An Algorithmic Introduction to Savings Circles

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Abstract

Rotating savings and credit associations (roscas) are informal financial organizations common in settings where communities have reduced access to formal financial institutions. In a rosca, a fixed group of participants regularly contribute sums of money to a pot. This pot is then allocated periodically using lottery, aftermarket, or auction mechanisms. Roscas are empirically well-studied in economics. They are, however, challenging to study theoretically due to their dynamic nature. Typical economic analyses of roscas stop at coarse ordinal welfare comparisons to other credit allocation mechanisms, leaving much of roscas' ubiquity unexplained. In this work, we take an algorithmic perspective on the study of roscas. Building on techniques from the price of anarchy literature, we present worst-case welfare approximation guarantees. We further experimentally compare the welfare of outcomes as key features of the environment vary. These cardinal welfare analyses further rationalize the prevalence of roscas. We conclude by discussing several other promising avenues.

1 Introduction

Rotating saving and credit associations (roscas) are financial institutions common in low- and middle-income nations, as well as immigrant and refugee populations around the world. In a rosca, a group of individuals meet regularly for a defined period of time. At each meeting, members contribute a sum of money into a pot, which is then allocated via some mechanism, such as a lottery or an auction. Recipients often use this money to purchase durable goods (e.g., farming equipment, appliances, and vehicles), to buffer shocks (e.g. an unexpected medical expense), or to pay off loans. Roscas often exist outside of legal frameworks and do not typically have a central authority to resolve disputes or enforce compliance. Instead, they provide a decentralized mechanism for peer-to-peer lending, where members who receive the pot earlier borrow from those who receive it later. They also create a structure for mutual support and community empowerment.

Roscas are used in over 85 countries and are especially prevalent in contexts where communities have reduced access to formal financial institutions (Aredo 2004; Bouman 1995a; Klonner 2002; La Ferrara 2002; Raccanello and Anand 2009). Roscas account for about one-half of Cameroon's national savings. Likewise, over one in six households in Ethiopia's highlands participate in *ekub*, the region's variant of roscas Bouman (1995a). Due to their ability to provide quick, targeted support within communities, roscas and other mutual aid organizations often play an instrumental role when communities experience shocks and disasters (Chevée 2021; Mesch et al. 2020; Travlou 2020).

Roscas are well-studied in the economics literature, with economic theory on the subject pioneered by Besley, Coate, and Loury (1993); Kovsted and Lyk-Jensen (1999); Kuo (1993) (see supplement for further related works). This line of work seeks to explain how roscas act as insurance, savings, and lending among members. While such studies have deepened our understanding of roscas, they are typically constrained in two main ways. First, the standard economic approach solves exactly for equilibria, which can be especially difficult due to the dynamic nature of roscas. Second, much of the existing theory focuses on coarse-grained comparisons between the welfare of roscas and other mechanisms for allocating credit. In part due to these coarse comparisons, this work often concludes that roscas allocate credit suboptimally, leaving open the question of why roscas are prevalent in practice.

In this work, we initiate an algorithmic study of roscas. Viewing roscas through the lens of approximation and using techniques from the price of anarchy literature, we study the welfare properties of rosca outcomes without directly solving for them. We specifically quantify the allocative efficacy of roscas: *how well do roscas coordinate saving and lending among participants with heterogeneous investment opportunities*? We show roscas enable a group's lending and borrowing in a way that approximately maximizes the groups' total utility. We do so under a wide range of assumptions on both participants' values for investment and the mechanisms used for allocating the pots. This robustness may provide one explanation for their prevalence.

Our work builds on the saving and lending formulation of Besley, Coate, and Loury (1994). We assume each participant seeks to purchase an investment, such as a durable good, but can only do so upon winning the rosca's pot. We analyze the welfare properties of typical pot allocation mechanisms, such as "swap roscas," where the participants are given an initial (e.g. random) allocation and then swap positions in an after-market through bilateral trade agreements. We also study the price of anarchy in auction-based roscas where, during each meeting, participants bid to decide a winner among those who have not yet received a pot. During each round, participants must weigh the value of investing earlier against the utility loss from spending to win that round.

Our technical contributions are as follows: For swap roscas, we prove that all outcomes guarantee at most a factor 2 loss. For auction-based roscas, we give full-information price of anarchy results: we study second-price sequential roscas and give a price of anarchy of 3 under a standard no-overbidding assumption. For first-price sequential roscas, we provide a ratio of $(2e + 1)e/(e - 1)$. Our work provides new applications of and extensions to the *smoothness* framework of Syrgkanis and Tardos (2013). However, due to the round-robin (i.e you can only win once) property of rosca allocations and the fact that all payments are redistributed to members, standard smoothness arguments do not immediately yield bounds in our setting. Via a new sequential composition argument, we show that roscas based on smooth mechanisms are themselves smooth, and we go beyond smoothness to bound the distortionary impact of redistributed payments on welfare. Our above results hold under the well-studied assumption of *quasilinear* utility for money. We extend most of our theoretical results to nonlinear utility functions, and also use simulations to consider the impact of nonlinear utilities in several natural families of swap roscas.

