Incomplete Argumentation Frameworks: Properties and Complexity

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Abstract

Dung's Argumentation Framework (AF) has been extended in several directions, including the possibility of representing unquantified uncertainty about the existence of arguments and attacks. The framework resulting from such an extension is called *incomplete AF* (iAF). In this paper, we first introduce three new satisfaction problems named *totality*, *determinism* and *functionality*, and investigate their computational complexity for both AF and iAF under several semantics.

We also investigate the complexity of credulous and skeptical acceptance in iAF under semi-stable semantics—a problem left open in the literature. We then show that any iAF can be rewritten into an equivalent one where either only (unattacked) arguments or only attacks are uncertain. Finally, we relate iAF to probabilistic argumentation framework, where uncertainty is quantified.

Introduction

Formal argumentation has emerged as one of the important fields in Artificial Intelligence (Bench-Capon and Dunne 2007; Simari and Rahwan 2009; Atkinson et al. 2017). In particular, Dung's abstract Argumentation Framework (AF) is a simple yet powerful formalism for modeling disputes between two or more agents (Dung 1995). An AF consists of a set of *arguments* and a binary *attack* relation over the set of arguments that specifies the interactions between arguments: intuitively, if argument a attacks argument b, then b is acceptable only if a is not. Hence, arguments are abstract entities whose status is entirely determined by the attack relation. An AF can be seen as a directed graph, whose nodes represent arguments and edges represent attacks.

Several argumentation semantics—e.g. *grounded* ($\mathtt{gr}$), *complete* ($\mathtt{co}$), *preferred* ($\mathtt{pr}$), *stable* ($\mathtt{st}$) (Dung 1995), and *semi-stable* ($\mathtt{sst}$) (Caminada 2006)—have been defined for AF, leading to the characterization of *σ-extensions*, that intuitively consist of the sets of arguments that can be collectively accepted under semantics $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$.

Various proposals have been made to extend the Dung framework with the aim of better modeling the knowledge to be represented. The extensions include Bipolar AF (Nouioua and Risch 2011; Nouioua 2013; Villata et al. 2012), AF with recursive attacks and supports (Cohen et al. 2015; Gottifredi et al. 2018; Cayrol et al. 2018), Dialectical framework (Brewka and Woltran 2010; Brewka et al. 2013), AF with preferences (Amgoud and Cayrol 1998; Modgil and Prakken 2013) and constraints (Coste-Marquis, Devred, and Marquis 2006; Arieli 2015; Alfano et al. 2021b), as well extensions for representing uncertain information. For the representation of uncertain information, two main extensions have been proposed: *incomplete AF* (iAF), where arguments and attacks may be uncertain (Baumeister et al. 2018, 2021), and *probabilistic AF* (PrAF), where arguments and attacks are associated with a probability (Dung and Thang 2010; Li, Oren, and Norman 2011; Hunter 2012). In this paper we focus on iAF, but we will also study its connection with PrAF.

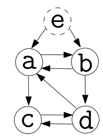

Figure 1: iAF Δ of Example 1.

Example 1. Consider an iAF $\Delta = \langle\{\mathtt{a},\mathtt{b},\mathtt{c},\mathtt{d}\}, \{\mathtt{e}\}, \{(\mathtt{a},\mathtt{b}), (\mathtt{b},\mathtt{a}), (\mathtt{a},\mathtt{c}), (\mathtt{b},\mathtt{d}), (\mathtt{c},\mathtt{d}), (\mathtt{d},\mathtt{c}), (\mathtt{d},\mathtt{a}), (\mathtt{e},\mathtt{a}), (\mathtt{e},\mathtt{b})\}, \emptyset\rangle$ whose corresponding graph is shown in Figure 1, where dashed nodes represent uncertain arguments (see Definition 1 for the formal definition). Δ describes the following scenario. A party planner invites Alice, Bob, Carl, David, and Erik to join a party. Due to their old rivalry (*i*) Alice replies that she will join the party if Bob, David and Erik do not; (*ii*) Bob replies that he will join the party if Alice and Erik do not; (*iii*) Carl replies that he will join the party if Alice and David do not; (*iv*) David replies that he will join the party if Bob and Carl do not; and (*v*) Erik replies that he is not sure that can join the party. This situation can be modeled by iAF Δ, where an argument $\mathtt{x}$ states that "*(the person whose names' initial is)* $\mathtt{x}$ *joins the party*", and argument $\mathtt{e}$="*Erik joins the party*" is uncertain. □

The semantics of an iAF is given by considering all *completions*, i.e. AFs obtained by removing consistent subsets of the uncertain elements, and for each completion its σ-extensions under a given semantics σ.

Example 2. Continuing with Example 1, the completions of Δ are as follows:

- $\Lambda_1 = \langle\{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}\}, \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a}), (\mathtt{a}, \mathtt{c}), (\mathtt{b}, \mathtt{d}), (\mathtt{c}, \mathtt{d}), (\mathtt{d}, \mathtt{c}), (\mathtt{d}, \mathtt{a}), (\mathtt{e}, \mathtt{a}), (\mathtt{e}, \mathtt{b})\}\rangle$;
- $\Lambda_2 = \langle\{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}, \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a}), (\mathtt{a}, \mathtt{c}), (\mathtt{b}, \mathtt{d}), (\mathtt{c}, \mathtt{d}), (\mathtt{d}, \mathtt{c}), (\mathtt{d}, \mathtt{a})\}\rangle$.

Completion Λ_1 is the AF obtained from Δ by keeping all the arguments and attacks, while completion Λ_2 is obtained from Δ by removing the uncertain argument e and, consistently with this, attacks (e, a) and (e, b) starting from e. □

Since most of the argumentation semantics defined for AF are *non-deterministic* (or *multiple-status*), the notions of credulous and skeptical acceptance have been defined. Thus, given a semantics σ, an argument is *credulously accepted* if it occurs in at least one σ-extension, whereas it is *skeptically accepted* if all σ-extensions contain it.

Acceptance problems have been recently extended to iAF: an argument is *possibly credulously* (resp. *skeptically*) accepted if there exists a completion where it is credulously (resp. skeptically) accepted; an argument is *necessarily credulously* (resp. *skeptically*) accepted if for all completions it is credulously (resp. skeptically) accepted.

Our main contributions are as follows.

Determinism, totality, and functionality for AF. We introduce three satisfaction problems for AF called *determinism* (DS), *totality* (TS), and *functionality* (FS). Informally, an argument is said to be *deterministic* if all extensions assign the same status (either accepted, rejected, or undefined) to it, whereas it is said to be *total* if for all extensions it is either accepted or rejected (i.e. it is never undefined). Totality is inspired by the criterion leading to stable semantics (Dung 1995). In fact, it requires the same property as the stable semantics but for a given goal argument (instead of all arguments). Specifically, while stable semantics forces the status of all arguments to be either accepted or rejected, totality requires this condition for a given goal argument under semantics σ. For instance, given an AF, we may be interested to know if an argument "*The defendant is guilty*" can be decided w.r.t. any σ-extension, i.e. if it is total. For AFs, determinism tells us if the status of a given argument is always the same (accepted/rejected/undecided), that is if we can safely make a decision or not based on the available knowledge. When both totality and determinism hold for a given argument, we say that it is *functional*. We study the complexity of the problems of checking determinism (DS), totality (TS), and functionality (FS) under several semantics for general and odd-cycle free AF (see Table 1).

Determinism, totality, and functionality for iAF. We extend the totality, deterministic and functional properties to iAFs. In this context, an argument a is *total* (resp. *deterministic*, *functional*) if it is total (resp. deterministic, functional) in *every* completion. Thus, for iAF, totality tells us if an argument can be decided whatever the considered scenario (i.e. completion/world) is, while determinism tells us if we can safely make a decision in every scenario (completion). Functionality combines totality and determinism and thus tells us if an argument can be firmly decided whatever the considered scenario is, that is if the information at hand is sufficient to make a decision irrespective of the possible world, as shown in the following example.

