Ranking Sets of Defeasible Elements in Preferential Approaches to Structured Argumentation: Postulates, Relations, and Characterizations

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Abstract

Preferences play a key role in computational argumentation in AI, as they reflect various notions of argument strength vital for the representation of argumentation. Within central formal approaches to structured argumentation, preferential approaches are applied by lifting preferences over defeasible elements to rankings over sets of defeasible elements, in order to be able to compare the relative strength of two arguments and their respective defeasible constituents. To overcome the current gap in the scientific landscape, we give in this paper a general study of the critical component of lifting operators in structured argumentation. We survey existing lifting operators scattered in the literature of argumentation theory, social choice, and utility theory, and show fundamental relations and properties of these operators. Extending existing works from argumentation and social choice, we propose a list of postulates for lifting operations, and give a complete picture of (non-)satisfaction for the considered operators. Based on our postulates, we present impossibility results, stating for which sets of postulates there is no hope of satisfaction, and for two main lifting operators presented in structured argumentation, Elitist and Democratic, we give a full characterization in terms of our postulates.

1 Introduction

Computational argumentation has established itself as a distinguished and vital research area within Artificial Intelligence (AI) that provides foundational approaches to nonmonotonic reasoning (Baroni et al. 2018; Bench-Capon and Dunne 2007), with heterogeneous applications, e.g., in legal reasoning, medical sciences, and e-government (Atkinson et al. 2017). In approaches to what is known as structured argumentation (Bondarenko et al. 1997; Modgil and Prakken 2013; García and Simari 2004; Besnard and Hunter 2008), formal reasoning processes are specified that give provably rational accounts (Caminada 2018) of what can be argued for, when faced with knowledge or beliefs that are possibly conflicting or inconsistent. Reasoning is carried out by formally defining how arguments can be constructed, and how arguments are conflicting, e.g., whether one argument is a counter-argument for another. Argumentation semantics (Baroni, Caminada, and Giacomin 2011) then state which sets of arguments can be deemed jointly acceptable together, e.g., because they form a non-conflicting viewpoint that defends itself against all counter-arguments.

Key to argumentative reasoning are preferential approaches (Beirlaen et al. 2018), which can reflect a wide variety of notions of argument strength, e.g., subjective and relative trust of arguments or societal values (Bench-Capon 2003). In major structured argumentation formalisms, preferences are applied to argumentative reasoning by considering preference relations on the components that constitute arguments. Arguments, e.g., in the structured formalisms of assumption-based argumentation (ABA) (Bondarenko et al. 1997) and ASPIC$^+$ (Modgil and Prakken 2013), are based on derivations (via rules) starting off from assumptions (premises). Importantly, some parts of arguments are deemed defeasible, e.g., assumptions or defeasible rules, while others are considered strict, i.e., logically sound deductions that cannot be attacked or countered. In this sense, defeasible elements of an argument present potential points of attacks or counters.

As illustrated in Figure 1, preferences are applied by considering a preference ordering over the defeasible elements, which are then lifted to comparisons of sets of defeasible elements. Using such a lifting, one inspects then certain defeasible elements of two arguments and ranks the arguments based on the ranking of their respective sets of defeasible elements. In ASPIC$^+$, e.g., one can define to consider the structure of arguments, and compare all or only some of the defeasible elements (e.g., only top-most defeasible rules).

In this way, structured argumentation formalisms are general frameworks that allow for a wide array of possible ways of inclusion of preferences. In particular, ASPIC$^+$ allows for a large family of such lifting operators that lift a ranking of the basic defeasible elements that can form arguments to rankings on sets of defeasible elements. While in recent years we have seen an increasing attention on research of preferential approaches to (structured) argumentation (Beirlaen et al. 2018), foundational and general research on preferences in argumentation is still lacking. We take up this opportunity and provide a foundational study of the key component of preferential reasoning in structured argumentation: we formally study the important lifting operation, from ranking of individual defeasible elements to ranking of sets.

