Combinatorial Civic Crowdfunding with Budgeted Agents: Welfare Optimality at Equilibrium and Optimal Deviation

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Abstract

Civic Crowdfunding (CC) uses the "power of the crowd" to garner contributions towards public projects. As these projects are non-excludable, agents may prefer to "free-ride," resulting in the project not being funded. Researchers introduce refunds for single project CC to incentivize agents to contribute, guaranteeing the project's funding. These funding guarantees are applicable only when agents have an unlimited budget. This paper focuses on a combinatorial setting, where multiple projects are available for CC and agents have a limited budget. We study specific conditions where funding can be guaranteed. Naturally, funding the optimal social welfare subset of projects is desirable when every available project cannot be funded due to budget restrictions. We prove the impossibility of achieving optimal welfare at equilibrium for any monotone refund scheme. Further, given the contributions of other agents, we prove that it is NP-Hard for an agent to determine its optimal strategy. That is, while profitable deviations may exist for agents instead of funding the optimal welfare subset, it is computationally hard for an agent to find its optimal deviation. Consequently, we study different heuristics agents can use to contribute to the projects in practice. We demonstrate the heuristics' performance as the average-case trade-off between the welfare obtained and an agent's utility through simulations.

Introduction

Local communities often find it beneficial to elicit contributions from their members for *public* good projects. E.g., the construction of markets, playgrounds, and libraries, among others (London 2021). These goods provide the local community with social amenities, generating social welfare. This process of generating funds from members towards community services is referred to as *Civic Crowdfunding* (CC). CC is instrumental in changing the interaction between local governments and communities. It empowers citizens by allowing participation in the design and planning of public good projects (Van Montfort, Siebers, and De Graaf 2021). Such democratization of public projects has led CC to become an active area of research (Diederich, Goeschl, and Waichman 2016; Goodspeed 2017; Chandra, Gujar, and Narahari 2017; Wang et al. 2021; Yan and Chen 2021; Cason, Tabarrok, and Zubrickas 2021). Moreover, introduction of web-based CC platforms (Kickstarter 2023; Spacehive 2023) has added to its popularity.

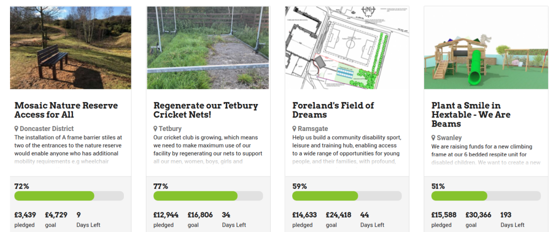

Figure 1: Example instance of Combinatorial Civic Crowdfunding (CC) (Spacehive 2023).

As depicted in Figure 1, typically, multiple projects are simultaneously available for CC. We refer to CC for multiple projects as *combinatorial* CC. Formally, CC comprises strategic agents who observe their valuations for the available public projects. Each project has a known target cost and deadline. Each agent contributes to the available projects as per its valuations and within its *budget*. The agent valuations are such that the overall sum is greater than the project's target cost, i.e., there is enough valuation (interest) for the project's funding. A project is *funded* when the agents' total contribution meets the target cost within the deadline. When funded, each agent obtains a quasi-linear utility equivalent to its valuation for the project minus its contribution. In turn, the community generates social welfare – the difference in the project's total valuation and cost.

Free-riding. The primary challenge in CC is due to the non-excludability of the public projects. That is, the citizens can avail a project's benefit without contributing to its funding. Consequently, strategic agents may free-ride and merely wait for others to fund the project. When the majority decides to free-ride, the project remains unfunded despite sufficient interest in its funding (Stroup 2000). To persuade strategic agents to contribute, researchers propose to provide additional incentives to them in the form of *refunds*.

Refunds. Zubrickas (2014) presents PPR, *Provision Point Mechanism with Refunds*, which employs the first such refund scheme. PPR assumes that a *central planner* keeps

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some refund budget aside. If the project is not funded, the planner returns the agent's contribution and an additional refund proportional to the agent's contribution. The refund scheme incentivizes the agents to increase their contribution to obtain a greater refund. The characteristic of the resulting game is that the public project is funded at equilibrium. However, PPR and subsequent works ((Damle et al. 2021) and references cited therein) assume that agents have an unlimited individual budget; in reality, the agents may have a limited budget. To this end, we aim to determine the funding guarantees for a subset of public projects that maximize social welfare within the available budget.

Our Approach and Contributions

This paper lays the theoretical foundation for combinatorial CC with budgeted agents. Table 1 presents the overview of our results, described in detail next.

Budget Surplus (BS). We first study the seemingly straightforward case of *Budget Surplus*, i.e., the overall budget across the agents is more than the projects' total cost. For this, it is welfare optimal to fund all the projects. Despite the surplus budget, we show that the projects' funding cannot be guaranteed at equilibrium (Theorem 2 and Corollary 1).

Subset Feasibility (SF). We observe that the budget distribution among the agents plays a significant role in deciding the funding status of the projects. Conditioning on the budget distribution, we introduce *Subset Feasibility* of a given subset of projects. We prove that Subset Feasibility coupled with Budget Surplus guarantees funding of every available project at equilibrium (Theorem 3), thereby generating the maximum possible social welfare.

Budget Deficit (BD). Trivially, in the case of *Budget Deficit* – when there is no Budget Surplus – one can only fund a subset of projects. It may be desirable that such a subset is welfare-maximizing within the budget. We refer to the funding of the socially welfare optimal subset at equilibrium as *socially efficient equilibrium*. For this case, we present the following results.

First, we show that, in general, achieving socially efficient equilibrium is impossible for any refund scheme (Example 1). Next, we prove that even with the stronger assumption of Subset Feasibility, it is still impossible to achieve socially efficient equilibrium (Theorem 4). Specifically, we prove that strategic deviations may exist for agents such that the optimal welfare subset remains unfunded. We then show that it is NP-Hard for an agent to find its optimal deviation, given the contributions of all the other agents (Theorem 6 and Corollary 2). Due to Theorem 4 and hardness of optimal deviation (Theorem 6), we construct five heuristics for the agent's contributions and empirically study their social welfare and agent utility through simulations (Figures 4, 5).

Related Work

Several works study the effect of agents' contribution to public projects (Wang et al. 2021; Chen, Tao, and Yu 2021; Soundy et al. 2021; Brandl et al. 2022). One way of modelling agent contribution is using *Cost Sharing Mechanisms* (CSMs) in CC (Moulin 1994). More concretely, CSMs focus on sharing the cost among the strategic agents to ensure that an efficient set of projects are funded (Moulin 1994; Moulin and Shenker 2001; Dobzinski et al. 2018; Dobzinski and Ovadia 2017; Birmpas, Markakis, and Schäfer 2019). The authors in (Wang et al. 2021; Ohseto 2000) model CSMs for non-excludable public projects and provide agent contributions that ensure specific desirable properties, e.g., individual rationality and strategy-proofness. However, these works do not guarantee funding at equilibrium since agents are strategic and CSMs do not offer refunds.