	(General) AF			odd-cycle free AF		
σ	TS	DS	FS	TS	DS	FS
gr	P	trivial	P	P	trivial	P
co	P	coNP-c	P	P	coNP-c	P
st	NP-c	DP-c	DP-c	trivial	coNP-c	coNP-c
pr	Π_2^p-c	Π_2^p-c	Π_2^p-c	trivial	coNP-c	coNP-c
sst	Π_2^p-c	Π_2^p-c	Π_2^p-c	trivial	coNP-c	coNP-c

Table 1: Complexity of TS, DS, and FS problems for AFs.

	(General) iAF			odd-cycle free iAF		
σ	TS	DS	FS	TS	DS	FS
gr	coNP-c	trivial	coNP-c	coNP-c	trivial	coNP-c
co	coNP-c	coNP-c	coNP-c	coNP-c	coNP-c	coNP-c
st	Π_2^p-c	Π_2^p-c	Π_2^p-c	trivial	coNP-c	coNP-c
pr	Π_2^p-c	Π_2^p-c	Π_2^p-c	trivial	coNP-c	coNP-c
sst	Π_2^p-c	Π_2^p-c	Π_2^p-c	trivial	coNP-c	coNP-c

Table 2: Complexity of TS, DS, and FS problems for iAFs.

Example 3. Continuing with Examples 1 and 2, suppose the party planner is interested to know if, whatever Erik decide to do, *he has sufficient information to decide* whether or not Alice (resp. Bob) joins the party. The preferred (stable/semi-stable) extensions of Λ_1 and Λ_2 are $\{\{\mathtt{c}, \mathtt{e}\}, \{\mathtt{d}, \mathtt{e}\}\}$ and $\{\{\mathtt{b}, \mathtt{c}\}\}$, respectively. As a and b are functional, the party planner concludes that a decision on the fact that Alice (resp. Bob) joins or not the party can be taken in both scenarios (Erik joins or not the party).

Note that such questions cannot be answered by possible/necessary credulous/skeptical acceptance. In fact, a is functional but not possible credulously accepted, and b is functional but not necessary skeptically accepted. □

We investigate the complexity of the problems of checking determinism, totality, and functionality for general iAF and odd-cycle free iAF. Our results are reported in Table 2.

Possible/Necessary Credulous/Skeptical Acceptance under Semi-Stable Semantics. We investigate the problems of (*i*) possible credulous (resp. skeptical) acceptance (PCA, resp. PSA), and (*ii*) necessary credulous (resp. skeptical) acceptance (NCA, resp. NSA) under semi-stable semantics—though studied for semantics $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}\}$, the investigation of the complexity of these problems was left open for semi-stable semantics in (Baumeister et al. 2018; Fazzinga, Flesca, and Furfaro 2020). We show that under semi-stable semantics PCA (resp. PSA) is Σ_2^p-complete (resp. Σ_3^p-complete), whereas NCA (resp. NSA) is Π_3^p-complete (resp. Π_2^p-complete). Thus, compared with the results for the preferred semantics (Baumeister et al. 2018), the complexity increases of one level of the polynomial hierarchy for credulous reasoning, while it does not change for skeptical reasoning and for problems DS, TS and FS (the results on last two rows of Table 2 coincide).

Equivalent Forms of iAF: arg-iAF, att-iAF, and farg-iAF. We show that an iAF Δ can be rewritten into an "equivalent" one Δ', such that for every σ-extension E of Δ, there is a σ-extension E' of Δ' coinciding with E (modulo some meta-arguments added in the rewriting). Particularly, Δ' can be of one of the following forms: $i)$ only arguments or only attacks are uncertain (Δ' is said to be an *arg-iAF* or *att-iAF*, respectively); or $ii)$ uncertainty is only on arguments that are not attacked by any other argument (Δ' is said to be an *farg-iAF*). We also show that an *arg-iAF* can be translated into an *farg-iAF*. It can be shown that the results in Table 2 also hold for these restricted forms of iAFs.

Relationship between iAF and PrAF. Finally, using the equivalence result between iAF and arg-iAF, we investigate the relationships between iAF and PrAF and relate the (possible/necessary credulous/skeptical) acceptance problems in iAF to probabilistic acceptance in PrAF. For instance, we show that, under the assumption that for each completion the existence of at least one extension is guaranteed, if an argument is necessarily skeptically accepted then its probabilistic acceptance is $\mathtt{1}$, whereas in all other cases if the argument is accepted then the probability of acceptance is in the interval $(\mathtt{0},\mathtt{1}]$. In our approach we derive a PrAF encoding a given iAF, and also show that computing the probability of acceptance of an argument is $FP^{\#P}$-hard even restricting to acyclic iAFs.

Proofs of our results are available in (Alfano et al. 2021c).

Preliminaries

In this section, we first review the Dung's framework and the incomplete one, and then recall complexity classes.

Argumentation Framework

An abstract *Argumentation Framework* (AF) (Dung 1995) is a pair $\langle A, R\rangle$, where A is a set of *arguments* and $R \subseteq A \times A$ is a set of *attacks*.

Given an AF $\Lambda = \langle A, R\rangle$ and a set $S \subseteq A$ of arguments, an argument $a \in A$ is said to be *i) defeated* w.r.t. S iff there exists $b \in S$ such that $(b, a) \in R$, and *ii) acceptable* w.r.t. S iff for every argument $b \in A$ with $(b, a) \in R$, there is $c \in S$ such that $(c, b) \in R$. The sets of arguments defeated and acceptable w.r.t. S are as follows (where Λ is understood):

- $\mathit{Def}(S) = \{a \in A \mid \exists b \in S \,.\, (b, a) \in R\}$;
- $\mathit{Acc}(S) = \{a \in A \mid \forall b \in A \,.\, (b, a) \in R \Rightarrow b \in \mathit{Def}(S)\}$.

Given an AF $\langle A, R\rangle$, a set $S \subseteq A$ of arguments is said to be *conflict-free* iff $S \cap \mathit{Def}(S) = \emptyset$. Moreover, $S \subseteq A$ is said to be a *complete* ($\mathtt{co}$) extension iff it is conflict-free and $S = \mathit{Acc}(S)$. A complete extension $S \subseteq A$ is said to be:

- *preferred* ($\mathtt{pr}$) iff it is $\subseteq$-maximal;
- *stable* ($\mathtt{st}$) iff it is a total preferred extension, i.e. a preferred extension such that $S \cup \mathit{Def}(S) = A$;
- *semi-stable* ($\mathtt{sst}$) iff it is a preferred extension with a maximal set of decided elements;
- *grounded* ($\mathtt{gr}$) iff it is $\subseteq$-minimal.

In the following, if not specified otherwise, σ denotes any semantics in $\{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$. For any AF Λ and semantics σ, $\sigma(\Lambda)$ denotes the set of σ-extensions of Λ. All the above-mentioned semantics except the stable admit at least one extension (i.e. $\sigma(\Lambda) \neq \emptyset$), and the grounded admits exactly one extension (i.e. $|\mathtt{gr}(\Lambda)| = 1$) (Dung 1995; Caminada 2006). The grounded semantics is called *deterministic* (or *unique status*), whereas the other semantics are called *non-deterministic* (or *multiple status*). The stable semantics is said to be *total* as each argument belongs to either E or $\mathit{Def}(E)$ (i.e. it is either *true* or *false*) for every extension E. For any AF Λ, it holds that $\mathtt{st}(\Lambda) \subseteq \mathtt{sst}(\Lambda) \subseteq \mathtt{pr}(\Lambda) \subseteq \mathtt{co}(\Lambda)$, and $\mathtt{gr}(\Lambda) \subseteq \mathtt{co}(\Lambda)$. An AF is *acyclic* (resp. *odd-cycle free*) if the associated graph is acyclic (resp. odd-cycle free). For acyclic AFs all the considered semantics coincide.