Towards a general treatment, we base our study on the methodology found in (computational) social choice (Brandt et al. 2016) and utility theory (Barberà, Bossert, and Pattanaik 2004), both fields which investigate ways of aggregating (rankings of) individual objects to (a ranking of) sets of objects, and apply such methods to the case of lifting operators for structured argumentation. Concretely, our main contributions are as follows.

- We collect lifting operators scattered throughout the literature that lift partial orderings over (defeasible) elements: we include the Elitist and Democratic operators from ASPIC$^+$ (Modgil and Prakken 2018), Hoare's and Smyth's operators (see, e.g., Brewka, Truszczynski, and Woltran (2010)), the well-known operators due to Kelly (1977), Fishburn (1972) and (Gärdenfors 1976) from social choice and further operators defined in the argumentation literature (Beirlaen et al. 2018; Young, Modgil, and Rodrigues 2016; Dyrkolbotn, Pedersen, and Broersen 2018). Additionally, we also consider the inverse of the Elitist and Democratic operator and a generalized form of the lexicographic orderings.
- We show relations among these operators, and, among other results, find that all operators extend Kelly's operator, yet diverge significantly in different directions.
- Based on existing works, we propose a list of postulates (properties of interest), together with new ones by us, and provide a complete picture which postulate is (not) satisfied by an operator, highlighting, e.g., that the property of being "reasonable-inducing" from Modgil and Prakken (2018), a sufficient condition for reaching certain desirable properties, is not satisfied, e.g., by Smyth's, Fishburn's, and Gärdenfors's operators.
- Based on our postulates, we give several impossibility results that state which (sets of) postulates cannot be satisfied together.
- We fully characterize the two main operators Elitist and Democratic via our postulates.

To give a clear picture, in Section 2 we give the formal background on how preferences are incorporated in ASPIC$^+$ and how lifting operators are used. We note that our results can also be applied to other formalisms, such as ABA$^+$ (Čyras and Toni 2016), an extension of ABA with preferences, and Defeasible Logic Programming (DeLP) (García and Simari 2004), which utilizes liftings. Section 3 presents the lifting operators, Section 4 shows properties and relations of the operators, Section 5 details our analysis regarding postulates and gives our characterization results.

2 Preferences in the ASPIC$^+$ Framework

For our contributions studying lifting operators, we present the context in the well-known ASPIC$^+$ (Modgil and Prakken 2018) framework, where they are used for defining preferences of arguments. ASPIC$^+$ is a general formal framework

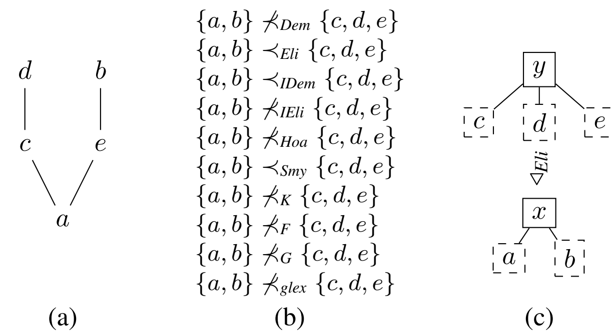

Figure 1: Ordering over defeasible elements $\leq$ (a) (higher is more preferred) is lifted to ordering over sets of defeasible elements $\prec_X$ under a variety of operators X (b). Finally an ordering $\triangleleft$ over arguments is induced (c), here shown for the Elitist ordering and two arguments.

for argumentation with several ingredients; due to space constraints we recap only the part required for understanding preferences in the framework, and refer the reader to Modgil and Prakken (2018) for details. We assume a language $\mathcal{L}$ composed of atoms x. One part of an ASPIC$^+$ framework is a knowledge base $\mathcal{K} \subseteq \mathcal{L}$ consisting of a defeasible part (called ordinary premises $\mathcal{K}_p$) and a non-defeasible part (axioms $\mathcal{K}_n$). Another part of ASPIC$^+$ is a set of rules over $\mathcal{L}$, denoted by $\mathcal{R}$. This set is composed of defeasible rules $a_1, \ldots, a_n \Rightarrow b$ and strict rules $a_1, \ldots, a_n \rightarrow b$. We denote the set of defeasible rules by $\mathcal{R}_d$ and the set of strict rules by $\mathcal{R}_s$. When we do not distinguish between strict or defeasible rules, we write $a_1, \ldots, a_n \rightsquigarrow b$. We restrict each set and rule to be finite.