Property	**Socially Efficient Equilibrium**
Budget Surplus	✗ (Corollary 1)
Budget Surplus + Subset Feasibility	✓ (Theorem 3)
Budget Deficit + Subset Feasibility	✗ (Theorem 4)

Table 1: Overview of our theoretical results.

In another line of work, funding of public projects is modeled as *Participatory Budgeting* (Aziz and Shah 2021). Brandl et al. (2022) study a model without targets costs and without quasi-linear utilities, applicable for making donations to long-term projects. Generally, in the PB literature, the utility of an agent is determined by the number of projects funded or the costs of the projects (e.g., (Aziz and Ganguly 2021; Sreedurga, Bhardwaj, and Narahari 2022)), whereas in CC, it is the difference between the agent's valuation and contribution.

For excludable public projects, Soundy et al. (2021) focus on effort allocation by strategic agents towards the project's completion. Contrarily, we focus on funding guarantees of non-excludable public projects with strategic agents.

CC with Refunds. In the seminal work, Bagnoli and Lipman (1989) present Provision Point Mechanism (PPM) for single project CC, without refunds. Consequently, PPM consists of several inefficient equilibria (Bagnoli and Lipman 1989; Healy 2006). Agents may also free-ride since the projects are non-excludable (Stroup 2000). To overcome such limitations, Zubrickas (2014) presents PPR, a novel mechanism that offers refunds proportional to contributions. Based on the attractive properties of PPR, other works propose different refund schemes for different agent models and strategy space (Damle et al. 2021; Chandra, Gujar, and Narahari 2016; Damle et al. 2019b,a). These works only focus on a single project with agents having unlimited budgets.

Among recent works, Padala, Damle, and Gujar (2021) attempt to learn equilibrium contributions when agents have a limited budget in combinatorial CC using Reinforcement Learning. However, the work does not provide any funding guarantees – welfare or otherwise – for the projects.

Chen, Tao, and Yu (2021) analyze the existence of cooperative Nash Equilibrium for funding a single public project using 'external investments.' Their work considers agents to have binary contributions, unlike our setting, where agent contributions are in $\mathbb{R}_+$. Moreover, in their utility structure, agents receive a fixed fraction of the total contribution when the project is funded; otherwise, their contribution is returned. That is, they do not model agent valuations.

We remark that while our work is motivated by the existing CC literature, it remains fundamentally different. To the best of our knowledge, we are the first to study the funding guarantees for combinatorial CC with budgeted agents. We also focus on an agent's equilibrium behavior and study the hardness of the optimal strategy for the agents.

Preliminaries

This section presents our CC model and important definitions. We also summarize PPR for the single project case.

Combinatorial CC Model

Let $P = \{1, \dots, p\}$ be the set of projects to be crowdfunded with target costs $T = \{T_1, \dots, T_p\}$. Let $N = \{1, \dots, n\}$ denote the set of agents interested in contributing to all projects. We consider a limited budget for each agent $\gamma = (\gamma_1, \dots, \gamma_n)$. Each agent i has a private valuation for the project j, denoted by $\theta_{ij} \geq 0$. We consider *additive* valuations, i.e., an agent i has a value of $\sum_{j \in M} \theta_{ij}$ for a funded subset $M \subseteq P$. Let $\vartheta_j = \sum_{i \in N} \theta_{ij}$ denote the total valuation in the system for the project j. An agent i contributes $x_{ij} \in \mathbb{R}_+$ to project j, s.t., $\sum_{j \in P} x_{ij} \leq \gamma_i$. The total contribution towards a project j is denoted by $C_j = \sum_{i \in N} x_{ij}$. The project is funded if $C_j \geq T_j$ by the deadline, and each agent gets the funded utility of $\theta_{ij} - x_{ij}$. If the project is unfunded ($C_j < T_j$), the agents are returned their contributions x_{ij} and in some mechanisms, additional refunds, as defined later.

Welfare Optimal. Ideally, when there is limited budget, it may be desirable to fund welfare optimal subset defined as follows. Note that, the welfare obtained from project j if funded is $\vartheta_j - T_j$ and zero otherwise (Börgers and Krahmer 2015; Chakrabarty and Swamy 2014)[1].

Definition 1 (Welfare Optimal). *A set of projects $P^\star \subseteq P$ is welfare optimal if it maximizes social welfare under the available budget, i.e.,*

$$P^\star \in \underset{M \subseteq P}{arg\,max} \sum_{j \in M} (\vartheta_j - T_j) \;\; s.t. \; \sum_{j \in M} T_j \leq \sum_{i \in N} \gamma_i. \quad (1)$$

We make the following observations based on Definition 1.

- Finding $P^\star$ requires public knowledge of ϑs, Ts and the value $\sum_{i \in N} \gamma_i$. Contrary to the PB or CSM literature, the aggregate valuation ϑ is assumed to be public knowledge in the CC literature (Zubrickas 2014; Chandra, Gujar, and Narahari 2016). Similarly, we also assume that the overall budget in the system $\sum_{i \in N} \gamma_i$ is public knowledge. This may be done by deriving the overall budget by aggregating citizen interest (Alegre 2020; HudExchange 2023).
- Computing $P^\star$ is NP-Hard as it can be trivially reduced from the KNAPSACK problem. However, note that our primary results focus on $P^\star$'s funding guarantees at equilibrium (and not actually computing it). Moreover, computing $P^\star$ may also not be a deal breaker as the number of simultaneous projects available will not be arbitrarily large. One may also employ FPTAS (Lawler 1977).

[1]All the results presented in this paper also hold if $P^\star \in \underset{M \subseteq P}{\arg\max} \sum_{j \in M} \vartheta_j$ s.t. $\sum_{j \in M} T_j \leq \sum_{i \in N} \gamma_i$.

Refund Scheme. We define the refund scheme for each project $j \in P$ as $R_j(B_j, x_{ij}, C_j) : \mathbb{R}^3_+ \to \mathbb{R}_+$ s.t. $r_{ij} = R_j(B_j, x_{ij}, C_j)$ is agent i's refund share for contributing x_{ij} to project j. The overall budget for the refund bonus $B_j > 0$ is public knowledge. Typically, if a project is unfunded, the agents receive r_{ij}, and zero refund otherwise. The total refunds distributed for project j can be such that $\sum_{i \in N} r_{ij} = B_j$ (e.g., (Zubrickas 2014)) or $\sum_{i \in N} r_{ij} < B_j$ (e.g., (Chandra, Gujar, and Narahari 2016; Damle et al. 2021)). Throughout the paper, we assume that $\sum_{i \in N} r_{ij} = B_j \; \forall j$.