In the following we consider the acceptability of *argument literals* (or simply *literals*). A literal is either an argument a or its negation $\neg a$. A set of literals S is said to be *consistent* if it does not contain two literals a and $\neg a$. We use $\neg S$ to denote the set $\{\neg a \mid a \in S\}$, and S^* to denote $S \cup \neg S$. To deal with literals, we define the notion of *fulfillment* of an extension. Given AF $\Lambda = \langle A, R\rangle$ and an extension E for it, the *fulfillment* of E is $E \cup \neg \mathit{Def}(E)$. Clearly, the fulfillment of an extension is a consistent set of literals. Herein, arguments in E are represented as positive literals (i.e. interpreted as *true*), while arguments defeated by E are represented as negative literals (i.e. interpreted as *false*).

In the rest of the paper, with a little abuse of notation, whenever we refer to an extension we mean its fulfillment.

For any AF $\Lambda = \langle A, R\rangle$, semantics σ, and literal $a \in A^*$, we say that a is *credulously* (resp. *skeptically*) accepted (under semantics σ), denoted as $CA_\sigma(\Lambda, a)$ (resp. $SA_\sigma(\Lambda, a)$) if a belongs to at least a (resp. every) σ-extension of Λ.

We use CA_σ (resp. SA_σ), or simply CA (resp. SA) whenever σ is understood, to denote the credulous (resp. skeptical) acceptance problem, that is, the problem of deciding whether a literal is credulously (resp. skeptically) accepted. Clearly, for the grounded semantics, which has exactly one extension, these problems are identical (i.e. $\mathsf{CA}_{\mathtt{gr}} \equiv \mathsf{SA}_{\mathtt{gr}}$).

Example 4. Consider the AF $\Lambda = \langle\{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}, \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a}), (\mathtt{a}, \mathtt{c}), (\mathtt{b}, \mathtt{c}), (\mathtt{c}, \mathtt{d}), (\mathtt{d}, \mathtt{c})\}\rangle$. Λ has 4 complete extensions: $E_0 = \emptyset$, $E_1 = \{\mathtt{a}, \neg\mathtt{b}, \neg\mathtt{c}, \mathtt{d}\}$, $E_2 = \{\neg\mathtt{a}, \mathtt{b}, \neg\mathtt{c}, \mathtt{d}\}$ and $E_3 = \{\neg\mathtt{c}, \mathtt{d}\}$. That is, $\mathtt{co}(\Lambda) = \{E_0, E_1, E_2, E_3\}$. E_0 is the grounded extension, E_1 and E_2 are stable (preferred and semi-stable) extensions. Thus, $\mathtt{a}$, $\mathtt{b}$, $\mathtt{d}$, as well as $\neg\mathtt{a}$, $\neg\mathtt{b}$, $\neg\mathtt{c}$, are credulously accepted for all semantics except the grounded; only $\mathtt{d}$ and $\neg\mathtt{c}$ are skeptically accepted for stable, preferred and semi-stable semantics. □

Incomplete Argumentation Framework

We now recall the incomplete AF (Baumeister et al. 2018).

Definition 1 (Incomplete AF)**.** An *incomplete (abstract) Argumentation Framework* (iAF) is a tuple $\Delta = \langle A, B, R, T\rangle$, where A and B are disjoint sets of arguments, and R and T are disjoint sets of attacks between arguments in $A \cup B$. Arguments in A and attacks in R are said to be *certain*, while arguments in B and attacks in T are said to be *uncertain*.

Certain arguments in A are definitely known to exist, while uncertain arguments in B are not known for sure: they may occur or may not. Analogously, certain attacks in R are definitely known to exist if both the incident arguments exist, while for uncertain attacks in T it is not known for sure if they hold, even if both the incident arguments exist.

An iAF compactly represents alternative AF scenarios, called *completions*.

Definition 2 (Completion)**.** A *completion* for an iAF $\Delta = \langle A, B, R, T\rangle$ is an AF $\Lambda = \langle A', R'\rangle$ where $A \subseteq A' \subseteq A \cup B$ and $R \cap (A' \times A') \subseteq R' \subseteq (R \cup T) \cap (A' \times A')$.

The set of completions of Δ is denoted by $comp(\Delta)$.

An iAF $\langle A, B, R, T\rangle$ is *acyclic* (resp. *odd-cycle free*) iff the AF $\langle A \cup B, R \cup T\rangle$ is acyclic (resp. odd-cycle free).

To define acceptability and properties satisfaction of literals for iAFs, we refine the definition of fulfillment of extensions. Given an iAF $\Delta = \langle A, B, R, T\rangle$ and an extension E for a completion $\Lambda = \langle A', R'\rangle$ of Δ, the *fulfillment* of E is $E \cup \neg Def(E) \cup \neg(B \setminus A')$. Herein, arguments in E are represented as positive literals (i.e. interpreted as *true*), while arguments attacked by E as well as uncertain arguments not occurring in Λ are represented as negative literals (i.e. interpreted as *false*).

Example 5. Consider the iAF Δ of Example 1 and the completion Λ_2 reported in Example 2. Λ_2 has only one preferred extension, $\{\mathtt{b}, \mathtt{c}\}$, whose fulfillment is $\{\neg\mathtt{a}, \mathtt{b}, \mathtt{c}, \neg\mathtt{d}, \neg\mathtt{e}\}$. □

Acceptance problems. Credulous and skeptical acceptance for iAF have been recently proposed in (Baumeister, Neugebauer, and Rothe 2018), where the goal, i.e. the element for which acceptance is checked, is an argument. As we focus on fulfillment of extensions, our goal is a literal.

Definition 3 (Possible/Necessary Credulous/Skeptical Acceptance)**.** Let $\Delta = \langle A, B, R, T\rangle$ be an iAF and $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$. Then, a literal $g \in A^* \cup B^*$ is said to be:

1. *possibly credulously accepted* under σ, denoted as $PCA_\sigma(\Delta, g)$, iff there exists a completion Λ of Δ and a σ-extension E of Λ such that $g \in E$;
2. *possibly skeptically accepted* under σ, denoted as $PSA_\sigma(\Delta, g)$, iff there exists a completion Λ of Δ such that g occurs in every σ-extension of Λ;
3. *necessarily credulously accepted* under σ, denoted as $NCA_\sigma(\Delta, g)$, iff for every completion Λ of Δ, there exists a σ-extension E of Λ such that $g \in E$;
4. *necessarily skeptically accepted* under σ, denoted as $NSA_\sigma(\Delta, g)$, iff for every completion Λ of Δ, g occurs in every σ-extension of Λ.

We use PCA_σ (resp. PSA_σ, NCA_σ, NSA_σ), or simply PCA (resp. PSA, NCA, NSA) whenever σ is understood, to denote the problem of deciding acceptance according to Item 1 (resp. 2, 3, and 4) of Definition 3. For the grounded semantics we have $\mathsf{PCA}_{\mathtt{gr}} \equiv \mathsf{PSA}_{\mathtt{gr}}$ and $\mathsf{NCA}_{\mathtt{gr}} \equiv \mathsf{NSA}_{\mathtt{gr}}$.

Example 6. Consider the iAF $\Delta = \langle\{\mathtt{a}, \mathtt{b}, \mathtt{d}\}, \{\mathtt{c}\}, \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a})\}, \emptyset\rangle$. Δ has 2 completions: $\Lambda_1 = \langle\{\mathtt{a}, \mathtt{b}, \mathtt{d}\}, \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a})\}\rangle$ and $\Lambda_2 = \langle\{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}, \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a})\}\rangle$. Under semantics $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$, Λ_1 has two extensions $E_1^{'} = \{\mathtt{a}, \neg\mathtt{b}, \neg\mathtt{c}, \mathtt{d}\}$ and $E_1^{''} = \{\neg\mathtt{a}, \mathtt{b}, \neg\mathtt{c}, \mathtt{d}\}$, while Λ_2 has two extensions $E_2^{'} = \{\mathtt{a}, \neg\mathtt{b}, \mathtt{c}, \mathtt{d}\}$ and $E_2^{''} = \{\neg\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$. Thus, $\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \neg\mathtt{a}, \neg\mathtt{b}, \neg\mathtt{c}$ satisfy PCA, $\mathtt{c}, \mathtt{d}, \neg\mathtt{c}$ satisfy PSA, $\mathtt{a}, \mathtt{b}, \mathtt{d}, \neg\mathtt{a}, \neg\mathtt{b}$ satisfy NCA and only $\mathtt{d}$ satisfies NSA. □

Observe that we consider acceptance of literals and, differently from (Fazzinga, Flesca, and Furfaro 2020) and (Baumeister et al. 2018), we also deal with $\mathtt{sst}$ semantics.