Arguments are constructed as follows in an inductive way.

- If $x \in \mathcal{K}$, then $A = x$ is an argument with $\texttt{Conc}(A) = x$.
- If $A_1, \ldots, A_n$ are arguments, $x_i = \texttt{Conc}(A_i)$ for $1 \leq i \leq n$, and $(x_1, \ldots, x_n \rightsquigarrow x) \in \mathcal{R}$, then $A = A_1, \ldots, A_n \rightsquigarrow x$ is an argument with $\texttt{Conc}(A) = x$.

We use the following shorthands for defeasible elements in an argument A: $\texttt{Prem}_\texttt{d}(A)$ denotes the ordinary premises (sub-arguments in $\mathcal{K}_p$), $\texttt{DefRules}(A)$ denotes the defeasible rules in A, and $\texttt{LastDefRules}(A)$ which is equal to the top-most rule if it is defeasible, and, otherwise, via a recursive definition, equals the top-most defeasible rules of the immediate sub-arguments.

Preferences in ASPIC$^+$ are defined via orderings on the defeasible elements. In fact, three levels of preferences are used in ASPIC$^+$: an ordering $\leq$ on defeasible elements, an ordering $\prec$ on sets of defeasible elements, and an ordering $\triangleleft$ on arguments. In ASPIC$^+$, two preorders $\leq_1$ and $\leq_2$ on $\mathcal{K}_p$ and $\mathcal{R}_d$, respectively, are used in order to define two liftings $\prec_1$ and $\prec_2$ on sets $D \subseteq \mathcal{K}_p$ and $D' \subseteq \mathcal{R}_d$. The focus of this work is on such lifting operators; we recall a wide variety of them (including the ones defined by Modgil and Prakken (2018)) in Section 3.

Based on $\prec$, one can define $\triangleleft$ (ranking on arguments), as follows, using either the so-called last-link principle or the weakest link principle. Using the weakest-link principle one compares all defeasible elements: argument B is more preferred than A ($A \lhd B$) if $\mathcal{R}_d(A) = \mathcal{R}_d(B) = \emptyset$, then $\texttt{Prem}_\texttt{d}(A) \prec \texttt{Prem}_\texttt{d}(B)$; if $\texttt{Prem}_\texttt{d}(A) = \texttt{Prem}_\texttt{d}(B) = \emptyset$, then $\mathcal{R}_d(A) \prec \mathcal{R}_d(B)$; else $\texttt{Prem}_\texttt{d}(A) \prec \texttt{Prem}_\texttt{d}(B)$ and $\mathcal{R}_d(A) \prec \mathcal{R}_d(B)$. Using the last link principle, we have $A \lhd B$ if $\texttt{LastDefRules}(A) \prec \texttt{LastDefRules}(B)$, or $\texttt{LastDefRules}(A) = \texttt{LastDefRules}(B) = \emptyset$ and $\texttt{Prem}_\texttt{d}(A) \prec \texttt{Prem}_\texttt{d}(B)$.

Preferences play a key role in argumentative conflict resolution in ASPIC$^+$: in certain cases if argument A attacks argument B, but $A \prec B$, the weaker argument A does not defeat B, and, e.g., one does not need to defend against the attack from A.

Example 1. *Consider atoms $\{a, b, c, d, e, x, y\}$ with ordinary premises $\{a, b, c, d, e\}$, two strict rules $a, b \rightarrow x$ and $c, d, e \rightarrow y$, leading to two arguments as shown in Figure 1(c). Dashed objects denote defeasible elements. With a preference order over the ordinary premises as shown in Figure 1(a), one can derive orderings over sets of defeasible elements (see subsequent section) shown in Figure 1(b), leading, under the weakest link principle, to the argument ordering shown in Figure 1(c) when using the particular Elitist operator.*