The CC literature also assumes that R is anonymous, i.e., refund share is independent of agent identity. Further, consider the following condition for a refund scheme, assuming R is differentiable w.r.t. x.

Condition 1 (Contribution Monotonicity (CM) (Damle et al. 2021)). *A refund scheme $R(x; \cdot)$ satisfies Contribution Monotonicity (CM) if it is strictly monotonically increasing with respect to the contribution x, i.e., $\frac{\partial R(x;\cdot)}{\partial x} > 0$.*

Agent Utilities and Important Definitions

Let $\mathcal{M}_{CC} = \langle P, N, \gamma, T, (\vartheta_j)_{j \in P}, (R_j)_{j \in P}, (B_j)_{j \in P} \rangle$ define a general combinatorial CC game. In this, the overall agent utility can be defined as.

Definition 2 (Agent Utility). *Given an instance of $\mathcal{M}_{CC}$, with agents having valuations $[\theta_{ij}]$ and contributions $[x_{ij}]$, the utility of an agent i for each project $j \in P$ is given by $\sigma_{ij}(\theta_{ij}, x_{ij}, r_{ij}, C_j, T_j) : \mathbb{R}^5_+ \to \mathbb{R}$*

$$\sigma_{ij}(\cdot) = \mathbf{1}_{C_j \geq T_j} \cdot \underbrace{(\theta_{ij} - x_{ij})}_{\text{Funded utility } \sigma^F_{ij}} + \mathbf{1}_{C_j < T_j} \cdot \underbrace{r_{ij}}_{\text{Unfunded utility } \sigma^U_{ij}}$$

where $\mathbf{1}_X$ is an indicator variable, such that $\mathbf{1}_X = 1$ if X is true and zero otherwise.

An agent i's utility for project j is either $\sigma^F_{ij} = \theta_{ij} - x_{ij}$ when j is funded, and $\sigma^U_{ij} = r_{ij}$ otherwise. Let $U_i(\cdot)$ denote the total utility an agent i derives, i.e., $U_i(\cdot) = \sum_{j \in P} \sigma_{ij}$. This incentive structure induces a game among the agents. As the agents are strategic, each agent aims to provide contributions that maximizes its utility. As such, we focus on contributions which follow pure strategy Nash equilibrium.

Definition 3 (Pure Strategy Nash Equilibrium (PSNE)). *A contribution profile $(x^*_{i1}, \dots, x^*_{ip})_{i \in N}$ is said to be Pure Strategy Nash equilibrium (PSNE) if, $\forall i \in N$,*

$$\sum_{j \in P} \sigma_{ij}(x^*_{ij}, x^*_{-ij}; \cdot) \geq \sum_{j \in P} \sigma_{ij}(x_{ij}, x^*_{-ij}; \cdot), \;\; \forall x_{ij}.$$

where x^*_{-ij} is the contribution of all agents except agent i.

Efficacy of PSNE Contributions. PSNE is the standard choice of solution concept in CC literature (Zubrickas 2014; Damle et al. 2021; Chandra, Gujar, and Narahari 2016; Damle et al. 2019b). Zubrickas (2014) shows that for an appropriate refund bonus (see Eq. 3), their PSNE strategies are the unique equilibrium of the mechanism. Moreover, Cason and Zubrickas (2017) empirically validate the effectiveness of these PSNE strategies using real-world experiments.

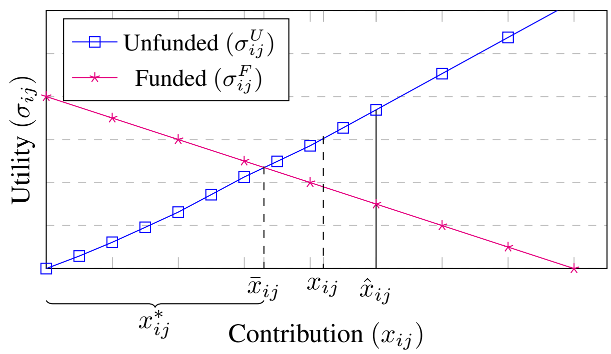

Figure 2: Utility vs. Contribution for agent i for project j.

Given the contributions $[x^*_{ij}]$, we can compute the set of the projects that are funded and unfunded at equilibrium. We refer to the funding of $P^\star$ at equilibrium as *socially efficient equilibrium*. We next define budget surplus.

Definition 4 (Budget Surplus (BS))**.** *Enough overall budget to fund each project $j \in P$, i.e., $\sum_{i\in N} \gamma_i \geq \sum_{j\in P} T_j$.*

We refer to the scenario $\sum_{i\in N} \gamma_i < \sum_{j\in P} T_j$ as Budget Deficit (BD). In CC literature, it is also natural to assume that $\vartheta_j > T_j, \forall j \in P$ (Zubrickas 2014). That is, there is sufficient interest in each available project's funding. Hence, when there is surplus budget, it is optimal to fund all the projects, i.e., $P^\star = P$.

Further, we assume that agents do not have any additional information about the funding of the public projects. This assumption implies that their belief towards the projects' funding is symmetric. This is a standard assumption in CC literature (Zubrickas 2014; Damle et al. 2021).

Single Project Civic Crowdfunding

Provision Point Mechanism with Refunds (PPR). For single project CC, i.e., $P = \{1\}$, Zubrickas (2014) proposes PPR which employs the following refund scheme $\forall i \in N$,

$$r^{PPR}_{i1} = R^{PPR}(x_{i1}, B_1, C_1) = \left(\frac{x_{i1}}{C_1}\right) B_1. \tag{2}$$

Each agent i's equilibrium contributions are defined such that its funded utility is greater than or equal to its unfunded utility, i.e., $\theta_{i1} - x^*_{i1} \geq r^{PPR}_{i1}$. We depict such a situation with Figure 2. The author shows that the project is *always* funded at equilibrium when $\vartheta_1 > T_1$, and that it is PSNE for each agent i to contribute x^*_{i1} or the amount left to fund the project, whichever is minimum. More formally,

Theorem 1 ((Zubrickas 2014))**.** *In PPR, with $\vartheta_1 > T_1$ and $B_1 > 0$, the set of PSNEs are $\{x^*_{i1} \mid x^*_{i1} \leq \bar{x}_{i1},\ \forall i; C_1 = T_1\}$ if $B_1 \leq \vartheta_1 - T_1$, where $\bar{x}_{i1} = \frac{T_1}{B_1+T_1}\theta_{i1}$. Otherwise, the set is empty.*

