contestants' efforts and the resulting qualities of their submissions. Even so, it seems plausible that the designer chooses an "ignorance is bliss" approach where she lowers the  $\tau$  value (thus, observes efforts' ranking less accurately) to encourage participation. Such situations occurred in practice. For example, according to (Wang 2010), "the International Table Tennis Federation (ITTF) changed the points scoring system for international matches from first to 21 to first to 11 in 2000. One reason for doing this is to reduce the accuracy level of the matches".

It is known that APA has full rent dissipation (Baye, Kovenock, and De Vries 1996) and in fact, as a corollary of (Ewerhart 2017), every Tullock contest with parameter  $\tau \geq 2$  has full rent dissipation. Thus, the class of Tullock contests contains maximal rent dissipation contests (namely, those with  $\tau \geq 2$ ). Also, it is a class of contests that have monotonically decreasing utility:

**Lemma 4.1** (Corollary of (Baye, Kovenock, and De Vries 1996; Schweinzer and Segev 2012; Ewerhart 2017)). *Let  $C_\tau$  be a Tullock contest with reward  $R$  and with parameter  $\tau \in [0, +\infty]$ . Then,  $\gamma_{C_\tau}(k) = R(\frac{1}{k} - \frac{k-1}{k^2}\tau)$  if  $\frac{k}{k-1} > \tau$  and  $\gamma_{C_\tau}(k) = 0$  if  $\frac{k}{k-1} \leq \tau$ . For  $\tau = +\infty$ ,  $\gamma_{C_\tau}(1) = R$  and  $\gamma_{C_\tau}(k) = 0$  for  $k \geq 2$ . As corollaries,*

- Every Tullock contest satisfies MDU.
- Any Tullock contest with  $\tau \geq 2$  has full rent dissipation.
- If  $\mathcal{S}$  is the set of all Tullock contests with parameter  $\tau$  in some range whose maximum  $\tau^{\max}$  is well defined and at most 2, then the Tullock contest with  $\tau^{\max}$  is the only contest in  $\text{MRD}(\mathcal{S})$ .

Theorem 3.1 therefore immediately implies:

#### Corollary 4.2.

1. Let  $\mathcal{T}_{R_i}$  be the set of all Tullock contests with reward  $R_i$ . Then, APA and any other Tullock contest with  $\tau \geq 2$  is a dominant contest for every designer in  $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{T}_{R_i})_{i=1}^m)$ .
2. If  $\mathcal{S}_i$  is the set of all Tullock contests with parameter  $\tau_i$  in some range whose maximum  $\tau_i^{\max}$  is well defined and at most 2. Then, the Tullock contest with  $\tau_i^{\max}$  is the only dominant contest for every designer in  $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ .

### 4.2 Contests with Varying Prize Structures

(Clark and Riis 1998) consider a formal model of prize structures, as follows. Given a total prize  $R_i$ , each designer  $i$  divides it into multiple prizes  $R_i^1, \dots, R_i^s$  after seeing the number of contestants  $k$  participating in her contest, with

$\tilde{p}_i = p_i(T_1, \dots, T_m)$  be the probability a contestant participates in contest  $T_i$  in the equilibrium of contestants, and let  $P = \text{Supp}(\mathbf{T}) = \{i : \tilde{p}_i > 0\}$  be the set of indices of contests in which contestants participate with positive probability when the contests are  $(T_1, \dots, T_m)$ . Then

1. for any contestant-symmetric SPE  $(C_1, \dots, C_m)$ ,  $p_i(C_1, \dots, C_m) = \tilde{p}_i$ .
2. if  $|P| \geq 2$ , then  $(C_1, \dots, C_m) \in \mathcal{S}_1 \times \dots \times \mathcal{S}_m$  is a contestant-symmetric SPE if and only if  $C_i \in \text{MRD}(\mathcal{S}_i), \forall i \in P$ .<sup>3</sup>
3. if  $|P| = 1$ , let  $P = \{i_0\}$ , then  $(C_1, \dots, C_m) \in \mathcal{S}_1 \times \dots \times \mathcal{S}_m$  is a contestant-symmetric SPE if and only if  $\gamma_{C_{i_0}}(n) = \gamma_{T_{i_0}}(n)$ .<sup>4</sup>

In the symmetric-reward case we can show that  $|P| = m$  which makes the statement shorter:

**Corollary 3.4.** *When  $R_1 = \dots = R_m$  (symmetric rewards),  $(C_1, \dots, C_m) \in \mathcal{S}_1 \times \dots \times \mathcal{S}_m$  is a contestant-symmetric SPE if and only if  $C_i \in \text{MRD}(\mathcal{S}_i)$  for all  $i \in \{1, \dots, m\}$ .*

Thus, the case of symmetric rewards is a "clear cut" while the general case is more involved. The following example demonstrates the need for this distinction using a setting with highly asymmetric rewards.