3 Lifting Operators

We survey and collect lifting operators from the literature in argumentation and other research areas. We focus on lifting operators that are defined on preorders, i.e., a binary relation that is reflexive and transitive, and return a strict partial order, i.e., a binary relation that is irreflexive, transitive, and asymmetric. Our focus on preorders (not total orderings) on defeasible elements is justified by their generality, and that it seems plausible that not all defeasible elements in an argumentation theory can be compared. Returning strict partial orders give us the key comparison between sets of defeasible elements (arguments) we require: whether a set (an argument) is *strictly* more preferred than another. Further, as in the current state of the art in, e.g., ASPIC$^+$, we focus on lifting operators that only take the set of defeasible elements and a preference ordering on them as input. In the following, we will use the symbols $\leq$ and $\preceq$ to refer to binary relations that are reflexive. Additionally, we use $<$ and $\prec$ to refer to binary relations that are irreflexive. Observe that for every preorder $\leq$, there is a corresponding strict partial order $<$ defined by $A < B$ iff $A \leq B$ and $B \not\leq A$. Furthermore, we write $A \sim B$ iff $A \preceq B$ and $B \preceq A$ hold.

In argumentation theories having an empty set of defeasible elements represents non-defeasibility (e.g., a logical deduction). Thus, we require that the empty set is handled differently: it cannot be less preferred than a non-empty set, and, if contained in a comparison, must be more preferred. We assume all subsequent operators to satisfy this condition without mentioning it in the definition explicitly and define them only on comparisons of non-empty sets. Some of the operators from outside the argumentation literature are in a straightforward fashion adapted to satisfy this condition.

First, we define what an operator is.

Definition 1. *An* operator *takes as input a set X and a preorder $\leq$ on X and outputs a strict partial order $\prec$ on the powerset of X.*

We begin with the Elitist and Democratic lifting operators from the ASPIC$^+$ framework (Modgil and Prakken 2018).

Definition 2 (Elitist)**.** *Let $\leq$ be a preorder on a set X. Define $\prec_{Eli}$ for two non-empty $A, B \subseteq X$ by $A \prec_{Eli} B$ iff $\exists a \in A$ s.t. $\forall b \in B$ we have $a < b$.*

That is, B is more preferred to A if there is at least one defeasible element in A that is (strictly) less preferred to each element in B. Switching the quantifications leads directly to the Democratic operator.

Definition 3 (Democratic)**.** *Let $\leq$ be a preorder on a set X. Define $\prec_{Dem}$ for two non-empty $A, B \subseteq X$ by $A \prec_{Dem} B$ iff $\forall a \in A$ we have $\exists b \in B$ s.t. $a < b$.*

We observe that $\prec_{Dem}$ is only irreflexive if the sets are finite, as, e.g., $\mathbb{N} \sim_{Dem} \mathbb{N}$. The Democratic operator could be considered a variant[1] of an operator called Hoare's operator (see, e.g., (Brewka, Truszczynski, and Woltran 2010)):

Definition 4 (Hoare)**.** *Let $\leq$ be a preorder on a set X. Define $\preceq_{Hoa}$ for two non-empty $A, B \subseteq X$ by $A \preceq_{Hoa} B$ iff $\forall a \in A$ we have $\exists b \in B$ s.t. $a \leq b$.*

This operator is not asymmetric as, e.g., $\{a\} \sim_{Hoa} \{b\}$ whenever $a \leq b$ and $b \leq a$. In accordance with our framework, we will only consider the corresponding strict partial order $\prec_{Hoa}$ defined by $A \prec_{Hoa} B$ if and only if $A \preceq_{Hoa} B$ and $B \not\preceq_{Hoa} A$. Hoare's operator is often considered together with its mirrored version, Smyth's operator (Brewka, Truszczynski, and Woltran 2010). Inspired by this, we also consider the mirrored version of the Democratic and the Elitist operator.