Complexity Classes

We recall here the main complexity classes used in the paper and, in particular, the definition of the classes Σ_k^p and Π_k^p, with $k \geq 0$ (see e.g. (Papadimitriou 1994)): (*i*) $\Sigma_0^p = \Pi_0^p =$ *P*; (*ii*) $\Sigma_1^p =$ *NP* and $\Pi_1^p =$ *coNP*; and (*iii*) $\Sigma_k^p = NP^{\Sigma_{k-1}^p}$ and $\Pi_k^p = co\Sigma_k^p$, $\forall k > 0$. *P* denotes the class of problems that can be solved in polynomial time (w.r.t. the size of the input) by a deterministic Turing machine. NP^C denotes the class of problems that can be solved in polynomial time using an oracle in the class C by a non-deterministic Turing machine. Under a standard complexity assumption, $\Sigma_k^p \subseteq \Sigma_{k+1}^p \subseteq$ *PSPACE* and $\Pi_k^p \subseteq \Pi_{k+1}^p \subseteq$ *PSPACE*. A language L is in the class *DP* iff there are two languages $L_1 \in$ *NP* and $L_2 \in$ *coNP* such that $L = L_1 \cap L_2$. *FP* is the class of the function problems that can be solved in polynomial time by a deterministic Turing machine. $FP^{\#P}$ is the class of functions computable by a polynomial-time Turing machine with a $\#P$ oracle. $\#P$ is the complexity class of the functions counting the number of accepting paths of a non-deterministic poly-time Turing machine (Valiant 1979).

Totality, Determinism and Functionality in AF

In this section we discuss desirable properties to be satisfied by literals w.r.t. a given semantics. As discussed in the introduction, these properties concern the fact that the status of a literal is *i*) never undefined (*totality*), *ii*) the same in all extensions (*determinism*), and *iii*) never undefined and the same in all extensions (*functionality*). They will be extended to the case of iAFs in a subsequent section.

Definition 4. Let $\Lambda = \langle A, R\rangle$ be an AF, $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$, and $g \in A^*$ a literal. We say that g is:

- *total* under σ, denoted as $TS_\sigma(\Lambda, g)$, if *i*) $\sigma(\Lambda) \neq \emptyset$ and *ii*) for each $E \in \sigma(\Lambda)$, either $g \in E$ or $\neg g \in E$;
- *deterministic* under σ, denoted as $DS_\sigma(\Lambda, g)$, if *i*) $\sigma(\Lambda) \neq \emptyset$, and *ii*) either $SA_\sigma(\Lambda, g)$ or $SA_\sigma(\Lambda, \neg g)$ or $(\neg CA_\sigma(\Lambda, g) \wedge \neg CA_\sigma(\Lambda, \neg g))$;
- *functional* under σ, denoted as $FS_\sigma(\Lambda, g)$, if g is total and deterministic under σ, that is *i*) $\sigma(\Lambda) \neq \emptyset$ and *ii*) either $SA_\sigma(\Lambda, g)$ or $SA_\sigma(\Lambda, \neg g)$.

Observe that only for $\sigma = \mathtt{st}$ we may have $\sigma(\Lambda) = \emptyset$: for all other semantics we have that $\sigma(\Lambda) \neq \emptyset$. We use TS_σ (resp. DS_σ, FS_σ) to denote the problem of deciding whether a goal is total (resp. deterministic, functional). The next theorem states the complexity of checking these properties.

Theorem 1. *For general AFs, it holds that:*

- TS_σ *is polynomial for* $\sigma \in \{\mathtt{gr}, \mathtt{co}\}$, *NP-complete for* $\sigma = \mathtt{st}$, *and* Π_2^p*-complete for* $\sigma \in \{\mathtt{pr}, \mathtt{sst}\}$*;*
- DS_σ *is trivial for* $\sigma = \mathtt{gr}$, *coNP-complete for* $\sigma = \mathtt{co}$, *DP-complete for* $\sigma = \mathtt{st}$, *and* Π_2^p*-complete for* $\sigma \in \{\mathtt{pr}, \mathtt{sst}\}$*;*
- FS_σ *is polynomial for* $\sigma \in \{\mathtt{gr}, \mathtt{co}\}$, *DP-complete for* $\sigma = \mathtt{st}$, *and* Π_2^p*-complete for* $\sigma \in \{\mathtt{pr}, \mathtt{sst}\}$.

Example 7. Considering the AF of Example 4, literals $\mathtt{c}$ and $\mathtt{d}$ are functional for $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$, whereas $\mathtt{a}$ and $\mathtt{b}$ are total, but not deterministic, for $\sigma = \mathtt{st}$. Obviously, for $\sigma = \mathtt{gr}$ all literals are deterministic, but not total. □

Theorem 2. *For odd-cycle free AFs, it holds that:*

- TS_σ *is polynomial for* $\sigma \in \{\mathtt{gr}, \mathtt{co}\}$*, trivial for* $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$*;*
- DS_σ *trivial for* $\sigma = \mathtt{gr}$*; coNP-complete for* $\sigma \in \{\mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$*;*
- FS_σ *is polynomial for* $\sigma \in \{\mathtt{gr}, \mathtt{co}\}$*, and coNP-complete for* $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$.

Proposition 1. *For any AF* $\Lambda = \langle A, R\rangle$*, literal* $g \in A^*$*, and semantics* $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{sst}\}$*, it holds that:*

- $\mathsf{DS}_\sigma(\Lambda, g) = false \Rightarrow \mathsf{SA}_\sigma(\Lambda, g) = false$ *(or, equivalently,* $\mathsf{SA}_\sigma(\Lambda, g) = true \Rightarrow \mathsf{DS}_\sigma(\Lambda, g) = true$*), and*
- $\mathsf{FS}_\sigma(\Lambda, g) \equiv \mathsf{SA}_\sigma(\Lambda, g) \vee \mathsf{SA}_\sigma(\Lambda, \neg g)$.

Concerning the previous results, notice that stable, preferred and semi-stable semantics coincide for odd-cycle free AFs, and thus the existence of a stable extension is always guaranteed (Dung 1995). Thus, Proposition 1 holds also for $\sigma = \mathtt{st}$ and odd-cycle free AFs.

Totality, Determinism and Functionality in iAF

The concepts of totality, determinism and functionality, defined in the context of AF can be extended to iAF as follows.

Definition 5 (Total, deterministic, functional literals)**.** Let $\Delta = \langle A, B, R, T\rangle$ be an iAF, $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$, and $g \in A^* \cup B^*$ a goal literal. We say that g is:

- *total* under σ, denoted as $TS_\sigma(\Delta, g)$, if for each $\Lambda \in comp(\Delta)$, *i)* $\sigma(\Lambda) \neq \emptyset$ and *ii)* for each $E \in \sigma(\Lambda)$, either $g \in E$ or $\neg g \in E$;
- *deterministic* under σ, denoted as $DS_\sigma(\Delta, g)$, if for each $\Lambda \in comp(\Delta)$, *i)* $\sigma(\Lambda) \neq \emptyset$, and *ii)* either $SA_\sigma(\Lambda, g)$ or $SA_\sigma(\Lambda, \neg g)$ or $(\neg CA_\sigma(\Lambda, g) \wedge \neg CA_\sigma(\Lambda, \neg g))$;
- *functional* under σ, denoted as $FS_\sigma(\Delta, g)$, if g is total and deterministic under σ, that is, for each $\Lambda \in comp(\Delta)$, *i)* $\sigma(\Lambda) \neq \emptyset$ and *ii)* either $SA_\sigma(\Lambda, g)$ or $SA_\sigma(\Lambda, \neg g)$.

The next result immediately derives from definitions.