Overall, this work aims to provide greater exposure for mutual aid organizations more generally and roscas to the algorithms community. In doing so, we present a case study showing how algorithmic game theory can provide a useful perspective for further understanding fundamental questions related to these financial organizations. Given their prevalence and efficacy, insights into roscas can help inform the design of other safety net programs, especially for communities that already commonly use roscas. As new technologies are introduced in low-access contexts, also, the need to understand existing, prevalent financial organizations is even more pressing. We close the paper with discussion of promising research directions.

2 Model and Preliminaries

A rosca consists of n participants and takes place over n discrete and fixed time periods, or *rounds*. During each round three things occur: (1) each participant contributes an amount p_0 into the rosca common pot, (2) a winner for the pot is decided among those who have not yet won, and (3) the winning participant is allocated the entire pot worth np_0. Typically, the contributions p_0 are decided ex ante during the formation of the rosca. As is common in previous literature, we will not model the selection process for p_0, but instead take it as given (c.f. Klonner 2001).

With the rosca contribution p_0 fixed, we can cast roscas as an abstract multi-round allocation problem, where every participant is allocated exactly one pot, and each pot is allocated to exactly one participant, illustrated in Alg 1. Each participant's value for the allocations is described by a real-valued vector $\mathbf{v}_i = (v_i^1, \dots, v_i^n)$, with v_i^t representing participant i's value for winning the pot in round t and having access to that money at that time. We denote allocations by $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_n)$, where $\mathbf{x}_i = (x_i^1, \dots, x_i^n)$ is an indicator vector, and $x_i^t = 1$ if and only if participant i receives the pot in round t. Based on a common observation from previous literature, we can further assume that values for allocation are non-increasing over time: i.e., for $t < t'$ and any i, $v_i^t \geq v_i^{t'}$. This follows if rosca funds are used to make lumpy investments, e.g., in a durable good, as is common in practice (Besley, Coate, and Loury 1993, 1994; Kovsted and Lyk-Jensen 1999; Klonner 2008). Participants prefer to own the good earlier rather than later, though different participants' values for owning the good earlier may vary.

Algorithm 1: Rosca Multi-Round Allocation

Constants: n: the number of participants and rounds in the rosca. p_0: amount contributed by each participant to the pot in each round of the rosca.
Inputs: Valuations $\mathbf{v}$, where v_i^t indicates the value to participant i of winning the pot in round t. $Alloc$ an allocation mechanism.

For each round $t \in \{1, 2, \dots, n\}$

1. Each participant contributes p_0 into the pot
2. $Alloc$ selects the winning participant (who has not yet won a round)
3. The winning participant receives the pot worth np_0
4. *Optional*: Some participants make payments based on $Alloc$, which are redistributed to the others as rebates.

2.1 Roscas with Payments

A variety of different pot allocation mechanisms are common in practice (see Ardener 1964; Bouman 1995b). This work considers roscas where participants make payments to influence their allocations, and assumes as a first-order approximation that participants are rational. Payments in roscas take the form $\mathbf{p} = (\mathbf{p}_1, \dots, \mathbf{p}_n)$, where $\mathbf{p}_i = (p_i^1 \dots, p_i^n)$. As participants' abilities to save money over time are typically limited, we assume participants' utilities are additively separable across rounds, but possibly nonlinear in money. That is, participant i with value vector $\mathbf{v}_i$ has utility for allocation $\mathbf{x}_i$ and payments $\mathbf{p}_i$ given by

$$u_i^{\mathbf{v}_i}(\mathbf{x}_i, \mathbf{p}_i) = \mathbf{x}_i \cdot \mathbf{v}_i - \sum\nolimits_t C(p_i^t),$$

for some disutility function C that is both increasing and satisfies $C(0) = 0$. In a given round, p_i^t could be positive, if the allocation mechanism requires i to make payments, or negative, if a different participant's payments are redistributed to i. We refer to the latter as *rebates*, and assume all payments are redistributed each round, i.e., $\sum_i p_i^t = 0$ for all t.

A participant who makes positive payments in round t has less money to spend in round t, and one who receives rebates in the form of negative payments has more to spend. The function $C(\cdot)$ describes participants' preferences for these changes in wealth. A more precise interpretation of $C(p_i^t)$ is as follows: assume that each participant i has a per-round income of w. Without participating in the rosca, they would receive a utility $U(w)$ from consumption of that income, for some increasing consumption utility function U. Upon contributing p_0 into the rosca pot each round, the participant's baseline consumption utility is $U(w - p_0)$. If the rosca's allocation procedure requires additional payments (or distributes rebates) of p_i^t, a participant's utility from consumption becomes $U(w-p_0-p_i^t)$. The disutility function C then represents the participant's difference in utility for consumption,

$$C(p) = U(w - p_0) - U(w - p_0 - p),$$

which is increasing.

A large body of anthropological and empirical work on roscas shows that participants in the same rosca tend to have similar economic circumstances (see Ardener 1964; Aredo 2004; Mequanent 1996). So, following the theory literature, we assume U and w (and hence C) are identical across participants, even if the value for receiving the pot differ between participants (Besley, Coate, and Loury 1994; Kovsted and Lyk-Jensen 1999; Klonner 2001). It is typical to assume consumption utility U is weakly concave, and hence C is weakly convex (Anderson and Baland 2002; Klonner 2003b, 2001). The special case of *quasilinear utilities*, where $C(p) = p$, is especially well-studied in the algorithmic game theory literature.

