Beyond Class-Conditional Assumption: A Primary Attempt to Combat Instance-Dependent Label Noise

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Abstract

Supervised learning under label noise has seen numerous advances recently, while existing theoretical findings and empirical results broadly build up on the class-conditional noise (CCN) assumption that the noise is independent of input features given the true label. In this work, we present a theoretical hypothesis testing and prove that noise in real-world dataset is unlikely to be CCN, which confirms that label noise should depend on the instance and justifies the urgent need to go beyond the CCN assumption.The theoretical results motivate us to study the more general and practical-relevant instance-dependent noise (IDN). To stimulate the development of theory and methodology on IDN, we formalize an algorithm to generate controllable IDN and present both theoretical and empirical evidence to show that IDN is semantically meaningful and challenging. As a primary attempt to combat IDN, we present a tiny algorithm termed self-evolution average label (SEAL), which not only stands out under IDN with various noise fractions, but also improves the generalization on real-world noise benchmark Clothing1M. Our code is released[1]. Notably, our theoretical analysis in Section 2 provides rigorous motivations for studying IDN, which is an important topic that deserves more research attention in future.

1 Introduction

Noisy labels are unavoidable in practical applications, where instances are usually labeled by workers on crowdsourcing platforms (Yan et al. 2014; Schroff, Criminisi, and Zisserman 2010). Unfortunately, Deep Neural Networks (DNNs) can memorize noisy labels easily but generalize poorly on clean test data (Zhang et al. 2017). Hence, how to mitigate the effect of noisy labels in the training of DNNs has attracted considerable attention recently. Most existing works, for their theoretical analysis or noise synthesizing in experiments, follow the class-conditional noise (CCN) assumption (Scott, Blanchard, and Handy 2013; Zhang and Sabuncu 2018; Menon et al. 2020; Ma et al. 2020), where the label noise is *in*dependent of its input features conditional on the latent true label.

Figure 1: Examples of *8* in MNIST (first row) and *Airplane* in CIFAR-10 (second row). The corruption probability dependent on the true class does not model such diverse features in each class.

In fact, instances with the same label can be entirely different, hence the probability of mislabeling should be highly dependent on the specific instance. As shown in the first row of Fig. 1, the second right image is likely to be mislabeled as the number 6 and the fourth right image is likely to be manually mislabeled as the number 7; in the second row, the last image is more likely to be mislabeled as the ship. In this paper, our *first contribution* (Section 2) is to present a theoretical hypothesis testing on the well-known real-world dataset, Clothing1M, to demonstrate the urgent need to go beyond the CCN assumption in practical applications. Meanwhile, we discuss the challenge of instance-dependent noise (IDN) with both theoretical and empirical evidence. Some pioneer efforts has been contributed to IDN, but most results are restricted to binary classification (Menon, van Rooyen, and Natarajan 2018; Bootkrajang and Chaijaruwanich 2020; Cheng et al. 2020) or based on assumptions such as the noise is parts-dependent (Xia et al. 2020).

To stimulate the development of theory and methodology on more practical-relevant IDN, we propose an algorithm to generate controllable IDN and present extensive characterizations of training under IDN, which is our *second contribution* (Section 3). Our *third contribution* (Section 4) is to propose an algorithm termed self-evolution average label (SEAL) to defend IDN, motivated by an experimental observation that the DNN's output corresponding to the latent true label can be activated with oscillation before memorizing noise. Specifically, SEAL provides instance-dependent label correction by averaging predictions of a DNN on each instance over the whole training process, then retrains a classifier using the averaged soft labels. The superior performance of SEAL is verified on extensive experiments, including synthetic/real-

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[1] https://github.com/chenpf1025/IDN

world datasets under IDN of different noise fractions, and the large benchmark Clothing1M (Xiao et al. 2015) with real-world noise.

2 From CCN to IDN - Theoretical Evidences

2.1 Preliminaries

Considering a c-class classification problem, let $\mathcal{X}$ be the feature space, $\mathcal{Y} = \{1, ..., c\}$ be the label space, $(X, Y) \in \mathcal{X} \times \mathcal{Y}$ be the random variables with distribution $\mathcal{D}_{X,Y}$ and $D = \{(x_i, y_i)\}_{i=1}^n$ be a dataset containing i.i.d. samples drawn from $\mathcal{D}_{X,Y}$. In practical applications, the true label Y may not be observable. Instead, we have an observable distribution of noisy labels $(X, \bar{Y}) \sim \bar{\mathcal{D}}_{X,\bar{Y}}$ and a dataset $\bar{D} = \{(x_i, \bar{y}_i)\}_{i=1}^n$ drawn from it. A classifier $f : \mathcal{X} \to \mathbb{P}^c$ is defined by a DNN that outputs a probability distribution over all classes, where $\mathbb{P}^c = \{s \in \mathbb{R}_+^c : \|s\|_1 = 1\}$. Unless specified, f denote the output after a softmax layer (Goodfellow, Bengio, and Courville 2016).

2.2 Beyond the CCN Assumption

The CCN assumption is commonly used in previous works, as clearly stated in theoretical analysis (Blum and Mitchell 1998; Yan et al. 2017; Patrini et al. 2016; Zhang and Sabuncu 2018; Xu et al. 2019; Menon et al. 2020; Ma et al. 2020) or inexplicitly used in experiments for synthetizing noisy labels (Han et al. 2018b; Yu et al. 2019; Arazo et al. 2019; Li, Socher, and Hoi 2020; Lukasik et al. 2020). Under the CCN assumption, the observed label $\bar{Y}$ is independent of X conditioning on the latent true label Y.