Definition 5 (Inverse Democratic)**.** *Let $\leq$ be a preorder on a set X. Define $\prec_{IDem}$ for two non-empty $A, B \subseteq X$ by $A \prec_{IDem} B$ iff $\forall b \in B$ we have $\exists a \in A$ s.t. $a < b$.*

Again, we can also consider a variant of this operator by replacing $<$ with $\leq$. We call (the strict part of) this operator Smyth and write $\prec_{Smy}$ for it. In order to see that Inverse Democratic is the inverse of the Democratic operator, we define the inverse $\leq^{-1}$ of a relation $\leq$ by $a \leq^{-1} b$ iff $a \geq b$. Then, we can see that for non-empty A, B, we have $A \prec_{IDem} B$ for a preorder $\leq$, iff $A \prec_{Dem}^{-1} B$ holds with respect to $\leq^{-1}$.

We can define an inverse of the Elitist operator similarly.

Definition 6 (Inverse Elitist)**.** *Let $\leq$ be a preorder on a set X. Define $\prec_{IEli}$ for two non-empty $A, B \subseteq X$ by $A \prec_{IEli} B$ iff $\exists b \in B$ s.t. $\forall a \in A$ we have $a < b$.*

The following operators are well known and often used in (computational) social choice, when studying strategyproofness (Barberà 2011; Brandt, Brill, and Harrenstein 2016), where such operators are often called extensions. Kelly's Extension could be considered as the "bare minimum" one requires when lifting a preference ordering.

[1]While this variant of Hoare's operator is less common, it is also considered outside of argumentation, for example in Benferhat, Lagrue, and Papini (2004) and, for linear orders, in Maly, Truszczynski, and Woltran (2019).

Definition 7 (Kelly's Extension)**.** *Let $\leq$ be a preorder on a set X. Define $\prec_K$ for two non-empty A, $B \subseteq X$ by $A \prec_K B$ iff $a < b$ holds for all $a \in A$ and $b \in B$.*

That is, under Kelly's operator, only if *every* $a \in A$ is ordered below all $b \in B$ we have a comparison in $\prec_K$.

Definition 8 (Fishburn's Extension)**.** *Let $\leq$ be a preorder on a set X. Define $\prec_F$ for two non-empty A, $B \subseteq X$ with $A \neq B$ by $A \prec_F B$ iff $a < b$, $a < x$ and $x < b$ holds for all $a \in A \setminus B$, $b \in B \setminus A$ and $x \in A \cap B$.*

Fishburn's operator relaxes Kelly's condition by allowing an 'overlap', as long as all elements that are just in A are ordered below all elements in B and all elements that are just in B are all ordered above all elements in A.

Definition 9 (Gärdenfors's Extension)**.** *Let $\leq$ be a preorder on a set X. Then for two non-empty A, $B \subseteq X$ with $A \neq B$ define $A \prec_G B$ iff one of the following holds:*

1. *$A \subset B$ and $a < b$ for all $a \in A$ and $b \in B \setminus A$.*
2. *$B \subset A$ and $a < b$ for all $a \in A \setminus B$ and $b \in B$.*
3. *Neither $A \subset B$ nor $B \subset A$ and $a < b$ for all $a \in A \setminus B$ and $b \in B \setminus A$.*

Intuitively, Gärdenfors's operator ignores the intersection of A and B completely and just demands that all elements that are just in A are smaller than all elements that are just in B (condition 3). This does not work if either $A \subseteq B$ or $B \subseteq A$, hence conditions 1 and 2 handle these cases.

Next we recall some further operators studied in the argumentation literature, The following operator appears in Beirlaen et al. (2018).

Definition 10. *Let $\leq$ be a preorder on a set X. Define $\preceq_{Dom}$ by $A \preceq_{Dom} B$ iff for all $a \in A$ there is a $b \in B$ for which $a \leq b$ and for all $b' \in B$ there is an $a' \in A$ for which $a' \leq b'$.*

This operator, which equals $\preceq_{Hoa} \cap \preceq_{Smy}$, is often called Plotkin's operator (Brewka, Truszczynski, and Woltran 2010). It is not asymmetric. In the following, we will again only consider the strict part of this operator.

In Beirlaen et al. (2018) two other operators are surveyed, called $\prec_{max\text{-}min}$ and $\prec_{min\text{-}min}$. We observe that, on finite sets, the former equals Kelly's extension, while the latter equals Inverse Democratic.

The following operator, called Disjoint Elitist (DEli) is studied by Young, Modgil, and Rodrigues (2016) and Dyrkolbotn, Pedersen, and Broersen (2018), which gives intuitive results in certain scenarios.