**Example 3.5.** *Consider  $m \geq 3$  contests and  $n$  contestants. Contest 1 has reward  $R_1 = 1$ , and each of others has reward  $R_j = \left(\frac{m-1}{m-2}\right)^{n-1} + 1$ . Each set  $\mathcal{S}_i$  contains all MDU contests (hence contains APA). Then for any contest  $C_1 \in \mathcal{S}_1$ ,  $(C_1, T_2, \dots, T_m)$  where  $T_j = \text{APA} \in \text{MRD}(\mathcal{S}_j)$  for  $j = 2, \dots, m$  is a contestant-symmetric SPE. In this equilibrium,  $p_1(C_1, T_2, \dots, T_m) = 0$ , and  $p_j(C_1, T_2, \dots, T_m) = \frac{1}{m-1} > 0$  for any  $j = 2, \dots, m$ .*

Finally, the equilibria in Theorem 3.1 are Pareto optimal for the contest designers:

#### Definition 3.6.

- For two strategy profiles  $\hat{C} = (\hat{C}_1, \dots, \hat{C}_m)$ ,  $C = (C_1, \dots, C_m) \in \mathcal{S}_1 \times \dots \times \mathcal{S}_m$  of the contest competition game  $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ , we say  $C$  is a Pareto improvement of  $\hat{C}$ , if  $u_i(C) \geq u_i(\hat{C})$  for all  $i \in \{1, \dots, m\}$  and  $u_i(C) > u_i(\hat{C})$  for at least one  $i \in \{1, \dots, m\}$ .
- A strategy profile  $\hat{C} = (\hat{C}_1, \dots, \hat{C}_m)$  is Pareto Optimal (PO) if there is no Pareto improvement of it.

**Theorem 3.7.** *The equilibria in Theorem 3.1 are PO.*

<sup>3</sup>If  $p_i(C_1, \dots, C_m) = 0$ , contest  $i$  could be anything since  $i$ 's utility, which is 0, cannot be improved by choosing any other contest  $C'_i$  as  $(C_i, C_{-i})$  is an equilibrium. Moreover, we show in the full version that  $p_i(C'_i, C_{-i})$  must be 0 as well, so the choice of  $C'_i$  does not affect the choices of contests of other designers.

<sup>4</sup>As  $p_{i_0}(C_1, \dots, C_m) = 1$ , with probability 1 there are  $n$  contestants in contest  $i_0$ , thus the contest success functions of contest  $i_0$  for  $k \neq n$  have no effect on the utility calculation for the contestants' best response and could be anything.

$\sum_{j=1}^s R_i^j = R_i$  and  $s \leq k$ .<sup>5</sup> Let  $e_\ell$  denote the effort a contestant  $\ell$  exerts in  $i$ 's contest. Let  $\tau > 0$  be a fixed parameter. All contestants in  $i$ 's contest win  $R_i^1$  with probability proportional to  $(e_\ell)^\tau$ ; after the winner of  $R_i^1$  is determined, one of the remaining contestants is randomly chosen to win  $R_i^2$  with probability again proportional to  $(e_\ell)^\tau$ ; and so on.<sup>6</sup> As long as pure-strategy symmetric effort-exerting equilibria for the contestants exist, setting  $s = 1$  and  $R_i^1 = R_i$  gives more utility to the designer than any other partition of the total reward. In our terminology, the contest with  $s = 1$ ,  $R_i^1 = R_i$  is an MRD contest. (It is unknown whether this holds if pure-strategy symmetric effort-exerting equilibria do not exist.) Clark and Riis (1998) give a sufficient condition for the existence of pure-strategy symmetric effort-exerting equilibrium:  $\tau \leq k/(k-1)$  and  $s \leq 0.63k$ . For example,  $\mathcal{S}_i$  may be the class of all contests that assign a single winner prize if there are up to 4 contestants, and three top winner prizes if there are 5 or more contestants. So, we obtain the following corollary:

**Corollary 4.3.** *Fix any  $0 < \tau \leq n/(n-1)$ . Let  $\mathcal{S}_i$  consist of multi-prize contests  $C_i$  that satisfy: for each  $k \geq 2$ , the number of divided prizes is at most  $s \leq 0.63k$ . Then, each designer choosing the MRD contest where  $s = 1$  and  $R_i^1 = R_i$  is an equilibrium of the contest competition game.*

Regardless of the prize structure, a larger  $\tau$  always gives a larger utility to a designer in the single-contest setting (Clark and Riis 1998). This holds as long as a pure-strategy symmetric effort-exerting equilibrium exists. Therefore, if  $\tau$  can be varied by the designers, a more general corollary holds:

**Corollary 4.4.** *Let  $\mathcal{S}_i$  consist of multi-prize contests  $C_i$  with adjustable parameter  $\tau$  that satisfy: for each  $k \geq 2$ , the number of divided prizes is at most  $s \leq 0.63k$ , and  $0 < \tau \leq n/(n-1)$ . Then, each designer choosing the MRD contest where  $s = 1$ ,  $R_i^1 = R_i$ , and  $\tau$  is the largest possible parameter, is an equilibrium of the contest competition game.*