Definition 1. *(CCN Model) Under the CCN assumption, there is a noise transition matrix $M \in [0, 1]^{c \times c}$ and we observe samples $(X, \bar{Y}) \sim \bar{\mathcal{D}} = \mathrm{CCN}(\mathcal{D}, M)$, where first we draw $(X, Y) \sim \mathcal{D}$ as usual, then flip Y to produce $\bar{Y}$ according to the conditional probability defined by M, i.e., $\Pr(\bar{Y} = q|Y = p) = M_{p,q}$, where $p, q \in \mathcal{Y}$.*

We have seen various specific cases of CCN, including uniform/symmetric noise (Ren et al. 2018; Arazo et al. 2019; Chen et al. 2019a; Lukasik et al. 2020), pair/asymmetric noise (Han et al. 2018b; Chen et al. 2019b), tri/column/block-diagonal noise (Han et al. 2018a). Since the noise transition process is fully specified by a matrix M, one can mitigate the effect of CCN by modeling M (Patrini et al. 2017; Hendrycks et al. 2018; Han et al. 2018a; Xia et al. 2019; Yao et al. 2019). Alternatively, several robust loss functions (Natarajan et al. 2013; Patrini et al. 2017; Zhang and Sabuncu 2018; Xu et al. 2019) have been proposed and justified. Many other works do not focus on theoretical analysis, yet propose methods based on empirical findings or intuitions, such as sample selection (Han et al. 2018b; Song, Kim, and Lee 2019; Yu et al. 2019), sample weighting (Ren et al. 2018) and label correction (Ma et al. 2018; Arazo et al. 2019).

Intuitively, CCN does not model such diverse features in each class, as illustrated by examples in Fig. 1. Theoretically, we can justify the need to go beyond the CCN assumption with the following theorem.

Theorem 1. *(CCN hypothesis testing) Given a noisy dataset with n instances, considering randomly sampling a validation set $\bar{V} = \{(x_i, \bar{y}_i)\}_{i=1}^m$, $m < n$, and training a network f on the rest instances. Let $f(\cdot)$ denote a hard prediction here and $\hat{er}_{\bar{V}}^{0-1}[f] = \sum_{i=1}^m \frac{1}{m}\mathbb{1}(f(x_i) \neq \bar{y}_i)$ be the validation error, where $\mathbb{1}(\cdot)$ is the indicator function. Let $w_p = \Pr[Y = p]$ be the fraction of samples per class. If the CCN assumption holds, we shall have*

$$\Pr\left[1 - \sum_{p=1}^{c} w_p \max_{q \in \mathcal{Y}} M_{p,q} - \hat{er}_{\bar{V}}^{0-1}[f] \geq \varepsilon\right] \leq e^{-2m\varepsilon^2} \tag{1}$$

Proof. Let $er_{\bar{\mathcal{D}}}^{0-1}[f] = \mathbb{E}_{(x,\bar{y})\sim\bar{\mathcal{D}}}\mathbb{1}(f(x) \neq \bar{y})$ be the expected error on noisy distribution. For any f, the CCN assumption implies $f(X)$ is independent of $\bar{Y}$ conditioning on Y, then we have,

$$\begin{aligned}
er_{\bar{\mathcal{D}}}^{0-1}[f] &= 1 - \mathbb{E}_{(x,\bar{y})\sim\bar{\mathcal{D}}}\mathbb{1}(f(x) = \bar{y}) \\
&= 1 - \sum_{p=1}^{c} w_p \Pr[f(X) = \bar{Y}|Y = p] \\
&= 1 - \sum_{p=1}^{c} w_p \sum_{q=1}^{c} \Pr[f(X) = q, \bar{Y} = q|Y = p] \\
&= 1 - \sum_{p=1}^{c} w_p \sum_{q=1}^{c} \Pr[f(X) = q|Y = p] \cdot M_{p,q} \\
&\geq 1 - \sum_{p=1}^{c} w_p \max_{q\in\mathcal{Y}} M_{p,q}.
\end{aligned}$$

Note that the error $\hat{er}_{\bar{V}}^{0-1}[f]$ is estimated on validation samples that are not used when training f, hence $\{\mathbb{1}(f(x_i) \neq \bar{y}_i)\}_{i=1}^m$ are m i.i.d. Bernoulli random variables with expectation $er_{\bar{\mathcal{D}}}^{0-1}[f]$. Using Hoeffding's inequality, we have, $\forall\varepsilon > 0$,

$$\begin{aligned}
&\Pr\left[1 - \sum_{p=1}^{c} w_p \max_{q\in\mathcal{Y}} M_{p,q} - \hat{er}_{\bar{V}}^{0-1}[f] \geq \varepsilon\right] \\
&\leq \Pr\left[er_{\bar{\mathcal{D}}}^{0-1}[f] - \hat{er}_{\bar{V}}^{0-1}[f] \geq \varepsilon\right] \leq e^{-2m\varepsilon^2}.
\end{aligned}$$