Definition 11. *Let $\leq$ be a preorder on a set X. Define $\prec_{DEli}$ by $A \prec_{DEli} B$ iff $\exists a \in A \setminus B$ s.t. $\forall b \in B \setminus A$ we have $a < b$.*

Observe that this operator is not transitive in the general case. Consider elements a, b, c, d such that $a < b$ and $c < d$. Then, this operator implies $\{a, c\} \prec_{DEli} \{b, c\}$ and $\{b, c\} \prec_{DEli} \{b, d\}$ but $\{a, c\} \not\prec_{DEli} \{b, d\}$. For using this operator further restrictions were imposed, but since we aim for general operators, we do not include operators that are non-transitive in the general case in our study.

Finally, we consider the famous lexicographic order, see e.g. (Fishburn 1974). Usually, the lexicographic order is only defined if we have a linear order on X. However, we observe that one way of defining the lexicographic order is $A \prec_{lex} B$ iff $\min(A \Delta B) \in A$. Here, Δ is the symmetric difference operator defined by $A \Delta B = (A \setminus B) \cup (B \setminus A)$. This definition also makes sense if we only have a preorder on X and hence can be used to generalize the lexicographic order to that setting.

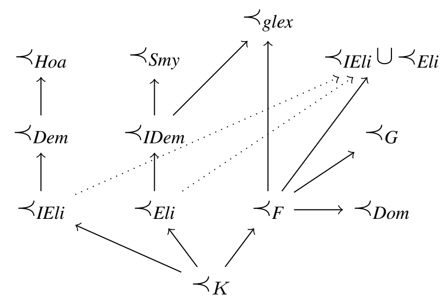

Figure 2: Relationship between operators. Arrow from operator a to b means a is contained in b. Dotted arrows for relationships that hold by definition.

Definition 12 (Generalized Leximin)**.** *Let $\leq$ be a preorder on a set X. Define $\prec_{glex}$ for two non-empty A, $B \subseteq X$ by $A \prec_{glex} B$ iff $\min(A \Delta B) \neq \emptyset$ and $\min(A \Delta B) \subseteq A \setminus B$.*[2]

It can be checked that $\prec_{glex}$ is a strict partial order and equals the usual lexicographic order if $\leq$ is a linear order. The leximax order could be generalized similarly. However, under the leximax order a set is preferred to all its subsets. In the argumentation domain, it is much more natural to prefer less defeasible elements, hence we will focus mostly on the leximin order.

4 Relations between Operators

We begin our formal analysis of the lifting operators by showing their relationships. We present relations between lifting operators $\prec_s$, $\prec_t$ when lifted from the same underlying preorder $\leq$ over the same underlying set X. Then, e.g., $\prec_s \subseteq \prec_t$ indicates that $A \prec_s B$ implies $A \prec_t B$ for all $A, B \subseteq X$. For brevity, in the following formal statements we tacitly assume presence of $\leq$ and X, without explicating them. The results are summarized in Figure 2.

First it is well known (see, e.g., Brandt and Brill (2011)), that we have $\prec_K \subseteq \prec_F \subseteq \prec_G$. Moreover, as we will see, all considered operators contain $\prec_K$, reflecting the very strict requirement of Kelly's operator. In particular, following from the definitions, the following relationships hold.

Proposition 1. *It holds that $\prec_K \subseteq \prec_{\mathcal{O}}$ for $\mathcal{O} \in \{Eli, IEli, F\}$.*

On the other hand, Elitist, and its inverse version do not extend Fishburn's operator, yet the union of Elitist and its inverse version fully contains Fishburn's operator.