To measure allocative performance of a rosca, we study the participants' total utility:

$$\textsc{Wel}^{\mathbf{v}}(\mathbf{x}, \mathbf{p}) = \sum\nolimits_i u_i^{\mathbf{v}_i}(\mathbf{x}_i, \mathbf{p}_i).$$

Following the interpretation of C in terms of consumption utility U, $\textsc{Wel}^{\mathbf{v}}(\mathbf{x}, \mathbf{p})$ represents the gain in utility to all participants for a given allocation $\mathbf{x}$ and payments $\mathbf{p}$, above the baseline total utility of $n^2U(w - p_0)$, obtained by each of the n participants obtaining utility $U(w - p_0)$ for n rounds. Among all possible matchings $\mathbf{x}$ and payment profiles $\mathbf{p} \in \mathbb{R}^{n\times n}$, the optimal welfare-outcome is given by the maximum-weight matching $\mathbf{x}^* = \operatorname{argmax}_{\mathbf{x}} \sum_i \mathbf{x}_i \cdot \mathbf{v}_i$ and payments $\mathbf{p}^* = (0, \ldots, 0)$, whose welfare is denoted $\mathrm{OPT}(\mathbf{v}) = \textsc{Wel}^{\mathbf{v}}(\mathbf{x}^*, \mathbf{p}^*)$.

To quantify the inefficiency of a rosca outcome $(\mathbf{x}, \mathbf{p})$, we study the approximation ratio $\mathrm{OPT}(\mathbf{v})/\textsc{Wel}^{\mathbf{v}}(\mathbf{x}, \mathbf{p})$. When rosca outcomes are equilibria of auctions, as in Section 3, this ratio is also known as the price of anarchy (PoA).

Roadmap. The remainder of this paper proceeds as follows: In Section 3, we prove a constant-approximation for auction roscas. We do the same for swap roscas in Section 4. Both sets of results focus on quasilinear utilities, where $C(p) = p$, and hence all welfare loss comes from allocative inefficiency. We extend these results to nonlinear utilities in the supplement. In Section 4, we further conduct experiments to study the impact of nonlinear utility on swap rosca welfare. We give directions for future work in Section 5.

3 Auction Roscas

Auctions are a common mechanism for allocating pots in roscas (Ardener 1964; Bouman 1995a; Klonner 2003a). Two major sources of variety in auction roscas are (1) when bids are solicited from participants and (2) the type of auction run for the bidding process. The bidding may occur either at the beginning, in which case a single (up-front) auction determines the full schedule of pot allocations, or sequentially, in which case a separate auction is held each period to determine the allocation for the corresponding pot. We consider sequential first- and second-price (equivalently, ascending- and descending-price) auctions, as well as up-front all-pay-style auctions. Payments are typically redistributed as rebates among all of the non-winning participants.

The fact that outcomes depend on participants' bidding behavior complicates our analysis. We assume participants play a *Nash equilibrium (NE)* of the rosca's auction game. That is, their bidding strategy maximizes their utility given the bidding strategies of other participants. Our analysis will use the *smoothness* framework of Syrgkanis and Tardos (2013), along with new arguments to handle rosca-specific obstacles. We assume participants have quasilinear utilities.

3.1 Proof Template: Up-Front Roscas

We begin our analysis by considering roscas with up-front bidding. In an up-front rosca, each participant i submits a bid b_i at the beginning of the rosca. Participants pay their bids, and are then assigned pots in decreasing order of their bids, with the highest participant receiving the pot in round 1, and so on. Each participant i's payments are redistributed evenly among the other participants in the form of reduced per-period payments into the rosca. Under quasilinear utilities, it is not relevant to the participants' utilities what round payments are made; the only relevant outcome is total payments, which we write as $p_i = \sum_t p_i^t$ when context allows. We can further assume per-period payments remain fixed and that the participants receive the redistributed payments up-front in the form of a rebate. We decompose the participants' total payments into their *gross payments* $\hat{p}_i$ and *rebates* $\hat{r}_i$, with $p_i = \hat{p}_i - \hat{r}_i$. Formally:

Definition 1. *In an* up-front rosca *with quasilinear participants, each participant i submits a bid b_i, with $\mathbf{b} = (b_1, \ldots, b_n)$. Let r_i denote the rank of participant i's bid. Allocations are $x_i^t(\mathbf{b}) = 1$ if $t = r_i$ and 0 otherwise. Participant i's gross payment is $\hat{p}_i(\mathbf{b}) = b_i$, and their rebate is $\hat{r}_i(\mathbf{b}) = \sum_{i'\neq i} b_{i'}/(n-1)$.*

Our auction rosca analyses all follow from a two-step argument. First, we use or modify the smoothness framework of Syrgkanis and Tardos (2013) to obtain a tradeoff between participants' utilities and their gross payments. Without rebates, typical auction analyses conclude by noting that high payments imply high welfare. However, because gross payments in roscas are redistributed, it could happen that both gross payments and rebates are high, but welfare is low. Our second step is to rule out this problem. For up-front roscas, we can demonstrate both steps simply.