□

Now we apply Theorem 1 to the widely used noise benchmark Clothing1M, which contains one million noisy training samples of clothing images in 14 classes. We keep $500k$ random samples from validation while train a ResNet-50 on the rest samples. After training, we get a validation error $\hat{er}_{\bar{V}}^{0-1}[f] = 0.1605$. The original paper (Xiao et al. 2015) provides additional refined labels and a noise confusion matrix that can be an estimator of M under the CCN assumption. Moreover, we estimate w_p using the proportion of labels on the $14k$ refined subset. In this way, we get $1 - \sum_{p=1}^{c} w_p \max_{q\in\mathcal{Y}} M_{p,q} - \hat{er}_{\bar{V}}^{0-1}[f] = 0.2212$. By substituting $\varepsilon = 0.2212$ and $m = 500k$ to Eq. (1), we get a **probability lower than $\mathbf{10^{-21250}}$**, which is statistically impossible. This contradiction implies that the CCN assumption does not hold on Clothing1M. In fact, this result is explainable by analyzing the difference between CCN and IDN. For

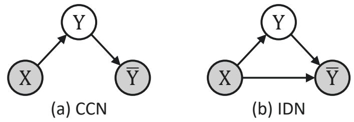


Figure 2: The graphical model of label noise.

CCN, label noise is independent of input features conditioning on the true class; hence the network can not generalize well on such independent noise. Thus, we expect a high validation error if the noise is CCN. While here, we obtain an empirical error much lower than expected; this implies that the network learned feature-dependent noise that can be generalized to the noisy validation set.

2.3 The IDN Model and Its Challenges

Now both theoretical evidences and intuitions imply that label noise should be dependent on input features, yet limited research efforts have been devoted to IDN. For binary classification under IDN, there have been several pioneer theoretical analysis on robustness (Menon, van Rooyen, and Natarajan 2018; Bootkrajang and Chaijaruwanich 2020) and sample selection methods (Cheng et al. 2020), mostly restricted to small-scale machine learning such as logistic regression. In the deep learning scenario, Xia et al. (2020) combat IDN by assuming that the noise is parts-dependent, with which they can estimate the noise transition for each part. Thulasidasan et al. (2019) investigate the co-occurrence of noisy labels with underlying features by adding synthetic features, such as the smudge, to mislabeled instances. However, this is not the typical realistic IDN where the noisy label should be dependent on inherent input features.

As presented in Definition 2, we can model instance-dependent label corruption, such that the noise transition matrix is a function of X. Note that both IDN and CCN consider close-set noise, as contrast to a specific label noise termed open-set noise (Wang et al. 2018), where the noisy instances does not belong to any considered classes. The graphical model of label noise is shown in Fig. 2. CCN can be seen as a degenerated case of IDN such that all instances have the same noise transition matrix.

Definition 2. *(IDN Model) Under the IDN model, $M : \mathcal{X} \rightarrow [0,1]^{c\times c}$ is a function of X. We observe samples $(X, \bar{Y}) \sim \bar{\mathcal{D}} = \mathrm{IDN}(\mathcal{D}, M)$, where first we draw $(X, Y) \sim \mathcal{D}$ as usual, then flip Y to produce $\bar{Y}$ according to the conditional probability defined by $M(X)$, i.e., $\Pr(\bar{Y} = q|Y = p) = M_{p,q}(X)$, where $p, q \in \mathcal{Y}$.*

Many existing robust loss functions (Natarajan et al. 2013; Patrini et al. 2017; Zhang and Sabuncu 2018; Xu et al. 2019) have theoretical guarantees derived from the CCN assumption but not IDN. Some sample selection algorithms (Malach and Shalev-Shwartz 2017; Han et al. 2018b; Yu et al. 2019; Li, Socher, and Hoi 2020), targeting at selecting clean samples from the noisy training set, work quite well under CCN. Though these methods does not directly rely on the CCN assumption, it can be more challenging to identify clean samples under IDN since the label noise is correlated with inherent input features that result in confusion.

Theoretically, *the optimal sample selection exists under CCN but may fail under IDN.* This is because under IDN, even if we select all clean samples accurately, the support of X can be different to its original support in clean distribution. While for CCN, it is possible to select an optimal subset. The key issue is whether the following holds for any $p \in \mathcal{Y}$.

$$supp(P(X|\bar{Y} = Y, Y = p)) \stackrel{?}{=} supp(P(X|Y = p)), \quad (2)$$

where $supp(\cdot)$ denotes the support of a distribution. For CCN, since X is independent of $\bar{Y}$ conditioning Y, the equality in Eq. (2) holds. While for IDN, Eq. (2) possibly does not hold. For example, if samples near the decision boundary are more likely to be corrupted, then the supports are different, which means learning with selected clean samples is statistically inconsistent (Cheng et al. 2020). More characterizations of IDN will be presented in the next section.

3 A Typical Controllable IDN

3.1 Enabling Controllable Experiments

The rapid advance of research on CCN not only attributes to simplicity of the noise model but also the simple generation process of synthetic noise. We are able to conduct experiments on synthetic CCN of any noise fraction by randomly flipping labels according to the conditional probability defined by M, which enable us to characterize DNNs trained with CCN (Arpit et al. 2017; Chen et al. 2019b), develop algorithms accordingly and quickly verify the idea. Similarly, it is desired to easily generate IDN with any noise fraction for any given benchmark dataset. A practical solution is to model IDN using DNNs' prediction error because the error is expected to be challenging for DNNs. To yield calibrated softmax output for IDN generation, Berthon et al. (2020) train a classifier on a small subset, calibrate the classifier on another clean validation set (Guo et al. 2017), then use predictions on the rest instances to obtain noisy labels. It does not generate noise for the whole dataset and the noise largely depends on the small training subset.