Proposition 2. *We find that $\prec_F \not\subseteq \prec_{Eli}$, $\prec_F \not\subseteq \prec_{IEli}$, and $(\prec_{Eli} \cap \prec_{IEli}) \not\subseteq \prec_F$, but $\prec_F \subseteq (\prec_{Eli} \cup \prec_{IEli})$.*

[2]Here, we define $\min(A) := \{a \in A \mid \nexists b \in A \text{ s.t. } b < a\}$

The union $\prec_{Eli} \cup \prec_{IEli}$ does not yield an interesting lifting operator according to our requirements as it is not transitive. Consider elements a, b, c, d, e such that $a < c$ and $c < d$ and no other comparisons are possible. $\{a, b\} \prec_{Eli} \{c\} \prec_{IEli} \{d, e\}$ but neither $\{a, b\} \prec_{Eli} \{d, e\}$ nor $\{a, b\} \prec_{IEli} \{d, e\}$.

In their definitions, Elitist and Democratic (and inverse Elitist and Democratic operator) are dual by switching quantifications. This is reflected in the following relation.

Proposition 3. *We have* $\prec_{Eli} \subseteq \prec_{IDem}$ *and* $\prec_{IEli} \subseteq \prec_{Dem}$.

These inclusions are, in general, strict. Hoare's and Smyth's operators extend this line of inclusions even further.

Proposition 4. *We have* $\prec_{Dem} \subseteq \prec_{Hoa}$ *and* $\prec_{IDem} \subseteq \prec_{Smy}$.

Again, these inclusions are, in general, strict. On the other hand, Gärdenfors's operator is not comparable to any of these operators.

Proposition 5. *We find that* $\prec_{G} \nsubseteq (\prec_{Hoa} \cup \prec_{Smy})$ *and* $(\prec_{Eli} \cap \prec_{IEli}) \nsubseteq \prec_{G}$.

Next we consider relations regarding $\prec_{Dom}$. By definition, we have $\preceq_{Dom} = \preceq_{Hoa} \cap \preceq_{Smy}$, but, nevertheless, $\prec_{Dom} \nsubseteq \prec_{Hoa}$, as for $a < b$ we have $\{a, b\} \prec_{Dom} \{b\}$ but $\{a, b\} \sim_{Hoa} \{b\}$. Similarly, $\prec_{Dom} \nsubseteq \prec_{Smy}$.

Proposition 6. *We have* $\prec_{Dom} \nsubseteq \prec_{Hoa}$*,* $\prec_{Dom} \nsubseteq \prec_{Smy}$ *and* $\prec_{Dom} \nsubseteq (\prec_{Eli} \cup \prec_{IEli})$ *while* $\prec_{Eli} \nsubseteq \prec_{Dom}$ *and* $\prec_{IEli} \nsubseteq \prec_{Dom}$ *and* $\prec_{F} \subseteq \prec_{Dom}$ *but* $\prec_{G} \nsubseteq \prec_{Dom} \nsubseteq \prec_{G}$.

That is, $\prec_{Dom}$ "branches off" from Fishburn's operator, but differently than Gärdenfors's and the family of operators for Democratic, Elitist, Hoare and their inverses.

Finally, we consider $\prec_{glex}$, which extends Smyth's and Fishburn's operators.

Proposition 7. *We find that* $\prec_{glex}$ *is not contained in any other operator, but* $\prec_{IDem} \subseteq \prec_{glex}$ *and* $\prec_{F} \subseteq \prec_{glex}$ *while, on the other hand* $\prec_{Smy}, \prec_{G}, \prec_{IEli}, \prec_{Dom} \nsubseteq \prec_{glex}$.