We have $\bar{x}_{i1} = \frac{T_1}{B_1+T_1}\theta_{i1}$ as the upper-bound of the equilibrium contribution, $\forall i \in N$. In PPR, the PSNE strategies in Theorem 1 are the unique equilibrium of the game when,

$$B_1 = \vartheta_1 - T_1 \implies \sum_{i\in N} \bar{x}_{i1} = T_1. \tag{3}$$

Funding Guarantees. For single project CC, Damle et al. (2021) show that the project is funded at equilibrium for any refund scheme that satisfies Condition 1. Trivially, one may observe that the refund scheme in PPR, i.e., $r^{PPR}_{i1} = \left(\frac{x_{i1}}{C_1}\right) B_1, \forall i$, also satisfies Condition 1. Damle et al. (2021) propose other refund schemes which satisfy Condition 1 and are exponential or polynomial in x. We remark that our results hold for any refund scheme satisfying Condition 1.

Funding Guarantees for Combinatorial CC under Budget Surplus

For CC under Budget Surplus (Def. 4) sufficient overall budget exists to fund all the projects. Theorem 2 shows that despite the sufficient budget, projects may not get funded as the set of equilibrium contributions for an agent may not exist. Unlike Theorem 1, agents may not have well-defined contributions satisfying PSNE. The non-existence results due to the uneven distribution of budget among the agents. Hence, agents with higher budgets exploit the mechanism to obtain higher refunds while ensuring the projects remain unfunded.

Theorem 2. *Given $(R_j)_{j\in P}$ which satisfy Condition 1, there are Budget Surplus (Def. 4) game instances of $\mathcal{M}_{CC}$ with $B_j = \vartheta_j - T_j, \forall j$ such that there is no equilibrium. That is, the set of equilibrium contributions may be empty.*

Proof. Consider P projects and N agents s.t. Def. 4 is satisfied, i.e., $\sum_{i\in N} \gamma_i \geq \sum_{j\in P} T_j$. We can easily construct game instances where there exists non-empty $N_1 \subset N$ s.t. $\sum_{i\in N_1} \gamma_i < \min_j T_j$. To satisfy Budget Surplus (Def. 4), $N_2 = N \setminus N_1$ must have enough budget so that the agents in $N_1 + N_2$ can fund all the projects.

Each agent i receives a funded utility $\sigma^F_{ij} = \theta_{ij} - x_{ij}$ for contributing x_{ij} towards project j. That is, as $x_{ij} \uparrow \implies \sigma^F_{ij} \downarrow$. The agent may also receive an unfunded utility of $\sigma^U_{ij} = r_{ij} = R_j(x_{ij}, B_j, \cdot)$ for project j. Since R_j is monotonically increasing (Condition 1), $x_{ij} \uparrow \implies \sigma^U_{ij} \uparrow$. We depict this scenario with Figure 2. Observe that σ^U_{ij} and σ^F_{ij} intersect at the upper-bound of the equilibrium contribution, $\bar{x}_{ij}$ (Theorem 1), where $\sigma^F_{ij} = \sigma^U_{ij}$. For any $x_{ij} > \bar{x}_{ij}$, $\sigma^U_{ij} > \sigma^F_{ij}$. The rest of the proof (see (Damle, Padala, and Gujar 2022)) shows that $\exists i \in N_2$ s.t. $\hat{x}_{ij} > \bar{x}_{ij}$ which in turn is not possible at equilibrium due to discontinuous utility structure at $\bar{x}_{ij}$. $\square$

Observe that if any project j is funded at equilibrium then the equilibrium set $(x^*_{i1}, \ldots, x^*_{ip})_{i\in N}$ can not be empty, contradicting Theorem 2. Corollary 1 captures this observation.

Corollary 1. *Given $(R_j)_{j\in P}$ satisfying CM (Condition 1), there are game instances of $\mathcal{M}_{CC}$ s.t. even with Budget Surplus (Def. 4), no project in P may be funded at equilibrium.*

With Corollary 1, we prove that Budget Surplus is not sufficient to fund every project at equilibrium. To this end, we next identify the sufficient condition to ensure the funding of every project, under Budget Surplus.

Subset Feasibility

With N_1 and N_2 in Theorem 2's proof, we assume a specific distribution on agents' budget. To resolve this, we introduce *Subset Feasibility* which assumes a restriction on each agent's budget distribution. Informally, if each agent i has enough budget to contribute $\bar{x}_{ij}$ (see Figure 2) for $j \in M$, $M \subseteq P$, then Subset Feasibility is satisfied for M. Formally,

Definition 5 (Subset Feasibility for M (SF$_M$))**.** *Given an instance of $\mathcal{M}_{CC}$ with $(R_j)_{j\in P}$ satisfying Condition 1, SF$_M$, $M \subseteq P$, is satisfied if, $\forall i \in N$ we have $\gamma_i \geq \sum_{j\in M} \bar{x}_{ij}$. Here, $\theta_{ij} - \bar{x}_{ij} = R_j(\bar{x}_{ij}, B_j, \cdot)$ (refer Figure 2).*

Claim 1. *Given any $(R_j)_{j\in P}$ whose equilibrium contributions satisfy $\sum_i \bar{x}_{ij} \geq T_j$, we have SF$_P \implies$ BS.*

Claim 1 follows from trivial manipulation. Similarly, we have BS $\not\Longrightarrow$ SF$_P$. From Claim 1, it is welfare optimal to fund every available project under SF_P. Theorem 3 indeed proves that under SF$_P$ each project $j \in P$ gets funded at equilibrium, thereby generating optimal social welfare.

Theorem 3. *Given $\mathcal{M}_{CC}$ and $(R_j)_{j\in P}$ satisfying CM (Condition 1) such that SF_P is satisfied, at equilibrium all the projects are funded, i.e., $C_j = T_j$, $\forall j \in P$ if $B_j \leq \vartheta_j - T_j$, $\forall j \in P$. Further, the set of PSNEs are:* $\left\{(x^*_{ij})_{j\in P} \mid \sigma^F_{ij}(x^*_{ij};\cdot) \geq \sigma^U_{ij}(x^*_{ij};\cdot), \forall j \in P,\ \forall i \in N\right\}$.

Theorem 3 implies that, under SF$_P$, the socially efficient equilibrium (with $P^\star = P$) is achieved. Intuitively, under SF$_P$ combinatorial CC collapses to simultaneous single projects; and thus, we can provide closed-form equilibrium contributions. However, SF$_P$ is a strong assumption and, in general, may not be satisfied. In fact, the weaker notion of Budget Surplus itself may not always apply. Therefore, we next study combinatorial CC with Budget Deficit.