Fact 1. *Let* Δ *be an iAF,* $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$ *and* g *a literal. We have that i)* $TS_\sigma(\Delta, g) \equiv TS_\sigma(\Delta, \neg g)$*, ii)* $DS_\sigma(\Delta, g) \equiv DS_\sigma(\Delta, \neg g)$*, and iii)* $FS_\sigma(\Delta, g) \equiv FS_\sigma(\Delta, \neg g)$ *hold.*

Example 8. Consider the iAF $\Delta = \langle A = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}, B = \{\mathtt{d}\}, R = \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a}), (\mathtt{b}, \mathtt{c}), (\mathtt{c}, \mathtt{c}), (\mathtt{d}, \mathtt{a}), (\mathtt{d}, \mathtt{b})\}, \; T = \emptyset\rangle$ reported in Figure 2, left side. Δ has two completions $\Lambda_1 = \langle A, R \setminus \{(\mathtt{d}, \mathtt{a}), (\mathtt{d}, \mathtt{b})\}\rangle$ and $\Lambda_2 = \langle A \cup B, R\rangle$, also shown in Figure 2 (center). For Λ_1 there are three complete extensions (recall that we refer to the fulfillments): $E_0 = \{\neg\mathtt{d}\}$, $E_1 = \{\mathtt{a}, \neg\mathtt{b}, \neg\mathtt{d}\}$ and $E_2 = \{\neg\mathtt{a}, \mathtt{b}, \neg\mathtt{c}, \neg\mathtt{d}\}$. The grounded extension is E_0, while E_1 is preferred, and E_2 is stable, preferred, and semi-stable. For Λ_2 there is only one complete extension $E_3 = \{\neg\mathtt{a}, \neg\mathtt{b}, \mathtt{d}\}$, which is grounded, preferred, and semi-stable. The goal $\mathtt{b}$ is $i)$ total only for semantics $\mathtt{pr}$ and $\mathtt{sst}$ (it is not total for $\mathtt{st}$ as Λ_2 does not have stable extensions) and $ii)$ deterministic for semantics $\mathtt{gr}, \mathtt{st}$ and $\mathtt{sst}$ (it is not deterministic for $\mathtt{pr}$ as the two preferred extensions, E_1 and E_2, of Λ_1 are such that E_1 contains $\neg\mathtt{b}$ while E_2 contains $\mathtt{b}$). Thus, $\mathtt{b}$ is functional for $\mathtt{sst}$ only. □

We use TS_σ (resp. DS_σ, FS_σ), or simply FS (resp. TS, DS) whenever σ is understood, to denote the problem of deciding whether a literal is total (resp. deterministic, functional). The complexity of the aforementioned problems is characterized in the following theorem.

Theorem 3. *For general iAFs, it holds that:*

- TS_σ *is coNP-complete for* $\sigma \in \{\mathtt{gr}, \mathtt{co}\}$ *and* Π_2^p*-complete for* $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$*;*
- DS_σ *is trivial for* $\sigma = \mathtt{gr}$*, coNP-complete for* $\sigma = \mathtt{co}$*, and* Π_2^p*-complete for* $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$*;*
- FS_σ *is coNP-complete for* $\sigma \in \{\mathtt{gr}, \mathtt{co}\}$ Π_2^p*-complete for* $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$.

The next proposition provides conditions under which unattacked arguments are functional.

Proposition 2. *For any iAF* $\Delta = \langle A, B, R, T\rangle$ *and unattacked argument* $g \in A \cup B$*, it holds that* g *is functional i) under semantics* $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{sst}\}$*, and ii) under semantics* $\sigma = \mathtt{st}$ *if* Δ *is odd-cycle free.*

Thus, under semantics $\sigma \in \{\mathtt{co}, \mathtt{gr}, \mathtt{pr}, \mathtt{sst}\}$, every unattacked argument is total and deterministic. However, this does not hold for general iAFs under stable semantics, as there could be completions prescribing no extensions.

We conclude this section by providing the complexity of the problems of deciding whether a literal is total, deterministic, and functional for the case of odd-cycle free iAFs.

Theorem 4. *For odd-cycle free iAFs, it holds that:*

- TS_σ *is coNP-complete for* $\sigma \in \{\mathtt{gr}, \mathtt{co}\}$ *and trivial for* $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$*;*
- DS_σ *is trivial for* $\sigma = \mathtt{gr}$*, coNP-complete for* $\sigma \in \{\mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$*;*
- FS_σ *is coNP-complete for* $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$.

The results of Theorem 4 complement those of Theorem 3 (see Table 2). Except for the grounded and complete semantics, the complexity of checking determinism and functionality decreases by one level of the polynomial hierarchy whereas checking totality becomes trivial.

Acceptance Problems for iAF

We first characterize the complexity of the so-called *verification problems* for iAF under the semi-stable semantics. To this end, we need to introduce the concepts of *possible* and *necessary extension* for iAF (Baumeister et al. 2018; Fazzinga, Flesca, and Furfaro 2020). Given an iAF Δ and a semantics σ, a (consistent) set of literals S is said to be a *possible* (resp. *necessary*) σ-extension for Δ if for at least one (resp. for every) completion Λ of Δ, S is a σ-extension of Λ.[1] The verification problem is then defined as follows. Given an iAF Δ, a semantics σ and a set of literals S,

[1]We use the definition of *possible* (resp. *necessary*) σ-extension introduced in (Fazzinga, Flesca, and Furfaro 2020) that revised the initial definition in (Baumeister et al. 2018).

the *possible* (resp. *necessary*) *verification problem* is deciding whether S is a possible (resp. necessary) σ-extension of Δ. It has been shown that the complexity of the possible and necessary verification problems is polynomial for $\sigma \in \{\texttt{gr}, \texttt{co}, \texttt{st}\}$, whereas it is Σ_2^p-complete (resp. coNP-complete) for the possible (resp. necessary) variant under the preferred semantics (Fazzinga, Flesca, and Furfaro 2020).

As stated next, the complexity of the verification problems for the semi-stable and preferred semantics coincides.

Theorem 5. *Given an iAF $\Delta = \langle A, B, R, T\rangle$ and a consistent set $S \subseteq A^* \cup B^*$, checking whether S is a possible (resp. necessary)* $\texttt{sst}$*-extension of Δ is Σ_2^p-complete (resp. coNP-complete).*

After characterizing the complexity of the verification problem, in the next theorem we consider the acceptance problems (cf. Definition 3) for semi-stable semantics.

Theorem 6. $\mathsf{PCA}_{\texttt{sst}}$ *(resp.* $\mathsf{PSA}_{\texttt{sst}}$, $\mathsf{NCA}_{\texttt{sst}}$, *and* $\mathsf{NSA}_{\texttt{sst}}$*) is Σ_2^p-complete (resp. Σ_3^p-complete, Π_3^p-complete, and Π_2^p-complete).*

Compared with the results for the other semantics (Baumeister et al. 2018), it turns out that deciding PCA and NCA for $\texttt{sst}$ is more costly than for any other semantics, while deciding $\mathsf{PSA}_{\texttt{sst}}$ and $\mathsf{NSA}_{\texttt{sst}}$ costs the same as deciding $\mathsf{PSA}_{\texttt{pr}}$ and $\mathsf{NSA}_{\texttt{pr}}$, respectively. Moreover, the results of Theorems 5 and 6 complement those in Table 2, showing that, though problems DS, TS, and FS under $\texttt{sst}$ and $\texttt{pr}$ have the same complexity, these semantics behave differently under credulous reasoning.

Equivalent Forms of iAFs

In this section we show that iAFs can be rewritten into equivalent iAFs where uncertainty is restricted to either attacks or (unattacked) arguments.

Definition 6 (arg-iAF and att-iAF)**.** An iAF $\Delta = \langle A, B, R, T\rangle$ is said to be *argument-incomplete* (arg-iAF for short) if $T = \emptyset$, whereas it is said to be *attack-incomplete* (att-iAF for short) if $B = \emptyset$.

Given an iAF Δ, we denote by $arg(\Delta)$ the arg-iAF derived from Δ by replacing every uncertain attack (a, b) with the certain attacks $(a, \alpha_{ab}), (\alpha_{ab}, \beta_{ab}), (\beta_{ab}, b)$, where α_{ab} (resp. β_{ab}) is a fresh certain (resp. uncertain) argument.