To stimulate the development of theory and methodology, we propose a novel IDN generator in Algorithm 1. Our labeler follows the intuition that 'hard' instances are more likely to be mislabeled (Du and Cai 2015; Menon, van Rooyen, and Natarajan 2018). Given a dataset $D = \{(x_i, y_i)\}_{i=1}^n$ with labels believed to be clean, we normally train a DNN for T epochs and get a sequence of networks with various classification performance. For each instance, if many networks predict a high probability on a class different to the labeled one, it means that it is hard to clearly distinguish the instance from this class. Therefore, we can compute the score of mislabeling $N(x)$ and the potential noisy label $\tilde{y}(x)$ as follow:

$$\begin{gathered} S = \sum_{t=1}^{T} S^t / T \in \mathbb{R}^{n\times c}, \\ N(x_i) = \max_{k\neq y_i} S_{i,k}, \quad \tilde{y}(x_i) = \arg\max_{k\neq y_i} S_{i,k}, \end{gathered} \quad (3)$$

where $S^t = [f^t(x_i)]_{i=1}^n$ is DNN's output at t-th epoch. The average prediction here reveals the DNN's confusion on instances throughout training. We flip the label of $p\%$ instances

Algorithm 1 IDN Generation.

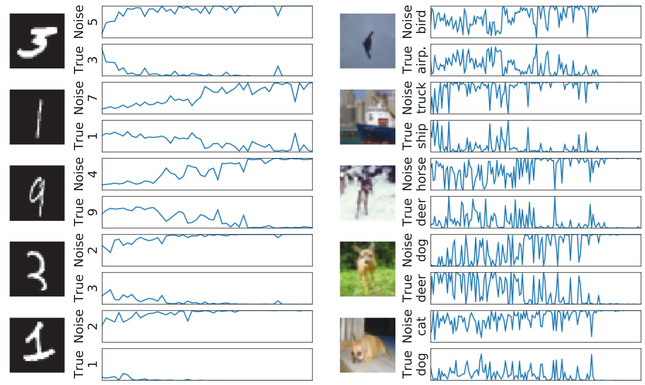
Input: Clean samples $D = \{(x_i, y_i)\}_{i=1}^n$, a target noise fraction p, epochs T.
Initialize a network f.
for $t = 1$ **to** T **do**
  for batches $\{(x_i, y_i)\}_{i \in \mathcal{B}}$ **do**
    Train f on $\{(x_i, y_i)\}_{i \in \mathcal{B}}$ using cross-entropy loss:
    $\mathcal{L}_{CE} = -\frac{1}{|\mathcal{B}|} \sum_{i \in \mathcal{B}} \log(f^t_{y_i}(x_i))$
  end for
  Record output $S^t = [f^t(x_i)]_{i=1}^n \in \mathbb{R}^{n \times c}$.
end for
Compute $N(x_i)$, $\tilde{y}(x_i)$ using $\{S^t\}_{t=1}^T$ (Eq. (3)).
Compute the index set $\mathcal{I} = \{p\% \arg\max_{1 \le i \le n} N(x_i)\}$.
Flip $\bar{y}_i = \tilde{y}_i$ if $i \in \mathcal{I}$ else keep $\bar{y}_i = y_i$.
Output: A dataset with IDN: $\bar{D} = \{(x_i, \bar{y}_i)\}_{i=1}^n$.

with highest mislabeling scores, where p is a target noise fraction. In essence, Algorithm 1 uses predictions of the DNN to synthetize noisy labels, while it stands out for being able to generate noisy labels of any noise ratio for the whole training set, requiring simply a single round of training on given labels. The noise is instance-dependent since it comes from the prediction error on each instance. Moreover, it is a typical challenging IDN since the error is exactly the class which is confusing for the DNN.

3.2 Characterizations of Training with IDN

To combat label noise, we can firstly characterize behaviors of DNNs trained with noise. For example, the memorization effect (Arpit et al. 2017) under CCN claims that DNNs tend to learn simple and general patterns first before memorizing noise, which has motivated extensive robust training algorithms. While our understanding on IDN is still limited. Here we present some empirical findings on IDN, to help researchers understand the behaviors of DNNs trained with IDN and to motivate robust training methods. We conduct experiments on MNIST and CIFAR-10 under IDN with varying noise fractions generated by Algorithm 1. For CCN, we use the most studied uniform noise. In all experiments throughout this paper, the DNN model and training hyperparameters we use are consistent. More details on experimental settings are summarized in Section 4.3 and Appendix A.

It is easier for DNNs to fit IDN. Firstly, let us focus on the training/testing curves in Fig. 4. For IDN and CCN with the same noise fraction, the training accuracy is higher under IDN. This implies that it is easier for DNNs to fit IDN. The finding is consistent with our intuition since noisy labels under IDN are highly correlated with input features that can mislead DNNs. In this sense, IDN is more difficult to mitigate because the feature-dependent noise is very confusing for DNNs, which can easily result in overfitting. Moreover, the peak testing accuracy before convergence, which implies the DNN learns general patterns first (Arpit et al. 2017), is much lower under IDN. This suggests that due to DNNs can fit IDN easily, the generalization performance degenerates at early stages of training. The observation is consistent with the findings on real-world noise presented by Jiang et al. (2020).



(a) MNIST (b) CIFAR-10

Figure 3: Examples of softmax outputs on the noisy label and latent true label. The x-axis is training epoch and the y-axis is DNN's output probability. The airp. is airplane for short.