Impossibility of Achieving Socially Efficient Equilibrium for Combinatorial CC under Budget Deficit

We now focus on the scenario when there is *Budget Deficit*, i.e., $\sum_{i\in N} \gamma_i < \sum_{j\in P} T_j$. In this scenario, only a subset of projects can be funded. Unfortunately, identifying the subset of projects funded at equilibrium is challenging. In CC, the agents decide which projects to contribute to based on their private valuations and available refund. This circular dependence of the equilibrium contributions and the set of funded projects make providing analytical guarantees challenging on the funded set. To analyze agents' equilibrium behavior and funding guarantees, we fix our focus on the subset of projects that maximize social welfare, i.e., $P^\star$ (Def. 1).

In this section, we first show that funding $P^\star \subset P$ at equilibrium is, in general, not possible for any $R(\cdot)$ satisfying Condition 1. Second, we prove that even with the stronger assumption of Subset Feasibility of the optimal welfare set, i.e., SF$_{P^\star}$, we may not achieve socially efficient equilibrium due to agents' strategic deviations. Last, we show that computing an agent i''s optimal deviation, given the contributions of the other agents $N \setminus \{i'\}$, is NP-Hard.

Procedure 1 Instance with $P = N = \{1, 2\}$ and fixed $R(\cdot)$

procedure GenerateValues($R(\cdot)$)
  $T_1 \leftarrow \mathbb{R}_+$
  Choose θ_{11} s.t. $\bar{x}_{11} < T_1 < \theta_{11}$ based on $R_1(\cdot)$
  Choose θ_{21} s.t. $\bar{x}_{21} := T_1 - \bar{x}_{11}$
  Set $T_2 = \bar{x}_{21}$, $\theta_{12} = 0$ and choose θ_{22} s.t.
    $\theta_{21} < \theta_{22} < \theta_{11} + \theta_{21} - x^*_{11}$ ▷ $P^\star = \{1\}$
  Set $\gamma_1 := \bar{x}_{11}$ and $\gamma_2 := \bar{x}_{21}$ ▷ *Satisfying SF$_{P^\star}$*
  return θ's, γ's, and T's ▷ *s.t. Agent 2 deviates*
end procedure

Welfare Optimality at Equilibrium

Consider the following example instance.

Example 1. *Let $P = \{1, 2\}$ and $N = \{1, 2\}$. Let $\gamma_1 = 1, \gamma_2 = 0, \theta_{11} = 1, \theta_{12} = 2$ and $\theta_{22} = 1$ with $\theta_{21} = 10$.*

In Example 1, the maximum funded utility agent 1 can receive from project 1 is 0 and unfunded utility $r_{11} < \theta_{11} = 1$. On the other hand, the agent obtains a utility of 1 when contributing to project 2. Hence at equilibrium, project 2 gets funded, although it is welfare optimal to fund project 1. Thus, socially efficient equilibrium is not achieved.

Example 1 is one pathological case where the agent with high valuation has zero budget, leading to sub-optimal outcome at equilibrium. Hence, we next strengthen the assumption on the budgets of the agents. Let $P^\star$ be the non-trivial welfare optimal subset and we assume that Subset Feasibility is satisfied for $P^\star$, i.e., SF$_{P^\star}$. Recall that with SF$_{P^\star}$, we assume that every agent has enough budget to contribute $\bar{x}_{ij}$ in $P^\star$. Theorem 4 shows that despite this strong assumption, achieving socially efficient equilibrium may not be possible.

Theorem 4. *Given an instance of $\mathcal{M}_{CC}$, a unique non-trivial $P^\star \subset P$ may not be funded at equilibrium even with Subset Feasibility for $P^\star$, SF$_{P^\star}$, for any set of $(R_j)_{j\in P}$ satisfying Condition 1.*

Proof. For $n = 2$ and $p = 2$, we construct an instance s.t. an agent has an incentive to deviate when $P^\star$ is funded. Procedure 1 presents the steps to construct the instance. We first select the target cost of the project 1, i.e. $T_1 \in \mathbb{R}_+$. Given $(R_j)_{j\in P}$ under Condition 1, we can always find an $\bar{x}_{11}$ for agent 1. At $\bar{x}_{11}$, funded utility is equal to unfunded utility (Figure 2). Trivially, $\bar{x}_{11} < \theta_{11}$. In (Damle, Padala, and Gujar 2022), we prove that the construction in Procedure 1 is always possible for any $R(\cdot)$ satisfying Condition 1. □

CCC with Budget Deficit: Optimal Strategy

Theorem 4 implies that P^* may not be funded at equilibrium even when P^* satisfies Subset Feasibility. In other words, w.l.o.g., an agent i' may have an incentive to deviate from any strategy that funds P^*. Motivated by such a deviation, we now address the question: *Given the total contribution by $N \setminus \{i'\}$ agents towards each project j, can the agent i' compute its optimal strategy?* We answer this question by (i) showing that such an optimal strategy may not exist if an agent's contribution space is continuous, i.e., $x \in \mathbb{R}_+$, and (ii) if contributions are discretized, then computing the optimal strategy is NP-Hard.

$$\max_{(x_{i'j})_{j\in P}} \sum_{j\in P} z_{i'j}\cdot(\theta_{i'j}-x_{i'j}) + (1-z_{i'j})\cdot R(x_{i'j},\cdot)$$

$$\text{s.t.} \sum_{j\in P} x_{i'j} \leq \gamma_{i'} \quad \textit{// Budget Constraint}$$

$$x_{i'j} \leq T_j - C_{j-i'}, \forall j \quad \textit{// Remaining Contribution}$$

$$\left.\begin{array}{l}(x_{i'j}-T_j+C_{j-i'})\cdot z_{i'j} \geq 0, \forall j\\ x_{i'j}-T_j+C_{j-i'} < z_{i'j}, \forall j\\ z_{i'j}\in\{0,1\}, \forall j\end{array}\right\} \textit{// Defining } \mathbf{1}_X$$

Figure 3: MIP-CC: Mixed Integer Program to calculate Agent i''s optimal strategy given the contributions of the remaining agents $N \setminus \{i'\}$.

MIP-CC: Mixed Integer Program for CC. We first describe the general optimization for an agent i' to compute its optimal strategy (i.e., contribution). For each $j \in P$, denote the aggregated contribution by agents in $N \setminus \{i'\}$ as $C_{j-i'}$. Now, for agent i''s optimal strategy, we need to maximize its utility given $T_j - C_{j-i'}$, $\forall j \in P$ and other variables such as the refund scheme R_j and bonus budget B_j. Figure 3 presents the formal MIP, namely *MIP-CC*, which follows directly from agent i' utility (Def. 2).