Analogously, we denote by $att(\Delta)$ the att-iAF derived from Δ as follows: for each uncertain argument b, make b certain and add an uncertain attack (α, b), where α is a fresh certain argument—it is sufficient to add only one fresh argument α.

Example 9. Consider the iAF $\Delta = \langle\{\texttt{b}, \texttt{c}\}, \{\texttt{a}\}, \{(\texttt{a}, \texttt{b}), (\texttt{b}, \texttt{a})\}, \{(\texttt{b}, \texttt{c})\}\rangle$. The arg-iAF derived from Δ is $\Delta' = \langle\{\texttt{b}, \texttt{c}, \alpha_{\texttt{bc}}\}, \{\texttt{a}, \beta_{\texttt{bc}}\}, \{(\texttt{a}, \texttt{b}), (\texttt{b}, \texttt{a}), (\texttt{b}, \alpha_{\texttt{bc}}), (\alpha_{\texttt{bc}}, \beta_{\texttt{bc}}), (\beta_{\texttt{bc}}, \texttt{c})\}, \emptyset\rangle$, whereas the att-iAF derived from Δ is $\Delta'' = \langle\{\texttt{b}, \texttt{c}, \texttt{a}, \alpha\}, \emptyset, \{(\texttt{a}, \texttt{b}), (\texttt{b}, \texttt{a})\}, \{(\texttt{b}, \texttt{c}), (\alpha, \texttt{a})\}\rangle$. $\square$

The transformations described above to eliminate uncertain attacks/arguments are inspired by those proposed in (Mantadelis and Bistarelli 2020) to eliminate attacks/arguments with probability less than 1 in probabilistic AF.

We now introduce a special class of arg-iAFs.

Definition 7 (farg-iAF)**.** An arg-iAF $\Delta = \langle A, B, R, \emptyset\rangle$ is said to be *fact-uncertain* (farg-iAF) iff $\forall (a, b) \in R$, $b \notin B$.

Observe that the iAF of Example 1 is an farg-iAF.

Given an arg-iAF Δ, $farg(\Delta)$ denotes the farg-iAF derived from Δ as follows: for each uncertain argument b which is attacked in Δ, make b certain and add the attacks $(b^u, b^c), (b^c, b)$, where b^c (resp. b^u) is a fresh certain (resp. uncertain) argument. With a little abuse of notation, for any iAF Δ we use $farg(\Delta)$ to denote $farg(arg(\Delta))$.

Example 10. Considering an arg-iAF $\Delta = \langle\{\texttt{m}, \texttt{r}\}, \{\texttt{f}, \texttt{w}\}, \{(\texttt{f}, \texttt{m}), (\texttt{m}, \texttt{f}), (\texttt{m}, \texttt{w}), (\texttt{w}, \texttt{r}), (\texttt{r}, \texttt{w})\}, \emptyset\rangle$, the derived farg-iAF Δ' is as follows: $\langle\{\texttt{f}, \texttt{m}, \texttt{w}, \texttt{r}, \texttt{f}^\texttt{c}, \texttt{w}^\texttt{c}\}, \{\texttt{f}^\texttt{u}, \texttt{w}^\texttt{u}\}, \{(\texttt{f}, \texttt{m}), (\texttt{m}, \texttt{f}), (\texttt{m}, \texttt{w}), (\texttt{w}, \texttt{r}), (\texttt{r}, \texttt{w}), (\texttt{f}^\texttt{u}, \texttt{f}^\texttt{c}), (\texttt{f}^\texttt{c}, \texttt{f}), (\texttt{w}^\texttt{u}, \texttt{w}^\texttt{c}), (\texttt{w}^\texttt{c}, \texttt{w})\}, \emptyset\rangle$. The completions of Δ' are obtained by selecting all subsets of the set $\{\texttt{f}^\texttt{u}, \texttt{w}^\texttt{u}\}$ of uncertain arguments. $\square$

Given an iAF $\Delta = \langle A, B, R, T\rangle$, let $\varphi \in \{arg, att, farg\}$, for any $\Lambda' = \langle A', R'\rangle \in comp(\varphi(\Delta))$, $af_\Delta(\Lambda')$ denotes the AF $\Lambda'' = \langle A'', R''\rangle \in comp(\Delta)$ with:

- $A'' = A \cup ((B \cap A') \setminus \{a \mid (\alpha, a) \in R' \vee a^u \notin A'\})$, and
- $R'' = (R \cap (A'' \times A'')) \cup ((T \cap (A'' \times A'')) \setminus \{(a, b) \mid (\beta_{ab} \notin A') \vee (\beta^c_{ab} \in A' \wedge \beta^u_{ab} \notin A')\}$.

Herein, the set $\{a \mid (\alpha, a) \in R' \vee a^u \notin A'\}$ is used to avoid considering arguments either (i) attacked by α in $comp(att(\Delta))$ or (ii) always false in $comp(farg(\Delta))$ as $a^u \notin A'$. Analogously, $\{(a, b) \mid (\beta_{ab} \notin A') \vee (\beta^c_{ab} \in A' \wedge \beta^u_{ab} \notin A')\}$ is used to avoid considering uncertain attacks that are chosen to not occur in either (i) $comp(arg(\Delta))$, as $\beta_{ab} \notin A'$, or (ii) $comp(farg(\Delta))$ as $(\beta^c_{ab} \in A' \wedge \beta^u_{ab} \notin A')$.

Example 11. Consider the iAF Δ of Example 9. Then, $farg(\Delta) = \langle A = \{\texttt{a}, \texttt{b}, \texttt{c}, \texttt{a}^\texttt{c}, \alpha_{\texttt{bc}}, \beta_{\texttt{bc}}, \beta^\texttt{c}_{\texttt{bc}}\}, B = \{\texttt{a}^\texttt{u}, \beta^\texttt{u}_{\texttt{bc}}\}, R = \{(\texttt{a}, \texttt{b}), (\texttt{b}, \texttt{a}), (\texttt{a}^\texttt{c}, \texttt{a}), (\texttt{a}^\texttt{u}, \texttt{a}^\texttt{c}), (\texttt{b}, \alpha_{\texttt{bc}}), (\alpha_{\texttt{bc}}, \beta_{\texttt{bc}}), (\beta_{\texttt{bc}}, \texttt{c}), (\beta^\texttt{c}_{\texttt{bc}}, \beta_{\texttt{bc}}), (\beta^\texttt{u}_{\texttt{bc}}, \beta^\texttt{c}_{\texttt{bc}})\}, T = \emptyset\rangle$. For $\Lambda = \langle A \cup \{\texttt{a}^\texttt{u}\}, R \setminus \{(\beta^\texttt{u}_{\texttt{bc}}, \beta^\texttt{c}_{\texttt{bc}})\}\rangle \in comp(farg(\Delta))$, containing the uncertain argument $\texttt{a}^\texttt{u}$ but not $\beta^\texttt{u}_{\texttt{bc}}$, $af_\Delta(\Lambda) = \langle\{\texttt{a}, \texttt{b}, \texttt{c}\}, \{(\texttt{a}, \texttt{b}), (\texttt{b}, \texttt{a})\}\rangle \in comp(\Delta)$ that contains the uncertain argument $\texttt{a}$ and does not contain the uncertain attack $(\texttt{b}, \texttt{c})$. $\square$

Lemma 1. *For any iAF Δ and $\varphi \in \{arg, att, farg\}$, $af_\Delta : comp(\varphi(\Delta)) \to comp(\Delta)$ is a surjective function.*

The next theorem states the 'equivalence' between iAFs and the iAFs derived by applying the previous mappings.

Theorem 7. *Let $\Delta = \langle A, B, R, T\rangle$ be an iAF, $\sigma \in \{\texttt{gr}, \texttt{co}, \texttt{st}, \texttt{pr}, \texttt{sst}\}$, and $\varphi \in \{arg, att, farg\}$. Then,*

- $comp(\Delta) = \{af_\Delta(\Lambda) \mid \Lambda \in comp(\varphi(\Delta))\}$*, and*
- $\sigma(\Lambda') = \{E \cap (A \cup B) \mid \Lambda \in comp(\varphi(\Delta)) \wedge \Lambda' = af_\Delta(\Lambda) \wedge E \in \sigma(\Lambda)\} \; \forall \Lambda' \in comp(\Delta)$.

