Introducing Uninformed Search with Tangible Board Games

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Abstract

Researchers have established the value of hands-on learning with tangible artifacts in mathematics and related fields. Inspired by this work, an assignment was developed for an undergraduate/graduate Artificial Intelligence course to introduce students to the formal representation of search. Students analyzed a familiar board game—e.g., Rush Hour or peg solitaire—using the standard approach to modeling an uninformed search process. The assignment was well-received by students, and analysis of their work yielded unexpected insights into the challenges students face in understanding how the formal problem model interacts with search algorithms. This paper introduces the theoretical motivations for the work, analyzes student work products, and makes recommendations for future extensions.

Background

Since the 1800s, mathematics educators have been using manipulatives in mathematics instruction. In 1989, Sowell published a meta-analysis of 60 studies on the effectiveness of using such materials with students ranging in age from kindergarten through college. The results showed that "mathematics achievement increased through the long-term use of concrete instructional materials." Effects were most evident when use of concrete materials was combined with symbolic instruction. Statistically significant effects were found for younger children with year-long use of manipulatives (e.g., beansticks, Cuisenaire rods, geoboards, or paper-folding). Also, learning outcomes improved when instruction was given by teachers knowledgable about appropriate use of manipulatives.

The education research community has been strongly shaped by the ideas of Jean Piaget, who demonstrated how children progress through stages of concrete manipulations (of tangible objects) on their way to "formal operations" on abstract, symbolic systems. For many years, researchers assumed this meant that learners essentially "graduated" from their to need use physical things to aid thought.

In their influential essay, "Epistemological Pluralism," Turkle and Papert (1990) argued for a "revaluation of the concrete." Even though children necessarily pass through stages of cognitive development that are tied to sensory stimuli, they argued that even advanced thinkers often make use of tactile, concrete modes of thought.

Whether or not directly inspired by Papert and Turkle's work, this "revaluation of the concrete" has taken root in a variety of forms. In the computer science community, Bell, Witten, and Fellows developed a hands-on introduction to the central ideas in computer science called "CS Unplugged" (1998). Their activities include topics such as binary numbers, algorithms, and cryptography, and use everyday classroom manipulatives to carry out exercises without computers at all.

More recently, the computer science education community has developed "kinesthetic learning activities," or KLAs, as a way to engage learners. In these activities, students in a classroom enact algorithms and other representations of abstract computational processes using whole body movement (Begel, Garcia, and Wolfman 2004).

Other CS educators have developed instructional activities that can be conducted with simple manipulatives such as stacking plastic cups and LEGO bricks. Research indicates that these activities are "effective in helping students develop mental models for mathematical concepts" in CS1 and CS2 courses (Bucci et al. 2000).

The operations research (OR) community has also explored the use of tangible board games for pedagogical purposes. DePuy and Taylor describe the use of four such games, including Rush Hour and peg solitaire, in undergraduate and graduate teaching (2007). They value the role of the games in encouraging engineering students' problem formulation skills, and report high levels of student motivation in the game-based assignments.

Introduction

Inspired by this body of work, an assignment was developed for a upper-level undergraduate/introductory-level graduate Artificial Intelligence course. In the assignment, students were asked to analyze popular board games for solution by a search process. Because the games are physical, tangible things, their rules are relatively simple, and students could literally play the games before and during their analytical work.

As the first topic in the AI course syllabus was search, having it serve as the very first assignment in the class was

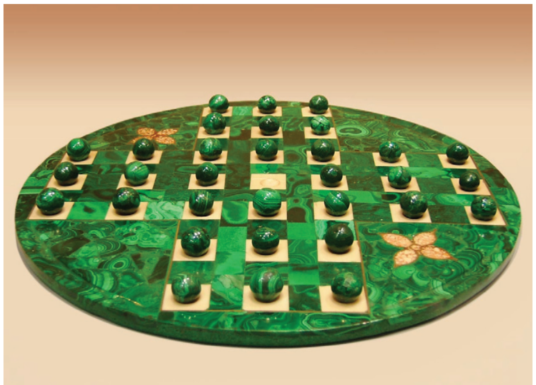

Figure 1: A version of peg solitaire with marbles. The goal is to jump marbles and remove them from the board (like Checkers) in such a way to leave exactly one marble when there are no moves remaining. *Photograph credit: Wikimedia Commons.*

logical. The assignment would be an "ice-breaker" between the instructor and his students, and students could perform an intellectually significant activity without being required to write code. (Subsequent assignments in the course design were all coding-intensive.)