MIP-CC: Optimal Strategy May Not Exist. We now show that MIP-CC (Figure 3) may not always admit well-defined contributions.

Example 2. *Let $P = \{1,2,3\}$ and $N = \{1,2\}$ s.t. both agents are* identical*, i.e., each $i \in N$ has the same value θ for each $j \in P$ and $\gamma_1 = \gamma_2$. Additionally $\forall j \in P$, $T_j = T$ and $B_j = \vartheta - T$,. Let the agents have budget s.t. $\gamma_i = \bar{x}_{i1}$ $\forall i \in N$, where $\bar{x}_{i1}$ is the upper bound equilibrium contribution for the single project case (see Figure 2).*

Theorem 5. *Given an instance of $\mathcal{M}_{CC}$ and for any set of $(R_j)_{j\in P}$ satisfying Condition 1, an agent i''s optimal strategy may not exist.*

Proof. In (Damle, Padala, and Gujar 2022), we show that for Example 2, given agent 1's contribution we can create an instance of $\mathcal{M}_{CC}$ s.t. zs in MIP-CC can be either $z_1 = \{1,0,0\}$ or $z_2 = \{0,0,0\}$. Then, agent 2's utility is $\theta - \gamma_2$ for z_1 with strategy $(\gamma_2, 0, 0)$ and $2B + R(\gamma_2 - \epsilon)$ for z_2 with strategy $(\gamma_2 - \epsilon, \epsilon/2, \epsilon/2)$, $\epsilon \leq 0$. As $\epsilon \downarrow$, agent 2's utility for z_2 increases. But for $\epsilon = 0$, only z_1 is possible and agent 2 receives $\theta - \gamma_2$ ($<$ utility for z_2). Due to this *discontinuity*, an optimal ϵ (i.e., optimal strategy) does not exist. □

MIP-CC-D. To overcome the above non-existence, we discretize the contribution space. An agent i can contribute $\kappa \cdot \delta$ where $\kappa \in \mathbb{N}^+$ and δ the smallest unit of contribution. With this restriction on an agent's contribution, the search space in MIP-CC (Figure 3) becomes finite. Consequently, agent i''s optimal strategy always exist. To distinguish MIP-CC with a discrete contribution space, we refer to it as *MIP-CC-D*.

MIP-CC-D: Finding Optimal Strategy is NP-Hard. We now show that solving MIP-CC for discrete contributions (i.e., MIP-CC-D) is NP-hard.

Theorem 6. *Given an instance of $\mathcal{M}_{CC}$ with discrete contributions and for any set of $(R_j)_{j\in P}$ satisfying Condition 1, computing optimal strategy for agent i', given the contributions of $N \setminus \{i'\}$, is NP-Hard.*

Proof. We divide the proof into two parts (see (Damle, Padala, and Gujar 2022)). In Part A, we design a MIP tuned for a specific case of combinatorial CC comprising identical projects with a refund scheme satisfying $\sum_j R(x_j,\cdot) = R(\sum_j x_j,\cdot)$. We prove that the MIP is NP-Hard by reducing it from KNAPSACK. In Part B, we show that this MIP reduces to MIP-CC-D. That is, any solution to MIP-CC-D can be used to determine a solution to MIP in polynomial time, implying that MIP-CC-D is also NP-Hard. □

Corollary 2. *Given an instance of $\mathcal{M}_{CC}$ with discrete contributions and for any set of $(R_j)_{j\in P}$ satisfying Condition 1, if all agents except i', $N \setminus \{i'\}$, follow a specific strategy that funds $P^\star \subset P$, then computing the optimal deviation for agent i' is NP-Hard.*

Experiments

Motivation. Theorem 4 proves that the optimal subset $P^\star$ may not be funded at equilibrium due to agents' strategic deviations. However, computing an agent's optimal deviation is also NP-Hard (Corollary 2). These observations highlight that computing closed-form equilibrium strategies in Budget Deficit Combinatorial CC, similar to Theorem 1 and Theorem 3, for agents is challenging. Given this challenge and the hardness of strategic deviations, agents may employ heuristics to increase utility (Zou, Gujar, and Parkes 2010; Lubin and Parkes 2012). We next propose five heuristics for agents to employ in practice and study their impact on agent utilities and the welfare generated.

Heuristics and Performance Measures

Heuristics. Given the conflict between agent utilities and $P^\star$'s funding (Theorem 4), we propose the following heuristics for agent $i \in N$, for each project $j \in P$, to employ in practice and observe their utility vs. welfare trade-off.

1. *Symmetric*: $x_{ij} = \min(\theta_{ij}, \gamma_i/m)$
2. *Weighted*: $x_{ij} = \left(\frac{\theta_{ij}}{\sum_{k\in P}\theta_{ik}}\right)\gamma_i$
3. *Greedy-θ*: Greedily contribute $x_{ij} = \bar{x}_{ij}$ in descending order of the projects sorted by $\theta_{ij}, \forall j$
4. *Greedy-ϑ*: Greedily contribute $x_{ij} = \bar{x}^*_{ij}$ in descending order of the projects sorted by $\frac{\vartheta_j}{T_j}, \forall j$
5. *OptWelfare*: $x_{ij} = \bar{x}_{ij}$, $\forall j \in P^\star$ and evenly distribute the remaining budget across $P \setminus P^\star$

Agents contribute the minimum amount of what is specified by the five heuristics and the amount left to fund the project. We consider OptWelfare as the *baseline* (preferred) heuristic since it generates optimal welfare, i.e., funds $P^\star$.