The memorization effect is less significant. The memorization effect (Arpit et al. 2017) is a critical phenomenon of DNNs trained with CCN: DNNs first learn *simple* and *general* patterns of the real data before fitting noise. It has motivated extensive robust training algorithms. The memorization effect is characterized by the testing accuracy and critical sample ratio (CSR) (Arpit et al. 2017) during training, where CSR estimates the density of decision boundaries. A sample x is a critical sample if there exists a $\hat{x}$, s.t.,

$$\arg\max_k f_k(x) \neq \arg\max_k f_k(\hat{x}), \text{ s.t., } \|x - \hat{x}\|_\infty \le r. \quad (4)$$

The curves of testing accuracy and CSR presented in Fig. 4 show typical characterizations of the memorization effect. Similar to CCN, the model achieves maximum testing accuracy before memorizing all training samples under IDN, which suggests that DNNs can learn general patterns first. Moreover, the CSR increases during training, suggesting that DNNs learn gradually more complex hypotheses. It is worth noting that under IDN, both peak testing accuracy and CSR are lower while the gap between peak and converged testing accuracy is smaller. On MNIST, the testing accuracy decreases at very early stage of training, suggesting that the memorizing of noise dominates learning of real data. Therefore, we conclude that the memorization effect still exists under IDN, but it is less significant compared to CCN.

Individual study: instance-level memorization. Apart from showing the memorization effect for the whole training set, we are interested in how memorization happens for individual instances. As an individual study, we train DNNs under 20% IDN and show examples in Fig. 3. We plot the entry of softmax output corresponding to the noisy label and true label throughout training. DNNs will eventually memorize the wrong label, while during training, the output corresponding to the true label can be largely activated with oscillation. The intensity of oscillation and the epoch when the memorization happens is quite different for each instance.

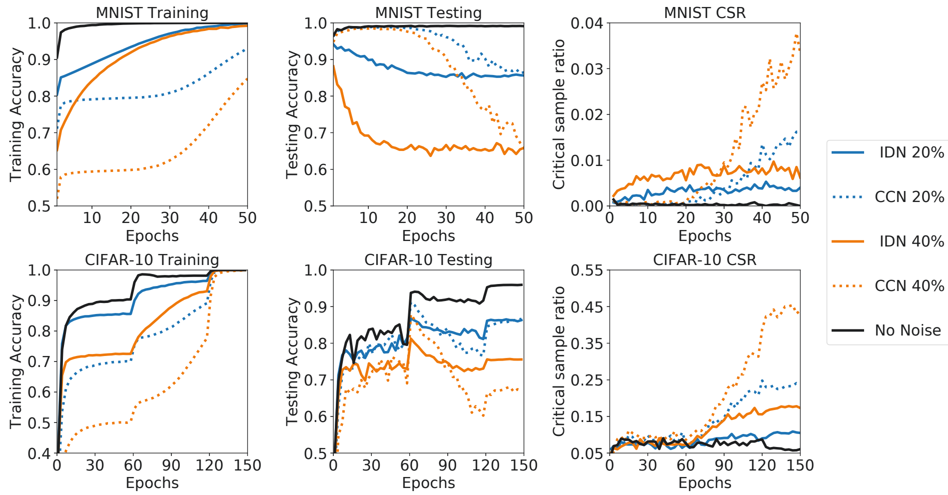


Figure 4: Training/testing accuracy and critical sample ratio throughout training on IDN and CCN with varying noise fractions.

Algorithm 2 An iteration of SEAL.

Input: Noisy samples $\bar{D} = \{(x_i, \bar{y}_i)\}_{i=1}^n$, epochs T, soft labels from the last iteration $\bar{S}$ (optional).
Initialize a network f.
if $\bar{S}$ is not available **then**
  # *The initial iteration*, use given noisy labels
  $\bar{S} = [e_{\bar{y}_i}]_{i=1}^n \in \mathbb{R}^{n\times c}$ # $e_{\bar{y}_i}$ is one-hot
end if
for $t = 1$ **to** T **do**
  for batches $\{(x_i, \bar{S}_i)\}_{i\in\mathcal{B}}$ **do**
    Train f on $\{(x_i, \bar{S}_i)\}_{i\in\mathcal{B}}$ using the loss:
    $\mathcal{L}_{SEAL} = -\frac{1}{|\mathcal{B}|}\sum_{i\in\mathcal{B}}\sum_{k=1}^{c}\bar{S}_{i,k}\log(f_k^t(x_i))$
  end for
  Record output $\bar{S}^t = [f^t(x_i)]_{i=1}^n \in \mathbb{R}^{n\times c}$.
end for
Update $\bar{S} = \sum_{t=1}^T \bar{S}^t / T \in \mathbb{R}^{n\times c}$.
Output: Trained f, $\bar{S}$ (can be used in next iteration).

4 SEAL: A Primary Attempt to Combat IDN

4.1 Methods

To mitigate the effect of label noise, we propose a practical algorithm termed self-evolution average label (SEAL). SEAL provides instance-dependent label correction by averaging predictions of a DNN on each instance over the whole training process, then retrains a classifier using the averaged soft labels. An iteration of SEAL is outlined in Algorithm 2 while we can apply SEAL with multiple iterations.