The first step follows from Lemma A.20 of Syrgkanis and Tardos (2013):

Lemma 1. *With quasilinear participants, any Nash equilibrium $\mathbf{b}$ of any up-front rosca with values $\mathbf{v}$ satisfies*

$$\sum\nolimits_i u_i^{\mathbf{v}_i}(\mathbf{b}) \geq \tfrac{1}{2}\mathrm{OPT}(\mathbf{v}) - \sum\nolimits_i \hat{p}_i(\mathbf{b}). \qquad (1)$$

The left hand side of (1) is the equilibrium welfare. It therefore suffices for the second step to upper bound the gross payments on the right hand side.

Lemma 2. *Let* $\mathbf{b}$ *be a Nash equilibrium of an up-front rosca with quasilinear participants and values* $\mathbf{v}$*. Then, for any participant* i*,* $\hat{p}_i(\mathbf{b}) \leq \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{b})$*.*

Proof. Assume for some i that $\hat{p}_i(\mathbf{b}) > \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{b})$. Then participant i must be overbidding. They could improve their utility by bidding 0, which, in an up-front rosca, does not change their rebates: $\hat{r}_i(0, \mathbf{b}_{-i}) = \hat{r}_i(\mathbf{b}) > \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{b}) - \hat{p}_i(\mathbf{b}) + \hat{r}_i(\mathbf{b})$. □

Since $\sum_i \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{b})$ is equal to the equilibrium welfare, Lemmas 1 and 2 together imply the following.

Theorem 1. *With quasilinear participants, every Nash equilibrium of an up-front rosca has PoA at most* 4*.*

3.2 Sequential Roscas

We now consider roscas with separate sequentially-held first- or second-price auctions for each pot as opposed to the single-auction format from the previous section.

Definition 2. *A* first-price rosca *runs a first-price auction in each round. That is, if the highest-bidding participant in round* t *among those who have not yet won is participant* i^**, with bid* $b^t_{i^*}$*, then* $x^t_{i^*} = 1$*,* $x^t_i = 0$ *for all other participants* i*. The gross payments are* $\hat{p}^t_{i^*} = b^t_{i^*}$ *and* $\hat{p}^t_i = 0$ *otherwise. The rebates are* $\hat{r}^t_i = b^t_{i^*}/(n-1)$ *for all* $i \neq i^*$*.*

Definition 3. *A* second-price rosca *runs a second-price auction in each round. That is, if the highest-bidding participant in round* t *among those who have not yet won is* i^**, with second-highest bid* $b^t_{(2)}$*, then* $x^t_{i^*} = 1$*,* $x^t_i = 0$ *for all other participants* i*. Gross payments are* $\hat{p}^t_{i^*} = b^t_{(2)}$ *and* $\hat{p}^t_i = 0$ *otherwise. Rebates are* $\hat{r}^t_i = b^t_{(2)}/(n-1)$ *for all* $i \neq i^*$*.*

Sequential auctions require a monitoring scheme in which the auctioneer discloses information about participants' bids after each round. Our results will hold for any deterministic monitoring scheme. A key subtlety is that participants' actions are now *behavioral strategies*: that is, at each stage, participants observe the disclosed history of play so far and can condition their future bids on this history. We denote the vector of behavioral strategies by $\mathbf{a} = (a_1, \ldots, a_n)$, and denote by b^t_i participant i's bid in a round t.

As with up-front roscas, we first derive a tradeoff between utility and gross payments, and second consider the impact of rebates. The sequential format complicates both steps. Our first step will follow from a novel composition argument, where we show that both first- and second-price roscas inherit a tradeoff from their single-item analogs. For second-price roscas, a standard no-overbidding assumption then bounds the auction's rebates and implies a welfare bound. For first-price roscas, we give a more involved analysis that bounds overbidding and yields an unconditional guarantee. Overbidding can both occur in equilibrium and harm welfare, so such an analysis is necessary.

Observe that, first- and second-price roscas can be thought of as the sequential composition n single-item auctions, with a rule excluding past winners. Formally:

Definition 4 (Round-Robin Composition)**.** *Given single-item auction* M*, the* n-item round-robin composition *of* M *is a multi-round allocation mechanism for* n *items using the following procedure: Each round* t*, each participant* i *who has not yet been allocated an item submits a bid* b^t_i*. The mechanism then runs* M *among the remaining participants to determine the allocation and payments for that round.*

The following definition of *smoothness*, adapted from Syrgkanis and Tardos (2013), lets us characterize both first- and second-price roscas with the same framework. For our purposes, it applies to any auction where in round t, each bidder who has not yet won submits a real-valued bid b^t_i, which we term *sequential single-bid auctions*. Note that this includes single-item auctions. We will show that smoothness of single-item auctions implies smoothness of their round-robin composition.