On Search

In the standard formulation provided by Russell and Norvig (2010), a problem is defined by five components:

- An **initial state**.
- A description of possible **actions** that may be taken. The set of actions is often a function of the current state of a **search agent**, which is traversing the problem space.
- A description of what an action accomplishes. The collection of state-action pairs is referred to as the **transition model**.
- A **goal test**, which determines whether a given state is a goal state. (There may be more than one goal state.)
- A **path cost function**, which specifies the cost of taking the action from a given state to the subsequent state.

Problems amenable to uninformed search are then those in which (1) solution methods only have access to this problem definition (i.e., there is no heuristic function to indicate if a given choice of action is leading closer to a solution), and (2) actions are deterministic (selecting an action always leads to the state specified in the transition model), and (3) the world is observable—the search agent always knows what state it is in.

Once the problem is represented in this way, then the standard set of search techniques—e.g., breadth-first search, depth-first search, uniform-cost search—may be applied.

There is a fair degree of abstraction in this standard formulation of search. Specifically, this approach separates the problem representation from the algorithm used to solve it. In a more naïve, intuitive approach, students may wish to commingle their local understanding of the problem and its domain with a search methodology.

Figure 2: Rush Hour game. The goal is to move the cars out of the way so that the car indicated by the arrow can exit through the opening that is horizontally to the right. *Photograph credit: Gail Depuy.*

Before presenting the canonical problem representation, the instructor might encourage students to first analyze a number of different problem domains, developing contextualized strategies for each. Then, these specific instances could be generalized to the canonical form, which separates problem representation from solution approach.

In lieu of this more time-consuming pedagogical approach, it seemed that having an assignment that focused squarely on problem representation (and ignored choice of search technique) would be valuable.

Figure 3 presents the assignment that was developed. As mentioned, the idea of using board games emerged from the theoretical arguments about the value of concrete objects in learning. Because such games are physical and have relatively simple rule sets, students could engage with a game directly (that is—play the game) to determine its rules. Or, students could select a game with which they are already familiar, and base their analysis on something that they already understood.

Table 1: Overall classification of student work.

Category	**Number**
Analysis plus solution strategy	9
Representational analysis	13
Weak representation and weak solution	2

Assignment 1. Based on the in-class discussion, analyze a children's game or puzzle that is amenable to solving by a search process. Good examples are the game Rush Hour, peg solitaire games, such as the Cracker Barrel peg-jump game, and other one-player board games. Play with the game for a while to make sure you understand it, and then write up a description, defining the following aspects of the game:

- Initial state. Does the game have only one initial state? Does it have multiple possible initial states?
- Set of actions. Are these a function of the current state? Explain how.
- Transition model. Given a particular state and the set of possible actions for this state, is the transition to a new state deterministic? Explain.
- Goal test. Is there a definite end state? Can you tell if you won or lost and the game is over?
- Step cost. Do different actions have different costs? Or are all actions equal in cost, and you can model any move with a cost of 1?

You should write about a two-page narrative for your chosen game. Also, if your chosen game isn't exactly something that fits the search model, try to modify its rules so that it does. Make it a search problem: observable environment, movement between discrete states, and an a priori known environment.

Figure 3: Game representation assignment as provided to students.

Table 2: Student game selection.

Game	Number
Rush Hour	13
peg solitaire	4
other physical game	5
virtual game	2

In lecture, the puzzle game "Rush Hour" was presented. This game consists of a 6×6 rectangular grid that is populated with cars (1×2 grid squares in size) and trucks (1×3 grid squares). The vehicles slide either horizontally or vertically depending on their initial orientation. The goal of the game is to work from a given starting configuration, sliding vehicles until one special car can slide horizontally out an opening at the edge of the grid (see Figure 2). The commercial version of the game includes a set of start configuration cards, with a graduated degree of solution difficulty.

Peg solitaire games were also discussed. In these games, a 2-dimensional array of holes is populated with pegs or marbles. At least one hole must be vacant at the beginning of the game. To play the game, a marker is used to jump over an adjacent one into a free hole; the jumped-over marker is then removed. The objective is the game is to select a sequence of moves such that exactly one marker is left when there are no remaining moves.

Students were then asked to complete the assignment. About eight copies of the Rush Hour game were distributed to students who wished to work with that game. Otherwise, students were asked to procure their own game to complete the assignment.

Results

The course had an enrollment of 24 students, a mixture of half upper-level undergraduates and graduate students. All students completed the homework. In the subsequent discussion, no distinction is made as to whether students were undergraduates or graduates.