Here we discuss the intuitions of SEAL. Without loss of generality, assume there exists a latent optimal distribution of true label for each instance. Let $S_i^* \in \mathbb{P}^c$ be the latent optimal label distribution of the i-th instance. S_i^* can be one-hot for a confident instance and be soft otherwise. Intuitively, we can image S_i^* as the output of an oracle DNN. Considering training a DNN on a c-class noisy dataset $\bar{D} = \{(x_i, \bar{y}_i)\}_{i=1}^n$ for sufficient many T epochs until converged, we let $f^t(x_i)$ be the output on x_i at t-th epoch. Based on the oscillations show in Figure 3, we roughly approximate the output on x_i at t-th epoch as

$$f^t(x_i) = \alpha_i^t \omega_i^t + (1-\alpha_i^t)e_{\bar{y}_i}, \tag{5}$$

where $t \in \{1, 2, \cdots, T\}$, $e_{\bar{y}_i}$ is the one-hot label, $\alpha_i^t \in [0,1]$ are coefficients dependent on instances and the network, $w_i^t \in \mathbb{P}^c$ are i.i.d. random vectors with $\mathbb{E}[w_i^t] = S_i^*$. The approximation may be inaccurate at the early stage of training because the network does not learn useful features and it is better to add a term for random predictions in the approximation. Still, we ignore this term because random predictions do not introduce bias toward any class and the effect is mitigated by taking the average. With the approximation, we intuitively compare with the prediction of a random epoch $f^\tau(x_i)$, where τ is a random epoch such that $\Pr[\tau = t] = 1/T, \forall t \in \{1, 2, \cdots, T\}$. Let $\bar{S} \in \mathbb{R}^{n\times c}$ be the soft labels obtained by SEAL and $\|\cdot\|$ denote a norm on $\mathbb{P}^c$, it is not difficult to see that for any training instance x_i,

$$\|\mathbb{E}[\bar{S}_i] - S_i^*\| \le \|e_{\bar{y}_i} - S_i^*\|, \tag{6}$$

$$\mathrm{var}(\bar{S}_{i,k}) \le \mathrm{var}(f_k^\tau(x_i)),\ \forall k \in \{1, 2, \cdots, c\}. \tag{7}$$

That is, SEAL yields instance-dependent label correction that is expected to be better than the given noisy labels and the label correction has lower variance due to taking the average.

We can run SEAL for multiple iteration to further correct the noise, termed as 'self-evolution'. We take the soft label (denoted as $\bar{S}_i^{[m]}, m \ge 0$) of the last iteration as input and output $\bar{S}_i^{[m+1]}$. Using similar approximation as Eq. (5) by replacing the training label $e_{\bar{y}_i}$ with $\bar{S}_i^{[m]}$, SEAL is expected to produce labels that gradually approach the optimal ones,

$$\|\mathbb{E}[\bar{S}_i^{[m+1]}] - S_i^*\| \le \|\bar{S}_i^{[m]} - S_i^*\|. \tag{8}$$

A concern of SEAL is the increased computational cost due to retraining the network. In experiments, we focus on verifying the idea of SEAL and we retrain networks from the scratch in each iteration to show the evolution under exactly the same training schedule, requiring scaled computational cost. While in practice, we may save computational cost by reserving the best model (e.g., using a noisy validation set) and training for less epochs.

4.2 SEAL v.s. Related Methods

Using predictions of DNNs has long been adopted in distillation (Hinton, Vinyals, and Dean 2015) and robust training algorithms that use label correction (Reed et al. 2015; Ma et al. 2018; Tanaka et al. 2018; Song, Kim, and Lee 2019; Nguyen et al. 2020; Arazo et al. 2019). SEAL provides an elegant solution that is simple, effective and has empirical and theoretical intuitions. Taking the average of predictions, motivated by the activation and oscillation of softmax output at the entry of true label, provides instance-dependent label correction. SEAL is different to vanilla distillation (Hinton, Vinyals, and Dean 2015): in the presence of label noise, simply distilling knowledge from a converged teacher network, which memorizes noisy labels, can not correct the noise.

Compared with existing label correction methods, SEAL simply takes the average rather than carefully tuning hyperparameters to ensure that (i) the DNN learns enough useful features and (ii) the DNN dose not fit too much noise. It is challenging to compromise between (i) and (ii) in learning with IDN. We have shown that the memorization on correct/corrupted labels can be quite different for each training instance. However, the above (i) and (ii) typically require carefully tuning in existing methods (Reed et al. 2015; Ma et al. 2018; Tanaka et al. 2018; Song, Kim, and Lee 2019; Nguyen et al. 2020; Arazo et al. 2019). For example, one usually needs to tune a warm-up epoch (Tanaka et al. 2018; Song, Kim, and Lee 2019; Arazo et al. 2019) before which no label correction is applied. A small warm-up epoch results in underfitting on useful features while a large one yields overfitting on noise. Worse still, one may need to tune an adaptive weight during training to determine how much we trust predictions of the DNN (Reed et al. 2015; Ma et al. 2018). As theoretically shown by Dong et al. (2019), the conditions are very strict for DNNs to converge and not to fit noise.

When implementing SEAL, there is no special hyperparameters other than the canonical hyperparameters such as the training epoch and learning rate. To determine these canonical hyperparameters, we simply need to examine the training accuracy on the noisy dataset. Since SEAL averages predictions throughout training, the label correction can be effective even if the DNN memorizes noise when converged. Therefore, our criterion of choosing hyperparameters is to make sure the training accuracy is converged and it is as high as possible. Moreover, the model architecture and training hyperparameters can be shared in each iteration of SEAL.