Definition 5. *Let* M *be a sequential single-bid auction. We say* M *is* (λ, μ_1, μ_2)*-smooth if for every value profile* $\mathbf{v}$ *and action profile* $\mathbf{a}$*, there exists a randomized action* $a^*_i(a_i, \mathbf{v})$ *for each* i *such that:*

$$\sum\nolimits_i (\mathbf{v}_i \cdot \mathbf{x}_i(a^*_i(a_i, \mathbf{v}), \mathbf{a}_{-i}) - p_i(a^*_i(a_i, \mathbf{v}), \mathbf{a}_{-i}) \geq \lambda \mathrm{OPT}(\mathbf{v}) - \mu_1 \sum\nolimits_i \hat{p}_i(\mathbf{a}) - \mu_2 \sum\nolimits_i B_i(\mathbf{a}),$$

where $B_i(\mathbf{a})$ *is* i*'s bid in the round where they win, or* 0 *if no such round exists.*

Syrgkanis and Tardos (2013) show that single-item first-price and second-price auctions are $(1 - 1/e, 1, 0)$-smooth, and $(1, 0, 1)$-smooth, respectively. However, the smoothness result they prove for a form of sequential composition fails to hold for round-robin composition, due to the cardinality constraint on allocations as in our setting. Here, we instead give a new composition argument tailored specifically to the rosca setting, that relies on values decreasing in time. Our composition result follows the following useful definition:

Definition 6. *A single-item mechanism* M *with allocation rule* $\mathbf{x}$ *and payments* $\mathbf{p}$ *is* strongly individually rational (IR) *if (1) for every profile of actions* $\mathbf{a}$*,* $x_i(\mathbf{a}) = 0$ *only if* $\hat{p}_i(\mathbf{a}) = 0$*, and (2) there exists an action* $\perp$ *such that for all* i *and* $\mathbf{a}_{-i}$*,* $\hat{p}_i(\perp, \mathbf{a}_{-i}) = 0$*.*

Lemma 3. *Let* M *be a strongly individually-rational single-item mechanism. If* M *is* (λ, μ_1, μ_2)*-smooth for* $\lambda \leq 1$ *and* $\mu_1, \mu_2 \geq 0$*, then its round-robin composition is* $(\lambda, \mu_1 + 1, \mu_2)$*-smooth as long as* $v^t_i \geq v^{t+1}_i$ *for all* i *and* t*.*

Our proof of this lemma, presented in the supplement, augments the main idea from the Syrgkanis and Tardos (2013) composition result with ideas from Kesselheim, Kleinberg, and Tardos (2015), who consider smoothness of non-sequential mechanisms for cardinality-constrained allocation environments. As a corollary of Lemma 3, we obtain that first- and second-price roscas are respectively $(1 - 1/e, 2, 0)$ and $(1, 1, 1)$-smooth.

We next analyze the impact of rebates. If no participant overbids, then payments (and hence rebates) are necessarily bounded by values, and we obtain a similar conclusion to Lemma 2. Moreover, we show in the supplement that a no-overbidding assumption is necessary for second-price roscas, as is often the case for auctions with second-price payments. The assumption we require is as follows:

Definition 7. *Action profile* $\mathbf{a}$ *satisfies* no-overbidding *if* $B_i(\mathbf{a}) \leq \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{a})$ *for every participant i.*

Theorem 2. *Let* M *be a strongly IR, single-item auction that is* (λ, μ_1, μ_2)*-smooth, with* $\lambda \leq 1$*. With quasilinear participants, every no-overbidding Nash equilibrium of the corresponding auction rosca with rebates has PoA at most* $(2 + \mu_1 + \mu_2)/\lambda$.

Proof. Lemma 3 implies that the rosca is $(\lambda, 1 + \mu_1, \mu_2)$-smooth before rebates. We can therefore write:

$$\begin{aligned}
\sum\nolimits_i u_i^{\mathbf{v}_i}(\mathbf{a}) &\geq \sum\nolimits_i u_i^{\mathbf{v}_i}(a_i^*, \mathbf{a}_{-i}) \\
&\geq \sum\nolimits_i (\mathbf{v}_i \cdot \mathbf{x}_i(a_i^*, \mathbf{a}_{-i}) - \hat{p}_i(a_i^*, \mathbf{a}_{-i})) \\
&\geq \lambda \mathrm{OPT}(\mathbf{v}) - (1 + \mu_1) \sum\nolimits_i \hat{p}_i(\mathbf{a}) \\
&\qquad - \mu_2 \sum\nolimits_i B_i(\mathbf{a}) \\
&\geq \lambda \mathrm{OPT}(\mathbf{v}) - (1 + \mu_1 + \mu_2) \sum\nolimits_i B_i(\mathbf{a}) \\
&\geq \lambda \mathrm{OPT}(\mathbf{v}) - (1 + \mu_1 + \mu_2) \sum\nolimits_i \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{a})
\end{aligned}$$

Since both $\sum_i u_i^{\mathbf{v}_i}(\mathbf{a})$ and $\sum_i \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{a})$ are equal to equilibrium welfare, the result follows. □

Corollary 1. *For quasilinear participants, any Nash equilibrium of the first-price rosca satisfying no-overbidding has PoA at most* $3e/(e-1)$.