Thus, any iAF Δ is equivalent to an arg-iAF (resp. farg-iAF, att-iAF) Δ' in the sense that there is mapping between the completions of $\varphi(\Delta)$ and the completions of Δ, and for any pair of AFs for which the mapping holds, the two AFs have the same (modulo arguments added in the rewriting) set of σ-extensions, for $\sigma \in \{\texttt{gr}, \texttt{co}, \texttt{st}, \texttt{pr}, \texttt{sst}\}$. This result entails that arg-iAFs (resp. farg-iAF, att-iAF) have the same expressivity of general iAFs, though arg-iAFs (resp. farg-iAF, att-iAF) have a simpler structure. The results of Theorem 7 with those of Theorems 3 and 4 entail that the

	a	¬a	b	¬b	c	¬c	d	¬d
gr	0	1/2	0	1/2	0	0	**1/2**	**1/2**
co	1/6	2/3	1/6	2/3	0	1/6	**1/2**	**1/2**
st	0	1/2	1/2	0	0	1/2	0	1/2
pr	**1/4**	**3/4**	**1/4**	**3/4**	0	1/4	**1/2**	**1/2**
sst	**0**	**1**	**1/2**	**1/2**	0	1/2	**1/2**	**1/2**

Table 3: Acceptance probability for literals of Example 8. Bold denotes total literals, under the given semantics.

complexity results for the problems of checking totality, determinism, and functionality for iAFs in Table 2 also hold for each restricted form of iAFs (att-iAF, arg-iAF, f-arg-iAF).

Finally, as shown in the next section, the restricted forms of iAF allow to establish a tight connection between iAF and Probabilistic AF, and simplify the presentation as it suffices to focus on arg-iAF and an analogous form of restricted Probabilistic AF where only arguments are uncertain.

Probabilistic Acceptance

Probabilistic Argumentation Framework (PrAF) has been investigated in the recent years (Li, Oren, and Norman 2011; Rienstra 2012; Fazzinga, Flesca, and Parisi 2015, 2016; Fazzinga, Flesca, and Furfaro 2019; Alfano et al. 2020a). Incomplete AF is tightly connected to PrAF, as every completion of an iAF corresponds to a so-called *possible world* in PrAF. Here we highlight this relationship and relate acceptance problems in iAF, to probabilistic acceptance in PrAF.

W.l.o.g. we focus on PrAF where only arguments are uncertain (and attacks are certain, i.e. their probability is 1), since as shown in (Mantadelis and Bistarelli 2020) a PrAF with probabilities on arguments and attacks can be transformed into an equivalent PrAF where only arguments may have probability lower than 1. Here, an argument $a \in A$ is viewed as a probabilistic event that is independent from events associated with other arguments $b \in A$ (with $b \neq a$).

Analogously to what is said above, since any iAF is equivalent to an arg-iAF, w.l.o.g. in the following we consider arg-iAFs and denote them by triples $\langle A, B, R\rangle$ (i.e. omitting the empty set of uncertain attacks).

Definition 8 (PrAF)**.** A Probabilistic Argumentation Framework (PrAF) is a triple $\langle A, R, P\rangle$ where $\langle A, R\rangle$ is an *AF*, and P is a function assigning a non-zero probability value to every argument in A, that is, $P : A \rightarrow (0, 1]$.

Observe that assigning probability equal to 0 to arguments is useless. Basically, the value assigned by P to any argument a represents the probability that a actually occurs. Moreover, every attack (a, b) occurs with conditional probability 1, that is, a attacks b whenever both a and b occur.

The meaning of a PrAF is given in terms of *possible worlds*. Given a PrAF $\nabla = \langle A, R, P\rangle$, a possible world of ∇ is an AF $\Lambda = \langle A', R'\rangle$ such that $A' \subseteq A$ and $R' = R \cap (A' \times A')$. We use $pw(\nabla)$ to denote the set of all possible worlds of ∇. An *interpretation* for a PrAF $\nabla = \langle A, R, P\rangle$ is a probability distribution function (PDF) I over the set $pw(\nabla)$ of the possible worlds. Each $\Lambda = \langle A', R'\rangle \in pw(\nabla)$ is assigned by I probability $I(\Lambda)$, where:

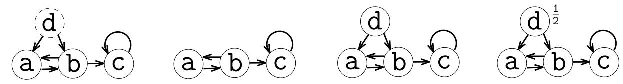

Figure 2: (From left to right:) iAF Δ of Example 8, its completions Λ_1 and Λ_2, and its derived PrAF Δ^p of Example 12.

$$I(\Lambda) = \prod_{a \in A} P(a) \cdot \prod_{a \in A \setminus A'} (1 - P(a)).$$

We now define a PrAF Δ^p encoding an iAF Δ.

Definition 9 (Derived PrAF)**.** Given an arg-iAF $\Delta = \langle A, B, R\rangle$, the PrAF derived from Δ is $\Delta^p = \langle A \cup B, R, P\rangle$, where $P\colon A \cup B \rightarrow \{\texttt{1/2}, \texttt{1}\}$ with $P(a) = \texttt{1}$ for $a \in A$ and $P(b) = \texttt{1/2}$ for $b \in B$.

It is easy to check that, given an arg-iAF $\Delta = \langle A, B, R\rangle$, for every $\Lambda \in pw(\Delta^p)$, $I(\Lambda)$ is equal to either 0 or $\frac{1}{2^{|B|}}$. As stated next, non-zero probability possible worlds of derived PrAF Δ^p one-to-one correspond to completions of Δ.

Proposition 3. *For any arg-iAF* Δ*,* $comp(\Delta) = \{\Lambda \mid \Lambda \in pw(\Delta^p) \wedge I(\Lambda) > 0\}$.

Example 12. Consider the arg-iAF Δ of Example 8 (see Figure 2). The derived PrAF $\Delta^p = \langle A \cup B, R, P\rangle$, with $P(x) = \texttt{1}$ for $x \in \{\texttt{a}, \texttt{b}, \texttt{c}\}$ and $P(\texttt{d}) = \texttt{1/2}$, is shown in Figure 2. There are only two possible worlds with probability greater than 0: $\Lambda_1 = \langle A, R \setminus \{(\texttt{d}, \texttt{a}), (\texttt{d}, \texttt{b})\}\rangle$ and $\Lambda_2 = \langle A \cup B, R\rangle$ with $I(\Lambda_1) = I(\Lambda_2) = \texttt{1/2}$. □

Given a PrAF, the *probabilistic acceptance* provides the probability that a given goal is accepted (Alfano et al. 2020a). Specifically, given a PrAF ∇, a semantics σ, and a goal literal g, the probability that g is accepted can be computed by associating to every extension $E \in \sigma(\Lambda)$, with $\Lambda \in pw(\nabla)$, a probability $Pr(E, \Lambda, \sigma)$ so that $\sum_{E \in \sigma(\Lambda)} Pr(E, \Lambda, \sigma) = 1$ (the sum of the probabilities of the σ-extensions of Λ is equal to 1). More in detail, a PrAF Δ^p derived from a given iAF Δ is considered, and a PDF over the set $\sigma(\Lambda)$ of the extensions of each possible world $\Lambda \in pw(\Delta^p)$ is required. This means that the condition $\sigma(\Lambda) \neq \emptyset$ must hold. To ensure this, *in the rest of this section, if not specified otherwise, whenever we write semantics* σ *and (arg-)iAF* Δ *we mean that either i)* $\sigma \in \{\texttt{gr}, \texttt{co}, \texttt{pr}, \texttt{sst}\}$ *and* Δ *is an (arg-)iAF without restrictions or ii)* $\sigma = \texttt{st}$ *and* Δ *is odd-cycle free*; in the latter case, the existence of an extension is guaranteed since $\texttt{st}(\Lambda) = \texttt{sst}(\Lambda) = \texttt{pr}(\Lambda) \neq \emptyset$ for all $\Lambda \in pw(\Delta^p)$.