4.3 Empirical Evaluation

Experimental setup. Our experiments focus on challenging IDN and real-world noise. We demonstrate the performance of SEAL on MNIST and CIFAR-10 (Krizhevsky and Hinton 2009) with varying IDN fractions as well as large-scale real-world noise benchmark Clothing1M (Xiao et al. 2015). We use a CNN on MNIST and the Wide ResNet 28×10 (Zagoruyko and Komodakis 2016) on CIFAR-10. On Clothing1M, we use the ResNet-50 (He et al. 2016) following the benchmark setting (Patrini et al. 2017; Tanaka et al. 2018; Xu et al. 2019). More details on experiments can be found in Appendix A.

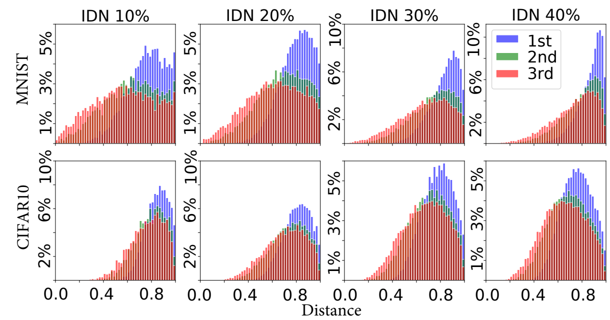


Figure 5: Histograms of the distance distribution. The distance is evaluated between the true label and the soft label obtained by SEAL in 1-3 iteration.

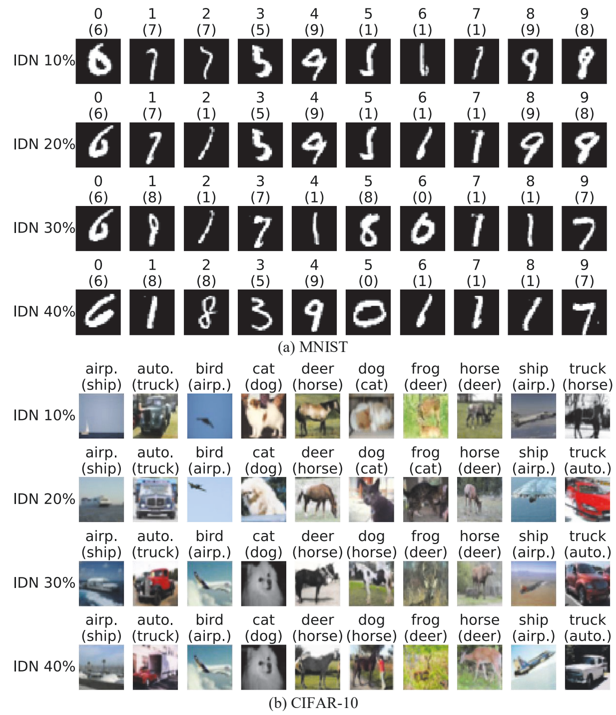


Figure 6: The noisy label and corrected label (in parentheses) obtained from SEAL.

SEAL corrects label noise. We first evaluate the distance between the true label and the soft label obtained by SEAL for corrupted instances, which is defined as

$$d(\bar{S}_i, y_i) = \|\bar{S}_i - e_{y_i}\|_1 / \|e_{\bar{y}_i} - e_{y_i}\|_1, \tag{9}$$

where the denominator is to normalize the distance such that $d(\bar{S}_i, y_i) \in [0, 1]$. Before applying SEAL, the label is initialized as the given label $e_{\bar{y}_i}$ and the distance concentrates at 1.0. In Fig. 5, we show histograms of the distance distribution for all corrupted instances. When applying SEAL iteratively, the distance distribution moves toward the left, suggesting that the updated soft label approaches the true label. This verifies that SEAL can correct label noise on varying datasets and noise fractions. To study individual instances, we de-

Method	10%	20%	30%	40%
CE	94.07 ±0.29	85.62 ±0.56	75.75 ±0.09	65.83 ±0.56
Forward	93.93 ±0.14	85.39 ±0.92	76.29 ±0.81	68.30 ±0.42
Co-teaching	95.77 ±0.03	91.07 ±0.19	86.20 ±0.35	79.30 ±0.84
GCE	94.56 ±0.31	86.71 ±0.47	78.32 ±0.43	69.78 ±0.58
DAC	94.13 ±0.02	85.63 ±0.56	75.82 ±0.58	65.69 ±0.78
DMI	94.21 ±0.12	87.02 ±0.42	76.19 ±0.64	67.65 ±0.73
SEAL	**96.75** ±0.08	**93.63** ±0.33	**88.52** ±0.15	**80.73** ±0.41

Table 1: Testing accuracy (%) on MNIST under instance-dependent label noise with different noise fractions.

Method	10%	20%	30%	40%
CE	91.25 ±0.27	86.34 ±0.11	80.87 ±0.05	75.68 ±0.29
Forward	91.06 ±0.02	86.35 ±0.11	78.87 ±2.66	71.12 ±0.47
Co-teaching	91.22 ±0.25	87.28 ±0.20	84.33 ±0.17	78.72 ±0.47
GCE	90.97 ±0.21	86.44 ±0.23	81.54 ±0.15	76.71 ±0.39
DAC	90.94 ±0.09	86.16 ±0.13	80.88 ±0.46	74.80 ±0.32
DMI	91.26 ±0.06	86.57 ±0.16	81.98 ±0.57	77.81 ±0.85
SEAL	**91.32** ±0.14	**87.79** ±0.09	**85.30** ±0.01	**82.98** ±0.05

Table 2: Testing accuracy (%) on CIFAR-10 under instance-dependent label noise with different noise fractions.

fine $\bar{N}(x)$-the confidence that a label needs correction and $\tilde{y}(x)$-the corrected label as follows.

$$\bar{N}(x_i) = \max_{k \neq \bar{y}_i} \bar{S}_{i,k}, \quad \tilde{y}(x_i) = \arg\max_{k \neq \bar{y}_i} \bar{S}_{i,k}. \tag{10}$$

In Fig. 6, we present examples of the highest $\bar{N}(x)$ in each class, with the given noisy label ($\bar{y}$, synthesized by Algorithm 1) and corrected label ($\tilde{y}$, in parentheses, obtained by Algorithm 2) marked on top of each image. The examples verify that SEAL can identify and correct noisy labels.