Performance Measures. To study the welfare vs. agent utility trade-off, we consider the following performance measures: (i) *Normalized Social Welfare* (SW_N) – Ratio of the welfare obtained and the welfare from $P^\star$ and (ii) *Normalized Agent Utility* (AU_N) – Ratio of the agent utility obtained

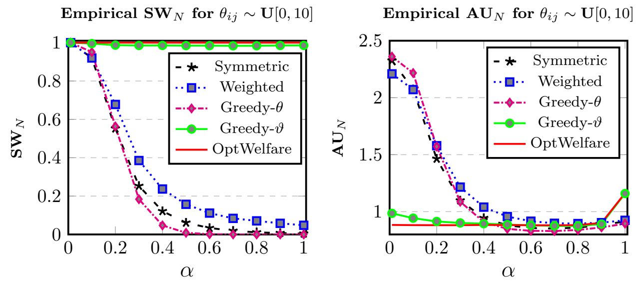

Figure 4: Empirical SW$_N$ and AU$_N$ for $\theta_{ij} \sim \mathbf{U}[0, 10]$

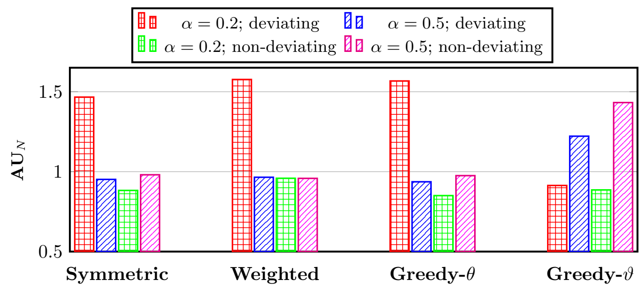

Figure 5: AU$_N$ for Agents Deviating (α) vs. Not $(1 - \alpha)$

w.r.t. to the utility when each agent has enough budget to play its PPR contribution $\forall\, j \in P$ (see Theorem 1).

We compare the heuristics when $\alpha \in (0, 1]$ fraction of the total agents *deviate*, i.e., choose heuristic $\in$ {Symmetric, Weighted, Greedy-θ, Greedy-ϑ}. The remaining $1 - \alpha$ fraction of agents use the baseline OptWelfare.

Simulation Setup and Results

Setup. We simulate the combinatorial CC game with $n = 100$, and $p = 10$ and PPR refund scheme (Eq. 2).[2] We sample θ_{ij}s, for each $i \in N$ and $j \in P$, using (i) Uniform Distribution, i.e., $\theta_{ij} \sim \mathbf{U}[0, 10]$, and (ii) Exponential Distribution, i.e., $\theta_{ij} \sim \text{Exp}(\lambda = 1.5)$. Here, λ is the rate parameter. When $\theta \sim \mathbf{U}[0, 10]$, we get agents whose per-project valuations differ significantly. For $\theta \sim \text{Exp}(\lambda = 1.5)$, the agents have approximately similar per-project valuations.

We ensure $\vartheta_j > T_j$ and $B_j \in (0, \vartheta_j - T_j]$ for each $j \in P$ and that the properties Budget Deficit and Subset Feasibility for $P^\star$ are satisfied. We run each simulation across 100k instances and observe the average SW$_N$ and AU$_N$ for each of the five heuristics. We depict our observations with Figures 4 and 5 when $\theta_{ij} \sim \mathbf{U}[0, 10]$. Results for $\theta_{ij} \sim \text{Exp}(\lambda = 1.5)$ are available in (Damle, Padala, and Gujar 2022).

Average SW$_N$ and AU$_N$. Figure 4 depicts the results when $\theta_{ij} \sim \mathbf{U}[0, 10]$. We make three main observations. First, deviating from the baseline heuristic (OptWelfare) is helpful only when few agents deviate, i.e., for smaller values of α. Despite such a deviation, we observe that the corresponding decrease in social welfare is marginal. On the other hand, the increase in α reduces the amount of the contributions, and the projects remain unfunded, reducing the social welfare and the agent utilities. Second, deviating from OptWelfare always increases the average agent utility – at the cost to the overall welfare. Third, Greedy-ϑ almost mimics OptWelfare, for both SW$_N$ and AU$_N$.

AU$_N$ for Deviating vs. Non-deviating Agents. In Figure 5, we compare the average utility for the agents who deviate versus those who do not. We let $\alpha = 0.2$ fraction of the agents deviate and follow the other four heuristics. From Figure 5, we observe that upon deviating to Symmetric, Weighted or Greedy-θ, the $\alpha = 0.2$ fraction of agents obtain higher AU$_N$ (red grid bars) compared to the remaining non-deviating agents who do not deviate (green grid bars). In contrast, Greedy-ϑ shows non-deviation to be beneficial. Since Greedy-ϑ performs close to OptWelfare, the AU$_N$ for deviation remains low compared to OptWelfare. Crucially, the deviation is not majorly helpful when many agents deviate. When $\alpha = 0.5$, we see comparable average AU$_N$ for agents who deviate and those who do not (blue lined vs. magenta lined bars, respectively). While deviating to Greedy-ϑ remains non-beneficial.

Discussion and Future Work. From Figures 4 and 5, we see that Greedy-ϑ performs similar to OptWelfare (which funds $P^\star$). Thus, as the number of projects p increases, to maximize social welfare, it may be beneficial for the agents to adopt Greedy-ϑ instead of deriving sophisticated strategies based on $P^\star$ (since computing $P^\star$ is NP-Hard).

Generally, it is challenging to determine PSNE contributions for combinatorial CC with budgeted agents. We propose four heuristics and study their welfare and agent utility trade-off. Future work can explore other heuristics that achieve better trade-offs and welfare guarantees. One can also study strategies that perform better on average such as Bayesian Nash Equilibrium. From the experiments, we observe that deviating from OptWelfare may increase agent utility. Thus, one can explore strategies such as ϵ-Nash Equilibrium, which approximates a worst-case ϵ increase in utility with unilateral deviation. Approximate strategies may also be desirable since finding optimal deviation is NP-Hard. However, the approximation must provide a desirable trade-off between agent utility and welfare.

Conclusion

This paper focuses on the funding guarantees of the projects in combinatorial CC. Based on the overall budget, we categorize combinatorial CC into (i) Budget Surplus and (ii) Budget Deficit. First, we prove that Budget Surplus is insufficient to guarantee projects' funding at equilibrium. Introducing the stronger criteria of Subset Feasibility guarantees the projects' funding at equilibrium under Budget Surplus. However, for Budget Deficit, we prove that the optimal welfare subset's funding can not be guaranteed at equilibrium despite Subset Feasibility. Next, we show that computing an agent's optimal strategy (and consequently, its optimal deviation), given the contributions of the other agents, is NP-Hard. Lastly, we propose specific heuristics and observe the empirical trade-off between agent utility and social welfare.

[2]The experimental trends presented remain same for different (n, p) pairs, such as $(50, 10)$, $(500, 10)$, and $(500, 20)$. The code is available at: github.com/magnetar-iiith/CCC.

References

Alegre, P. 2020. Case study 2-Porto Alegre, Brazil: participatory approaches in budgeting and public expenditure management. http://web.worldbank.org/archive/website00522/WEB/PDF/SDN71.PDF. Accessed: 2023-03-21.

Aziz, H.; and Ganguly, A. 2021. Participatory Funding Coordination: Model, Axioms and Rules. In *Algorithmic Decision Theory (ADT)*, 409–423.