Definition 10 (Probabilistic Acceptance)**.** Given an arg-iAF $\Delta = \langle A, B, R\rangle$ and a literal $g \in A^* \cup B^*$, the probability $PrA^{\sigma}_{\Delta}(g)$ that g is acceptable w.r.t. semantics σ is:

$$PrA^{\sigma}_{\Delta}(g) = \sum_{\substack{\Lambda \in pw(\Delta^p) \wedge \\ E \in \sigma(\Lambda) \wedge g \in E}} I(\Lambda) \cdot Pr(E, \Lambda, \sigma)$$

where $Pr(\cdot, \Lambda, \sigma)$ is a PDF over the set $\sigma(\Lambda)$.

Hereafter, we consider the uniform PDF assigning to every σ-extension of Λ the same probability ($1/n$ where n is the number of σ-extensions). Our results still hold for other PDFs over $\sigma(\Lambda)$, such as that used in (Alfano et al. 2020a).

Given an arg-iAF Δ and a literal g, the problem of computing the value $PrA^{\sigma}_{\Delta}(g)$ (under a given semantics σ) is denoted by PrA_σ, or simply PrA whenever σ is understood. We recall that PrA_σ is defined after choosing an arbitrary but fixed PDF over the set of extensions. As stated next, PrA_σ is $\mathsf{FP}^{\#\mathsf{P}}$-hard, regardless of the chosen PDF and semantics σ.

Theorem 8. *PrA_σ is $\mathsf{FP}^{\#\mathsf{P}}$-hard, even for acyclic arg-iAF and for any chosen PDF.*

The following propositions highlight the relations between iAFs and PrAFs (with uniform PDF over extensions).

Proposition 4. *For any goal g, we have that:*

- *$PCA_\sigma(\Delta, g)$ is false iff $PrA^\sigma_{\Delta^p}(g) = 0$;*
- *$NSA_\sigma(\Delta, g)$ is true iff $PrA^\sigma_{\Delta^p}(g) = 1$.*

The connection between iAF and PrAF is also investigated in (Baumeister et al. 2021), where the PrAF associated to an iAF assigns a probability in $(0, 1)$ to each uncertain argument. Moreover, PCA, PSA, NCA, and NSA are related to the concept of *probabilistic credulous/skeptical acceptance* (Fazzinga, Flesca, and Furfaro 2018), which is different from that of probabilistic acceptance of Definition 10. Indeed, the probability that an argument g is credulously (resp. skeptically) accepted is the sum of the probabilities of the possible worlds where g is credulously (resp. skeptically) accepted, according to a given semantics σ. Hence, the probability of a world Λ is added to the summation iff g belongs to at least one (resp. every) σ-extension of Λ. In contrast, with the aim of offering a more granular approach, Definition 10 uses the probabilities assigned to σ-extensions by the PDF $Pr(\cdot, \Lambda, \sigma)$. For instance, taking the PrAF Δ^p of Example 12 (see Figure 2), under the complete semantics the probabilistic skeptical (resp. credulous) acceptance of $\mathtt{b}$ is $\mathtt{0}$ (resp. $\mathtt{1/2}$), while $PrA^{\mathtt{co}}_{\Delta^p}(\mathtt{b}) = \mathtt{1/6}$ as $\mathtt{b}$ belongs to one of three extensions of one of the two worlds (cf. Table 3). Although the conditions of Proposition 4 are similar to those identified in (Baumeister et al. 2021), they refer to different notions of probabilistic acceptance. In fact, while probabilistic skeptical and credulous acceptance define an interval, Definition 10 provides a precise value in that interval.

The next proposition considers acyclic arg-iAFs, a subclass of odd-cycle free iAFs.

Proposition 5. *Let $\Delta = \langle A, B, R\rangle$ be an acyclic arg-iAF and g a goal. It holds that:*

- *$PSA_\sigma(\Delta, g) \equiv PCA_\sigma(\Delta, g)$ and $NSA_\sigma(\Delta, g) \equiv NCA_\sigma(\Delta, g)$;*
- *$PSA_\sigma(\Delta, g)$ is true iff $PrA^\sigma_{\Delta^p}(g) \geq \frac{1}{2^{|B|}}$;*
- *$NSA_\sigma(\Delta, g)$ is false iff $PrA^\sigma_{\Delta^p}(g) \leq 1 - \frac{1}{2^{|B|}}$.*

Our last result relates the satisfaction of properties (e.g. totality) to probabilistic acceptance.

Theorem 9. *A goal g is:*

- *total iff $PrA^\sigma_\Delta(g) + PrA^\sigma_\Delta(\neg g) = 1$;*
- *deterministic iff $\forall \Lambda \in comp(\Delta)$, $\sum_\alpha Pr(E, \Lambda, \sigma) = 1$, where either (i) $\alpha = E \in \sigma(\Lambda) \wedge g \in E$ or (ii) $\alpha = E \in \sigma(\Lambda) \wedge \neg g \in E$ or (iii) $\alpha = E \in \sigma(\Lambda) \wedge (g \notin E \wedge \neg g \notin E)$.*

Example 13. Consider again the iAF of Example 8. Assuming a uniform distribution for the probabilities of extensions, the probabilistic acceptance of literals is as reported in Table 3 (where bold means that the literal of the column is total under the semantics of the row). □

Conclusions and Future Work

We have presented the total, deterministic and functional properties for AFs and iAFs, and provided complexity bounds for the problems of checking whether these properties hold under five well-known argumentation semantics. We also identified equivalent forms of iAFs (in terms of extensions), investigated possible and necessary variants of the credulous and skeptical acceptance problems for iAFs, and addressed the possible and necessary verification problems under semi-stable semantics (left open in previous work). We have also explored the relationship between acceptance problems in iAFs and probabilistic acceptance in PrAFs.

As iAF generalizes Partial AF (Coste-Marquis et al. 2007; Baumeister et al. 2021), allowing to model the problem of merging AFs representing subjective views of several agents, we believe that checking totality, determinism, and functionality in such context may help in understanding agents' points of view.

As future work we plan to extend our investigation to other semantics, such as ideal (Dung, Mancarella, and Toni 2007) and eager semantics (Caminada 2007). We also plan to consider other notions of acceptance that, as totality/determinism/functionality, require the existence of an extension (e.g. skeptical acceptance under stable semantics requiring non-emptyness (Dunne and Wooldridge 2009)).

We envisage that SAT-based approaches for computing credulous and skeptical acceptance (Järvisalo 2018) could be used to define algorithms for checking functionality, totality and determinism since these properties can be defined in terms of credulous and skeptical acceptance for AFs. More elaborated solutions need to be investigated for iAF.

Considering the connections between iAFs and Control AFs (Neugebauer, Rothe, and Skiba 2021; Gaignier et al. 2021; Dimopoulos, Mailly, and Moraitis 2018; Mailly 2020; Niskanen, Neugebauer, and Järvisalo 2020), we plan to investigate totality, determinism, and functionality in that context. Finally, given the inherent dynamic nature of argumentation and the typical high computational complexity of most of the reasoning tasks (Alfano et al. 2020a), there have been efforts toward the investigation of incremental techniques that use AF solutions (e.g. extensions, skeptical acceptance) at time t to recompute updated solutions at time $t + 1$ after that an update (e.g. adding/ removing an attack) is performed (Greco and Parisi 2016a,b; Alfano, Greco, and Parisi 2017, 2019, 2021; Alfano and Greco 2021; Doutre and Mailly 2018). These approaches have been extended to argumentation frameworks more general than AFs (Alfano, Greco, and Parisi 2020; Alfano et al. 2020b; Alfano, Greco, and Parisi 2018a,b; Alfano et al. 2021a, 2018); in (Odekerken, Borg, and Bex 2020) *stability* in a dynamic structured argumentation setting is studied, where so-called future setups can be seen as completions of an iAF. Following these lines of research, we plan to investigate incremental techniques for checking totality, determinism, and functionality of goal arguments in dynamic iAFs.

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