Broadly, the students' work can be classified into three categories:

1. *Performed a representational analysis, and also described a search solution strategy.* In this case, students analyzed their game per the five criteria laid out in the Russell and Norvig text, and also described an appropriate search methology that could be used to solve it, given the particulars of the game itself. Nine students submitted work in this category.
2. *Analyzed the game per the assignment parameters.* In this case, students described their selected game per the given canonical form. Just over half of the work fell into this category (13 of 24 cases).
3. *Failed to analyze the game, and instead attempted to describe a search procedure for solving it.* In this case, students did not use the problem representation approach, and instead described a search procedure that might be applied to solving their game of choice. Two students submitted work in this category.

The results are summarized in Table 1.

There are a couple of interesting results from this initial analysis. First, nearly half of the students (11 of 24) were not comfortable with performing analysis only. Nine of these 11 did extra work and went beyond the boundaries of the assignment to think specifically about how they should solve their particular search problem. This suggests that the strong separation of problem formulation followed by solution strategy is not a natural or satisfying approach for many students.

Regarding the two students whose work is characterized as "failed to analyze the game," both had a weak mixture of an attempt at characterization and an attempt at describing a search procedure. Neither aspects were described well. This initial assignment ultimately served as a predictor of these two individuals' overall class performance, which was subpar. (It should be noted that student work for this assignment was not reviewed at this level of detail until after the course was completed, so this observation is retrospective rather than prospective.)

Beyond the initial analysis, several other aspects of students' work are noteworthy: details of game selection and analysis, physical vs. virtual games, misconceptions revealed in students' work, and the role of tangibility. These are now discussed.

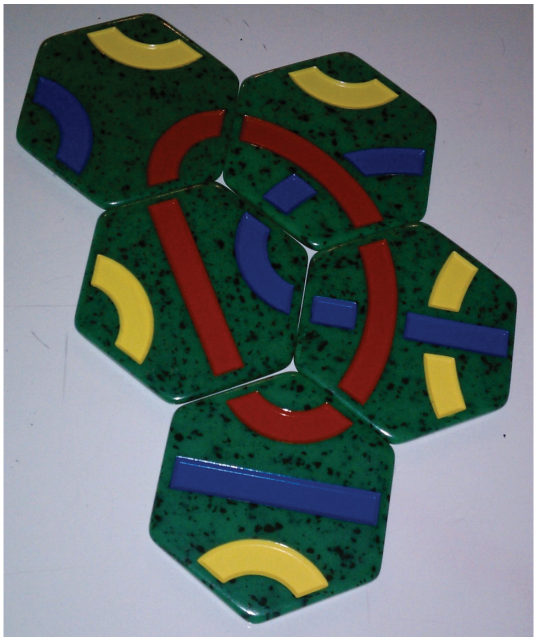

Figure 4: Tantrix, a visual-spatial puzzle game. These five tiles form a circuit.

Game Selection

Table 2 summarizes student choice of game. Not surprisingly, most students chose the games discussed in class—Rush Hour and a version of peg solitaire. It was already known to them that these games would map well into the problem representation, and these were familiar and popular games.

Of the self-selected games, five were physical board games and two were virtual games (i.e., computer-based puzzle games). Of the physical games, two will be discussed: Tantrix and Snakes & Ladders.

Figure 4 illustrates the puzzle game *Tantrix*, a puzzle game consisting of hexagonal tiles, each having three paths of red, yellow, and blue. The goal of the game is to make a closed color circuit connecting the paths, using a particular number of tiles (at least three, but not more than ten).

The student who selected this game was one of the nine in the analysis plus search process category. He was typical of those in this category in that he interwove the discussion of the representation with a discussion of a search implementation strategy. In particular, based on the student's prior expertise, he presented a search algorithm strategy based on a state machine.

One student selected Snakes & Ladders (also known as Chutes & Ladders) for his game. This is a game of chance for two or more players, where players take turns moving their marker on the playing board. To move, a player rolls a set of dice and advances one's marker that many squares on the playing board. If the marker lands at the base of a ladder, the marker is transported to the top of the ladder, skipping ahead toward the goal at the end of the board. On the other hand, if the marker arrives at the top of a snake/chute, the marker slides down to an earlier position on the board.

To convert this game into a search problem, the student made the following changes:

- It is converted into a one-player game.
- The player can elect any die roll that is desired, and then moves the marker by this amount. The set of possible die rolls becomes the set of allowable actions from any state.

With these changes, the game fit perfectly into canonical problem representation. The student additionally manually solved the game, presenting an optimal solution consisting of seven steps.

Virtual Games

Two of the students selected virtual games, after having obtained permission first. The two games were *Flood-It*, a single player game for the iPhone in which the player must flood the game board with one color within a certain number of steps, and *Snake*, the classic video game in which the player attempts to eat "apples" on a two-dimensional grid while avoiding poison squares and the player's own ever-expanding tail. Both games mapped well into the search problem formulation.