Corollary 2. *For quasilinear participants, any Nash equilibrium of the second-price rosca satisfying no-overbidding has PoA at most* 3.

3.3 Relaxing No-Overbidding

The no-overbidding assumption in the previous section rules out behavior where participants overbid in early rounds to induce others to bid high in later rounds, thereby resulting in high rebates. When this behavior is extreme, participants' payments could conceivably far exceed their values, which in turn complicates the smoothness-based approach. The following example gives a Nash equilibrium of a first-price rosca where overbidding leads to welfare loss.

Example 1. *Consider three participants, with* $\mathbf{v}_1 = (1, 0, 0)$*,* $\mathbf{v}_2 = (2, 2, 0)$*, and* $\mathbf{v}_3 = (2, 2, 0)$*. The following behavioral strategies form a Nash equilibrium. Participant 1 bids 2 in round 1. Participants 2 and 3 bid 1 in round 1. If participant 1 bids less than 2 in round 1, participants 2 and 3 bid 0 in round 2. Otherwise, they bid 2. The optimal welfare is then* 4*, but the equilibrium welfare is* 3.[1]

Despite the loss exhibited in Example 1, we can obtain a constant price of anarchy for first-price roscas without an overbidding assumption. Lemma 4 below shows that overbidding cannot drive payments much higher than equilibrium welfare. The lemma extends the following logic: In equilibrium, the participant who wins in the final round has no competition, and is therefore making zero payments. Consequently, the participant who wins in the second-to-last round cannot expect any rebates from round n, and therefore has no incentive to overbid. This, in turn, limits the rebates due the participant who wins the round before that, and so on. These limits on rebates limit the extent of overbidding that might occur. Throughout this section, we index participants such that in round t, the winner is participant t.

Lemma 4. *Fix a Nash equilibrium of a first-price rosca. Then:*

$$\hat{p}_t^t \leq v_t^t + \tfrac{1}{n-1} \sum_{t'=t+1}^{n} v_{t'}^{t'} (\tfrac{n}{n-1})^{t'-t-1}.$$

We provide the proof in the supplementary materials.

Theorem 3. *In any Nash equilibrium of the first-price rosca, the PoA is at most* $(2e+1)e/(e-1)$.

The result follows from summing the bounds on $\hat{p}_i(\mathbf{a})$ from Lemma 4, which can be arranged to obtain an upper bound of $e \sum_i \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{a})$ of the total gross payments. The theorem then follows from applying smoothness as before.

3.4 Extension to Nonlinear Utilities

In the supplement, we extend the price of anarchy results above beyond quasilinear utilities. With arbitrary convex cost for payments C, the setting comes to resemble hard budgets, for which the price of anarchy is known to be poor. We parametrize our results by upper (β) and lower (α) bounds on the slope C'. We give performance guarantees which scale linearly with the ratio β/α. For up-front roscas, our bounds are unconditional, while for sequential roscas, we assume an analogous no-overbidding condition to the quasilinear version.

4 Swap Roscas

Several common rosca formats eschew competition between participants in allocating pots. Examples include roscas based on random lottery allocations or those based on seniority or social status (Anderson, Baland, and Moene 2009; Kovsted and Lyk-Jensen 1999). To improve total welfare, it is common practice for participants to engage in an *aftermarket* by buying or selling their assigned allocations when it is mutually beneficial, i.e., by swapping rounds in the rosca (Mequanent 1996).

In this section, we formally define these swap roscas and show that, for participants with quasilinear utilities ($C(p) = p$), this aftermarket is guaranteed to converge to an outcome that yields at least half of the optimal welfare. We then present experimental results showing that this guarantee is often better, even for strictly convex C.

4.1 Theoretical Analysis

As is common in the literature and in practice, we assume that the aftermarket occurs via a series of two-agent swaps (Mequanent 1996; Bouman 1995b; Ardener 1964). We assume these swaps can occur at any round t. We denote by $\mathbf{p}^t$ the vector of payments for round t, which are initialized to 0

[1] This example does not satisfy the refinement of subgame perfection, though our welfare guarantees do not need this restriction.

for each round and updated as swaps occur. A swap occurs if and only if it is utility-improving for two participants under some set of payments. Formally:

Definition 8. *Given initial allocation* $\mathbf{x}$ *and payments* $\mathbf{p}^t$ *at round* t*, a* swap *is given by a pair of participants* i*,* i' *assigned to rounds* $j, j' \geq t$*, respectively, and a payment* $\hat{p}$*. A swap is valid if* $v_i^{j'} - C(p_i^t + \hat{p}) > v_i^j - C(p_i^t)$ *and* $v_{i'}^j - C(p_{i'}^t - \hat{p}) > v_{i'}^{j'} - C(p_{i'}^t)$*.*

Upon executing a swap, set $x_i^j, x_{i'}^{j'} \leftarrow 0$*,* $x_i^{j'}, x_{i'}^j \leftarrow 1$*,* $p_i^t \leftarrow p_i^t + \hat{p}$*, and* $p_{i'}^t \leftarrow p_i^t - \hat{p}$*.*