Aziz, H.; and Shah, N. 2021. Participatory budgeting: Models and approaches. In *Pathways Between Social Science and Computational Social Science*, 215–236. Springer, Cham.

Bagnoli, M.; and Lipman, B. L. 1989. Provision of public goods: Fully implementing the core through private contributions. *The Review of Economic Studies*, 56(4): 583–601.

Birmpas, G.; Markakis, E.; and Schäfer, G. 2019. Cost sharing over combinatorial domains: Complement-free cost functions and beyond. In *ESA*, 1–17.

Börgers, T.; and Krahmer, D. 2015. *An introduction to the theory of mechanism design*. Oxford University Press, USA.

Brandl, F.; Brandt, F.; Greger, M.; Peters, D.; Stricker, C.; and Suksompong, W. 2022. Funding public projects: A case for the Nash product rule. *Journal of Mathematical Economics*, 99: 102585.

Cason, T. N.; Tabarrok, A.; and Zubrickas, R. 2021. Early refund bonuses increase successful crowdfunding. *Games and Economic Behavior*, 129: 78–95.

Cason, T. N.; and Zubrickas, R. 2017. Enhancing fundraising with refund bonuses. *Games and Economic Behavior*, 101: 218–233.

Chakrabarty, D.; and Swamy, C. 2014. Welfare maximization and truthfulness in mechanism design with ordinal preferences. In *Proceedings of the 5th conference on Innovations in Theoretical Computer Science*, 105–120.

Chandra, P.; Gujar, S.; and Narahari, Y. 2016. Crowdfunding Public Projects with Provision Point: A Prediction Market Approach. In *ECAI*, 778–786.

Chandra, P.; Gujar, S.; and Narahari, Y. 2017. Referral-Embedded Provision Point Mechanisms for Crowdfunding of Public Projects. In *AAMAS*, 642–650.

Chen, Y.; Tao, B.; and Yu, F.-Y. 2021. Cooperation in Threshold Public Projects with Binary Actions. In *IJCAI*, 104–110.

Damle, S.; Moti, M. H.; Chandra, P.; and Gujar, S. 2019a. Aggregating Citizen Preferences for Public Projects Through Civic Crowdfunding. In *AAMAS*, 1919–1921.

Damle, S.; Moti, M. H.; Chandra, P.; and Gujar, S. 2019b. Civic Crowdfunding for Agents with Negative Valuations and Agents with Asymmetric Beliefs. In *IJCAI*, 208–214.

Damle, S.; Moti, M. H.; Gujar, S.; and Chandra, P. 2021. Designing Refund Bonus Schemes for Provision Point Mechanism in Civic Crowdfunding. In *PRICAI*, 18–32.

Damle, S.; Padala, M.; and Gujar, S. 2022. Combinatorial Civic Crowdfunding with Budgeted Agents: Welfare Optimality at Equilibrium and Optimal Deviation. *arXiv*, abs/2211.13941.

Diederich, J.; Goeschl, T.; and Waichman, I. 2016. Group size and the (in) efficiency of pure public good provision. *European Economic Review*, 85: 272–287.

Dobzinski, S.; Mehta, A.; Roughgarden, T.; and Sundararajan, M. 2018. Is Shapley cost sharing optimal? *Games and Economic Behavior*, 108: 130–138.

Dobzinski, S.; and Ovadia, S. 2017. Combinatorial cost sharing. In *ACM EC*, 387–404.

Goodspeed, R. 2017. Community and urban places in a digital world. *Digital World. City & Community*, 16(1): 9–15.

Healy, P. J. 2006. Learning dynamics for mechanism design: An experimental comparison of public goods mechanisms. *Journal of Economic Theory*, 129(1): 114–149.

HudExchange. 2023. Participatory Budgeting. https://www.hudexchange.info/programs/participatory-budgeting/. Accessed: 2023-03-21.

Kickstarter. 2023. Kickstarter — Wikipedia, The Free Encyclopedia. https://en.wikipedia.org/w/index.php?title=Kickstarter. Accessed: 2023-03-21.

Lawler, E. L. 1977. Fast approximation algorithms for knapsack problems. In *18th Annual Symposium on Foundations of Computer Science (sfcs 1977)*, 206–213. IEEE.

London, M. 2021. The Impact of Community Crowdfunding. https://www.london.gov.uk/what-we-do/regeneration/funding-opportunities/make-london/impact-community-crowdfunding. Accessed: 2023-03-21.

Lubin, B.; and Parkes, D. C. 2012. Approximate strategyproofness. *Current Science*, 1021–1032.

Moulin, H. 1994. Serial cost-sharing of excludable public goods. *The Review of Economic Studies*, 61(2): 305–325.

Moulin, H.; and Shenker, S. 2001. Strategyproof sharing of submodular costs: budget balance versus efficiency. *Economic Theory*, 18(3): 511–533.

Ohseto, S. 2000. Characterizations of strategy-proof mechanisms for excludable versus nonexcludable public projects. *Games and Economic Behavior*, 32(1): 51–66.

Padala, M.; Damle, S.; and Gujar, S. 2021. Learning Equilibrium Contributions in Multi-project Civic Crowdfunding. In *WI-IAT*, 368—-375.

Soundy, J.; Wang, C.; Stevens, C.; and Chan, H. 2021. Game-theoretic Analysis of Effort Allocation of Contributors to Public Projects. In *IJCAI*, 405–411.

Spacehive. 2023. Spacehive - Crowdfunding For Local Projects. https://www.spacehive.com/. Accessed: 2023-03-21.

Sreedurga, G.; Bhardwaj, M. R.; and Narahari, Y. 2022. Maxmin Participatory Budgeting. *arXiv preprint arXiv:2204.13923*.

Stroup, R. L. 2000. Free riders and collective action revisited. In *Public Choice Essays in Honor of a Maverick Scholar: Gordon Tullock*, 137–150. Springer, Boston, MA.

Van Montfort, K.; Siebers, V.; and De Graaf, F. J. 2021. Civic Crowdfunding in Local Governments: Variables for Success in the Netherlands? *Journal of Risk and Financial Management*, 14(1): 8.

Wang, G.; Guo, R.; Sakurai, Y.; Ali Babar, M.; and Guo, M. 2021. Mechanism Design for Public Projects via Neural Networks. In *AAMAS*, 1380—-1388.

Yan, X.; and Chen, Y. 2021. Optimal Crowdfunding Design. In *AAMAS*, 1704–1706.

Zou, J.; Gujar, S.; and Parkes, D. 2010. Tolerable manipulability in dynamic assignment without money. In *AAAI*, 947–952.

Zubrickas, R. 2014. The provision point mechanism with refund bonuses. *Journal of Public Economics*, 120: 231–234.