SEAL improves generalization under IDN. We conduct experiments on MNIST and CIFAR-10 with IDN of varying noise fractions, compared with extensive baselines including (i) cross-entropy (CE) loss; (ii) Forward (Patrini et al. 2017), which trains a network to estimate an instance-*in*dependent noise transition matrix then corrects the loss; (iii) Co-teaching (Han et al. 2018b), where two classifiers select small-loss instances to train each other; (iv) Generalized Cross Entropy (GCE) loss, which is a robust version of CE loss with theoretical guarantee under CCN; (v) deep abstaining classifier (DAC) (Thulasidasan et al. 2019), which gives option to abstain samples depending on the cross-entropy error and an abstention penalty; (vi) Determinant based Mutual Information (DMI), which is an information-theoretic robust loss function. The number of iterations is 10 on MNIST and 3 on CIFAR-10. SEAL consistently achieves the best generalization performance, as shown in Table 1 and Table 2, where we report the accuracy at the last epoch and repeat each experiment three times.

Method	Testing accuracy
CE∗	68.94
Forward∗	69.84
Co-teaching	70.15
GCE∗	69.09
Joint Optimization∗	72.16
DMI∗	72.46
CE	69.07
SEAL	**70.63**
DMI	72.27
SEAL (DMI)	**73.40**

Table 3: Testing accuracy (%) on Clothing1M. The $*$ marks published results.

SEAL improves generalization under real-world noise. Clothing1M (Xiao et al. 2015) is a large-scale real-world dataset of clothes collected from shopping websites, with noisy labels assigned by the surrounding text. Following the benchmark setting (Patrini et al. 2017; Tanaka et al. 2018; Xu et al. 2019), the training set consists of $1M$ noisy instances and the additional validation, testing sets consist of $14K$, $10K$ clean instances. The number of SEAL iterations is 3. In Table 3, we present the test accuracy. By default, SEAL is implemented with normal cross-entropy, where we see 1.56% absolute improvement. Notably, SEAL also improves advanced training algorithms such as DMI (Xu et al. 2019) when we use DMI at the first iteration.

5 Conclusion

In this paper, we theoretically justify the urgent need to go beyond the CCN assumption and study IDN. We formalize an algorithm to generate controllable IDN which is semantically meaningful and challenging. As a primary attempt to combat IDN, we propose a method SEAL, which is effective for both synthetic IDN and real-world noise.

Notably, our theoretical analysis in Section 2 provides rigorous motivations for studying IDN. Learning with IDN is an important topic that deserves more research attention in future.

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A More Details on Experiments

On **MNIST**, we use a convolution neural network (CNN) with the standard input 28×28 and 4 layers as follows: [conv 5×5, filters 20, stride 1, relu, maxpool /2]; [conv 5×5, filters 50, stride 1, relu, maxpool /2]; [fully connect 4*4*50→500, relu]; [fully connect 500→10, softmax]. Models are trained for 50 epochs with a batch size of 64 and we report the testing accuracy at the last epoch. For the optimizer, we use SGD with a momentum of 0.5, a learning rete of 0.01, without weight decay.

On **CIFAR-10**, we use the Wide ResNet 28×10. Models are trained for 150 epochs with a batch size of 128 and we report the testing accuracy at the last epoch. From Fig 4 in the main paper, we can see that the epoch of 150 is sufficient to achieve 100% training accuracy. For the optimizer, we use SGD with a momentum of 0.9 and a weight decay of 5×10^{-4}. The learning rate is initialized as 0.1 and is divided by 5 after 60 and 120 epochs. We apply the standard data augmentation on CIFAR-10: horizontal random flip and 32×32 random crop after padding 4 pixels around images. The standard normalization with mean=(0.4914, 0.4822, 0.4465), std=(0.2023, 0.1994, 0.2010) is applied before feeding images to the network.

On **Clothing1M**, following the benchmark setting (Patrini et al. 2017; Tanaka et al. 2018; Xu et al. 2019), we use the ResNet-50 pre-trained on ImageNet and access the clean validation set consisting of $14K$ instances to do model selection. Models are trained for 10 epochs with a batch size of 256 on the noisy training set consisting of $1M$ instances. For the optimizer, we use SGD with a momentum of 0.9 and a weight decay of 10^{-3}. We use a learning rate of 10^{-3} in the first 5 epochs and 10^{-4} in the second 5 epochs in all experiments except for DMI (Xu et al. 2019), where the learning rate is 10^{-6} and 0.5×10^{-6} according to its original paper. We apply the standard data augmentation: horizontal random flip and 224×224 random crop. Before feeding images to the network, we normalize each image with mean and std from ImageNet, i.e., mean=(0.485, 0.456, 0.406), std=(0.229, 0.224, 0.225). Considering that a pre-trained model and a clean validation are accessed in all methods, we do not reinitialize our model in each SEAL iteration. Instead, we start the training on top of the best model from the last iteration.

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