Note that with quasilinear participants, all valid swaps must strictly improve allocative efficiency since $C(p) = p$. That is, $v_i^{j'} + v_{i'}^j > v_i^j + v_{i'}^{j'}$, and the validity of a swap does not depend on the initial payments $\mathbf{p}^t$. We then study roscas of the following form:

Definition 9. *A* swap rosca *starts from an initial allocation* $\mathbf{x}$ *and initial payments* $\mathbf{p} = \{\mathbf{p}^t\}_{t=1}^n$ *of* 0 *for each participant and round. At each round* $t = 1, \ldots, n$*, participants execute valid swaps and we update the allocation and payment accordingly. We do so until there are no valid swaps.*

Note that for non-linear C, new swaps may become valid moving from round t to $t+1$, as each new round's payments reset to 0. For quasilinear participants, however, Definition 9 executes all swaps in round 1. In this case, the resulting allocation is guaranteed to be stable to pairwise swaps.

Definition 10. *An allocation* $\mathbf{x}$ *is* swap-stable *if for all participants* i, i' *assigned to* j, j'*, we have that* $v_i^j + v_{i'}^{j'} \geq v_i^{j'} + v_{i'}^j$*.*

For quasilinear participants, swap-stability is guaranteed regardless of the initial allocation. Convergence of the swap process follows from the fact that the total allocated value $\sum_i \mathbf{v}_i \cdot \mathbf{x}_i$ strictly increases each swap and that the number of allocations is finite.

Theorem 4. *For quasilinear participants, the welfare approximation for every swap rosca is at most* 2*.*

Proof. Without loss of generality, assume that the welfare-optimal allocation assigns each participant i to be allocated the pot in round i, so the optimal welfare is $\sum_i v_i^i$. Now let $\pi(i)$ denote the *round* when participant i is allocated the pot in the swap rosca's final allocation, and $\pi^{-1}(i)$ the *participant* allocated the pot in round i. Note that π and π^{-1} are bijections. Furthermore, under quasilinear utilities, all payments between participants are welfare-neutral, and hence the rosca welfare is given by $\sum_i v_i^{\pi(i)}$.

For any participant i, note that swap-stability implies

$$v_i^{\pi(i)} + v_{\pi^{-1}(i)}^i \geq v_i^i + v_{\pi^{-1}(i)}^{\pi(i)} \geq v_i^i.$$

Summing over all participants i, we get

$$\sum\nolimits_i v_i^{\pi(i)} + \sum\nolimits_i v_{\pi^{-1}(i)}^i \geq \sum\nolimits_i v_i^i.$$

Since π and π^{-1} are bijections, both sums on the lefthand side are equal to the rosca welfare, and the righthand side is the optimal welfare, giving us a 2-approximation. $\square$

Example 1 in the appendix shows that this bound is tight.

4.2 Experimental Results

The results presented so far partially rationalize the prevalence of auction and swap roscas. However, two limitations prevent a comprehensive view of roscas' allocative efficiency. First, the worst-case nature of our theoretical results give little detail about outcomes in *typical* instances. Second, our results hold only under quasilinear utilities, which may be less realistic for extremely vulnerable participants.

This section complements our theoretical results with computational experiments that shed light on these latter questions for swap roscas. We simulate swap roscas under natural instantiations of participants' values, and with participants' costs for payments taking a well-studied but non-linear form. We find that the approximation ratio of these roscas in more typical scenarios is significantly better than the worst-case ratio, even after relaxing quasilinearity. Our experiments also allow us to study the way rosca performance changes as participants' values for their payments become more convex. In particular, we use *constant relative risk aversion (CRRA)* utilities, given by

$$C(p; W, a) = (1-a)^{-1}(W^{1-a} - (W-p)^{1-a}),$$

where the parameter W represents the participant's starting wealth, and a governs the convexity of the function, with $a = 0$ being quasilinear. For $a > 0$, CRRA utilities have a vertical asymptote at $p = W$, as participants are unable to spend beyond their means. We choose W to be less than many of our participants' maximum values for the rosca pot. This is intended to capture that most participants cannot afford the durable good without the rosca (Anderson and Baland 2002). Note that as $a \to 1$, $C(p; W, a) \to \ln(W) - \ln(W - p)$. We choose CRRA utilities because they are standard for modeling preferences for wealth in economics (see, e.g. Romer 1996).

We give two sets of experimental results. In each, we run 9- and 30-person roscas (typical sizes for small- and medium-sized roscas), and compare three quantities: the optimal welfare under our selected value profile, the expected approximation ratio of a random allocation before any swaps, and the approximation ratio for a swap rosca run from a random allocation. Our swap roscas are simulated according to the description in Section 4. For a pair of participants i and j for whom there exists a valid swap, there are generally many payments which will incentivize a swap and we choose the smallest such payment.