It may be the case that for many of today's students, virtual games serve the same role that physical games have for those of us who are older, and thus are equally "tangible" as games that have physical pieces. This is an active area of research; e.g., Manches, O'Malley, and Benford (2010) have explored the role of physical representations in solving number problems, comparing young children's use of physical and virtual materials.

Student Misconceptions

Several student narratives revealed interesting conceptual errors. These were all clustered around the separation of the search task into problem representation and solution algorithm components, and were specifically focused on the transition model.

Rush Hour. In a Rush Hour analysis, a student perceived the fact that the search agent may choose which car to move, and given that car, how many steps, as an example of non-determinism: "You can shift the vehicle for one to four grids according to the length of the vehicle and the free grids available for that vehicle. And secondly, the vehicle that you can shift is not deterministic. . . . In conclusion, the transition action is not deterministic."

Peg Solitaire. In an analysis of the transition model for peg solitaire, one student described it as follows: "To win the game, every move must be evaluated *a priori* in terms of whether subsequent moves will result in winning the game. This will eliminate a number of options, leaving behind options from which any one chosen at random will result in a win. In other words, it appears that the game has both deterministic and random elements to it."

Tantrix. The student who introduced the Tantrix game described a transition model in which new tiles are placed, tested for extending an existing path, and re-tried (possibly including back-tracking involving the removal of previously placed tiles).

In all three of these cases, students are trying to perform the work that ultimately is carried out by the search algorithm in the transition step. They are thinking concretely about their particular search problem, rather than abstracting it to the canonical form and allowing the search algorithm itself to handle multiple branches from a given state.

All three of these examples come from the category where students went "above and beyond" the parameters of the assignment and attempted to describe search algorithms in addition to the formal representation. Thus, some of the 13 students who satisfied the precise requirements of the assignment, and did not describe a search algorithm, may also have held these conceptual errors.

Based on conversation and questions that occurred in lecture, it is suspected that the confusion of these three students was representative of a larger group.

It is interesting to note that the operations research community has its own set of problem domains which the AI community considers "search," but OR has their own representation and solution techniques, which are different than ours. For example, Jeffersona et al. (2006) performed a detailed formal analysis of peg solitaire, comparing solution approaches in the OR community—e.g., integer programming—to classic AI search techniques. So our representation style is exactly that—a choice among several possible approaches—and one which is not inherently evident.

Role of Tangibility

The question that inspired this work was whether providing a tangible exercise would improve student learning. Of the 24 completed assignments, all showed evidence that students had thought concretely about the problem posed by the game. In particular, students described in some detail what it would be like to actually play their game. This was often done in the context of a specific game scenario, and sometimes included an illustration of the game situation; e.g.:

- In describing Rush Hour: *If there is a car in the lower left hand corner of the board on the bottom row of length 3, there are only 3 possible resulting states: the car is in columns 2, 3, and 4; the car is in columns 3, 4, and 5, and the car is in columns 4, 5, and 6.*
- In describing the Snake video game, accompanying a hand-drawn illustration: *A right turn would land the player in a wall, losing the game. A left turn allows the player to eat a red apple—the main objective.*
- In describing the Flood-It video game, also with a hand-drawn illustration: *The player chose the action to change the color of the owned area to red. This means the owned area now includes the adjacent red square.*

Even the two students whose work was characterized as having a weak representation and weak solution described the process of playing their game. This suggests the central activity of the assignment was accessible to all students.

Discussion and Conclusions

The original motivation for the assignment was to (1) have an accessible introduction to search, (2) allow students to think concretely about a specific problem, and (3) encourage formal analysis without requiring students to write code.

In all of these ways, the assignment did succeed. Further, students found it engaging, with (as mentioned before) nearly half of them going beyond the strict requirements. This is consistent with results reported by other researchers. Also, it is encouraging that that nearly a third of the students chose games of their own when they were not required to do so.

Even though students did often treat their games in a concrete fashion (e.g., describing the rules contextually, or explaining a specific game state), it is difficult to go further in identifying how tangible media might have served in students' learning. Students shared results in class, and having the physical materials (plus a document camera to project images to the whole classroom) did create interest. Still, it is hard to identify how the tangible materials might have been valuable at a conceptual level.

The assignment had the unexpected result of revealing confusion in the transition model, where a number of students tried to include the work that is actually accomplished by the search algorithm. It is likely that this is a pervasive issue, indicating that more attention should be paid to this in classroom instruction.

The assignment definitely served its role as a friendly first assignment in the class. In thinking about how to change it, I am pleased that many students went beyond the specific parameters, so I would be inclined against discouraging that.

In future semesters, I would be likely to use it again, and I would structure game selection in such a way to encourage more personal student choice.

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