## 5 Welfare Optimality

Throughout this section, let  $\mathbf{C} = (C_1, \dots, C_m)$  be a tuple of contests. Denote the sum of designers' expected utilities, the sum of contestants' expected utilities, and their sum by

$$W_D(\mathbf{C}) = \sum_{i=1}^m u_i(\mathbf{C}), \quad W_C(\mathbf{C}) = n \sum_{i=1}^m p_i \beta(C_i, p_i),$$

$$W_S(\mathbf{C}) = W_D(\mathbf{C}) + W_C(\mathbf{C})$$

where  $p_i = p_i(\mathbf{C})$ . We show that the welfare in the equilibrium of Theorem 3.1 is optimal in several natural cases:

**Theorem 5.1.** *Consider a contest competition game  $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$  and fix some  $T_i \in \text{MRD}(\mathcal{S}_i)$*

<sup>5</sup>(Azmat and Möller 2009) consider a similar model but assume that designers choose prize structures before knowing the number of contestants. We allow to condition the prize structure on the realized number of contestants which is more general.

<sup>6</sup>This is a valid CSF in our model as  $f_\ell^k(e_1, \dots, e_k)$  specifies the expected fraction of the total prize contestant  $\ell$  wins. For the prize structures described here,  $f_\ell^k(e_1, \dots, e_k) = (\pi_1 R_i^1 + \dots + \pi_s R_i^s) / R_i$  where  $\pi_j$  is the prob. that  $\ell$  wins the  $j$ 'th prize.

that satisfies MDU. Then the equilibrium  $(T_1, \dots, T_m)$  maximizes  $W_S$  in each one of the following cases:

1. *Unrestricted contest design: for all  $i$ ,  $\mathcal{S}_i = \mathcal{C}_{R_i}$ .*
2. *APA is a possible contest:  $\forall i, \text{APA} \in \mathcal{S}_i$ . (APA can be replaced with any other full rent dissipation contest.)*
3. *A symmetric CCG:  $R_1 = \dots = R_m = R$  and  $\mathcal{S}_1 = \dots = \mathcal{S}_m = \mathcal{S} \subset \mathcal{C}_R$ .*
4. *An MRD-symmetric CCG:  $R_1 = \dots = R_m = R$  and  $\text{MRD}(\mathcal{S}_1) = \dots = \text{MRD}(\mathcal{S}_m)$ .*

(The second case generalizes the first, and the fourth case generalizes the third since every symmetric CCG is also MRD-symmetric.) The full version shows that these conclusions do not always hold outside of the four cases above.

Although the total welfare is not always maximized at the equilibrium of Theorem 3.1, the contestants' welfare is always minimized at these equilibria, as the next theorem shows.<sup>7</sup>

**Theorem 5.2.** *Fix some  $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ . Let  $\mathbf{T} = (T_1, \dots, T_m)$  be one of the equilibria in Theorem 3.1, i.e.,  $T_i \in \text{MRD}(\mathcal{S}_i)$  and  $T_i$  satisfying MDU. Then for any  $\mathbf{C} \in \mathcal{S}_1 \times \dots \times \mathcal{S}_m$ ,  $W_C(\mathbf{T}) \leq W_C(\mathbf{C})$ .*

Theorems 5.1 and 5.2 together immediately imply:

**Corollary 5.3.** *Consider a contest competition game  $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$  and fix some  $T_i \in \text{MRD}(\mathcal{S}_i)$  satisfying MDU. Then the equilibrium  $(T_1, \dots, T_m)$  maximizes  $W_D$  in the four cases of Theorem 5.1.*

Thus, for example, (APA, ..., APA) maximizes the designers' welfare and minimizes the contestants' welfare in the case of unrestricted contest design.

## 6 Discussion

The bottom line of our formal analysis is that under our assumptions a contest designer can ignore competition and focus solely on increasing the effort of the contestants that arrive to the contest. This conclusion advances the state-of-art as, a-priori, the contest competition game can yield outcomes that depend on the number of designers, contestants, and equilibrium selection. Game-theoretically, our conclusion is compelling as it is supported by Pareto-optimality and even by dominant strategies.

The full version further explores which assumptions are necessary and which can be relaxed for the above to still hold. This gives both a prescription for designers as well as a basis on which further research to study alternative assumptions can develop. E.g., the assumption of linear and symmetric cost of effort is discussed, where we conjecture (with supporting evidence) that our conclusions will continue to hold for risk-loving contestants, while the results may qualitatively change for risk-averse contestants.<sup>8</sup>

<sup>7</sup>Designers' PO does not immediately imply minimal contestants' welfare since (1) the game is not constant-sum as  $W_S(\mathbf{C})$  depends on  $p_i$ 's and (2) PO outcomes need not necessarily maximize the aggregate designers' utility.

<sup>8</sup>In our model, each contestant chooses, possibly in a random way, exactly one contest to participate in. When a contestant participates in only one contest, assuming linear cost of effort is the same as assuming risk-neutrality.

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