4.3 Experiment: CRRA Utilites

Our first experiment fixes a profile of participant values and studies the performance of swap roscas as the convexity parameter a and starting wealth W vary. The value profile, comprised of 9 participants, features 6 with *cutoff values* of the form $v_i^t = \overline{v}$ for all $i \leq \hat{t}$ for some $\hat{t}$, and three participants with values which are roughly linearly decreasing in time. The average maximum value among cutoff participants is 5, which matched the average value for linearly decreasing values. We give all value profiles explicitly in the supplement. We consider values of a ranging from 0 (quasilinear) to 2 (very convex), focusing on smaller values, as larger values of a tend to represent very similar, extreme functions. We take W in the range $\{1, \ldots, 5\}$, as this puts participants' wealth levels generally below their values for the rosca pot. Welfare values are averaged over $10,000$ simulation runs, each starting with a random initial allocation that participants can pay to improve through swaps. Results for this simulation can be found in Table 1.

		W				
		1	**2**	**3**	**4**	**5**
	0	1.035	1.034	1.035	1.034	1.035
	0.1	1.121	1.119	1.070	1.067	1.063
	0.2	1.122	1.121	1.080	1.074	1.074
	0.3	1.122	1.119	1.118	1.086	1.081
a	**0.5**	1.122	1.124	1.121	1.119	1.120
	0.75	1.122	1.122	1.123	1.121	1.121
	1	1.124	1.122	1.121	1.123	1.122
	1.5	1.123	1.123	1.122	1.122	1.123
	2	1.122	1.125	1.123	1.122	1.123

Table 1: Swap Rosca Performance Under Different CRRA Parameters (OPT = 45, random baseline ratio = 1.601)

Across all values of W, the approximation ratio of swap roscas generally worsens (increases) as the level of convexity a increases. Intuitively, this is likely due to the fact that since C is convex, a participant receiving payments for a swap values them less than the participant offering the payments. Consequently, swaps are less likely to occur, even if they would lead to improved allocative efficiency. Meanwhile, the effect of W depends on the level of convexity a. When $0 < a < 0.5$, participants with higher wealth W have more money to spend on swaps, making swaps more likely to occur and hence improve allocative efficiency. Thus, approximation ratios improve (decrease) with higher W. However, as convexity increases, the disincentive to swap caused by convexity overcomes the benefit of having greater wealth with which to pay for swaps, and the approximation ratios no longer change with W. For all parameter values chosen, however, swap roscas led to a marked improvement over the approximation ratio from random allocation alone, suggesting that even under extreme convexity, participants are able to identify local improvements to social welfare. We also repeat this experiment with a 30-participant rosca using similar value profiles and observe the same trends. We present the results in the supplement.

4.4 Experiment: Distributional Diversity

Our second set of experiments, discussed in more detail in the supplement, varies the distribution of values across the population of participants, again for 9- and 30-person roscas. This allows us to study the way the distribution of need across a population impacts rosca welfare. We find that performance is insensitive to wide inequality in values of participants in the population.

5 Discussion and Conclusion

Roscas are complex and varied social institutions, significant for their integral role in allocating financial resources worldwide. In this work, we focus specifically on the allocative efficacy of roscas as lending and saving mechanisms. We derive welfare guarantees for roscas under a variety of allocation protocols and show that many commonly-observed roscas provide a constant factor welfare approximation to the optimal allocation. This guarantee, we believe, gives partial explanation for the ubiquity of roscas. In addition to these specific results, our work also serves as proof of concept for the potential for techniques from algorithmic game theory to help us better understand roscas and, more generally, how communities self-organize to create opportunity. We highlight ideas for further exploration below.

First, our work modeled the savings aspect of roscas, though roscas are also used as insurance when participants experiencing unanticipated needs may bid to obtain the pot earlier than they may have otherwise planned (Calomiris and Rajaraman 1998; Klonner 2003b, 2001). There remain many gaps in our understanding of roscas when participants' values and incomes evolve stochastically over time.

Another challenge is understanding the tension between allocative efficiency and wealth inequality. Participants with valuable investment opportunities might not bid as aggressively if their low wealth causes them to value cash highly. This is exacerbated when participants experience income shocks, which are often experienced by economically vulnerable individuals (Abebe, Kleinberg, and Weinberg 2020; Nokhiz et al. 2021). Ethnographic work shows that altruism plays a significant role in alleviating this tension (Klonner 2008; Sedai, Vasudevan, and Pena 2021). Roscas often serve a dual role of community-building institutions. Consequently, participants tend to observe signals about each others' shocks, and act with mutual aid in mind (Klonner 2008; Mequanent 1996).

Though roscas often work outside formal institutions, studies show that "rosca enforcement" is not often an issue. For instance, (Smets 2000; Van den Brink and Chavas 1997) show that early recipients of the pot rarely default, in part due to strong community norms and standards. These considerations often go unaccounted for in theoretical studies of roscas. A deeper understanding of community norms and standards can shed more light on rosca enforcement mechanisms and robustness.

Finally, there are many questions on how aspects of the population and environment govern the performance of roscas: i.e., under what conditions would one prefer one type of rosca over another? Similarly, how do roscas perform when their members evolve over time, e.g., with some participants joining part way through the rosca and potentially holding more leverage? Likewise, rosca formation is known to be crucial, with many roscas preferring individuals with similar socio-economic backgrounds. Modeling and examining the rosca formation process can improve our understanding of the interaction between the rosca formation process and their functionality, efficacy, and robustness.

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