Example 2. *Let* $X = \{a, b, c, d, e, f, g\}$ *and let* $\leq$ *be given by* $a < c < e < g$ *and* $b < d < e$*. Then,* $\{a, b\} \prec_{\mathcal{O}} \{c, d\}$ *holds for example for* $\mathcal{O} = Dem$ *or* $\mathcal{O} = Hoa$ *as* $a < c$ *and* $b < d$ *holds. However,* $\{a, b\} \not\prec_{IEli} \{c, d\}$ *as no element in* $\{c, d\}$ *is preferred to all elements in* $\{a, b\}$*. In contrast,* $\{a, b\} \prec_{K} \{e\}$ *as* $a, b < e$ *and hence the same holds for all operators. Now,* $\{a, b\} \prec_{\mathcal{O}} \{e, f\}$ *holds for* $\mathcal{O} = IEli$ *and hence also for* $\mathcal{O} = Dem$ *as* $a, b < e$ *but not for* $\mathcal{O} = Smy$ *as neither* $a \leq f$ *nor* $b \leq f$*. Furthermore, we have* $\{a, b\} \not\prec_{glex} \{e, f\}$ *as* $f \in \min(\{a, b\} \Delta \{e, f\})$*. Next, consider* $\{a, c, e\} \prec_{\mathcal{O}} \{c, e\}$*. It is easy to check that this is satisfied for* $\mathcal{O} = F$ *and* $\mathcal{O} = Eli$ *but not by* $\mathcal{O} = Dem$ *as there is no* $x \in \{a, c, e\}$ *such that* $e < x$*. Additionally, we have* $\{a, c, e\} \prec_{Dom} \{c, e\}$ *as* $a \leq a, c, e$ *and* $a, c, e \leq e$ *holds, implying* $\{a, c, e\} \preceq_{Dom} \{c, e\}$ *and* $c, e \not\leq a$ *implying* $\{c, e\} \not\preceq_{Dom} \{a, c, e\}$*. Finally, consider* $\{a, c, g\} \prec_{\mathcal{O}} \{a, e, g\}$*. This only holds for* $\mathcal{O} = glex$ *and* $\mathcal{O} = G$*. In particular,* $\{a, c, g\} \not\prec_{Dom} \{a, e, g\}$ *because* $\{a, c, g\} \preceq_{Dom} \{a, e, g\}$ *and* $\{a, e, g\} \preceq_{Dom} \{a, c, g\}$ *hold.*

5 Postulates on Lifting Operations

In this section, we will define several postulates on lifting operators, discuss situations in which these postulates are desirable and show several impossibility results. These postulates state properties that should be satisfied by an operator $\prec$ for all sets X and preorders $\leq$. For brevity, we again omit in the postulates and formal statements the underlying order $\leq$ on X. In the following, we use A, B to denote non-empty subsets of X. We summarize our findings in Table 1.

First of all, if we compare two sets of premises that both contain only one premise, then we naturally expect the comparison between the sets to equal the comparison between their single elements. This property is often called the extension rule or just extension (Barberà, Bossert, and Pattanaik 2004). All operators we consider satisfy this postulate.

Extension rule If $A = \{x\}$, $B = \{y\}$ then $A \prec B$ iff $x < y$.

In most cases, we only want to set a preference between two sets A and B if there is sufficient "reason" to do so: a basic requirement is then that there is an element a in A and an element b in B such that the preference between a and b explains the preference between A and B.

Comparability $A \prec B$ implies $\exists a \in A, b \in B$ s.t. $a < b$.

It is possible that in some situations we also accept other "reasons" to prefer one set to another. For example, if we value a small set of premises we could set $A \prec B$ whenever $B \subsetneq A$ or even $|B| < |A|$. Such operators would be, as we call them, not (solely) comparison-based. The operators that we consider that show such a behavior are $\prec_{Hoa}$, $\prec_{Smy}$ and $\prec_{glex}$, where $\prec_{glex}$ sets $A \prec_{glex} B$ whenever $A \supsetneq B$, $\prec_{Hoa}$ sets $A \prec_{Hoa} B$ whenever $A \subsetneq B$ and $B \not\preceq_{Hoa} A$, i.e., if there is a $b \in B$ such that $b \not\leq a$ for all $a \in A$ and $\prec_{Smy}$ sets $A \prec_{Smy} B$ whenever $A \supsetneq B$ and $B \not\preceq_{Smy} A$. Consequently, these operators do not satisfy Comparability.

The following is a rephrasing from Modgil and Prakken (2018), which if satisfied (together with irreflexivity and transitivity) is a sufficient condition forming a class of lifting operators satisfying further desirable properties in ASPIC^{+}.[3]

Reasonable inducing If $(\cup_{1 \leq i \leq n} A_i) \prec B$ then for some i, $1 \leq i \leq n$, we have $A_i \prec B$.

Within postulates to follow, an alternative name for this would be weak left decomposition. It is interesting to note, that reasonable inducing essentially rules out setting $A \prec B$ for all $B \subsetneq A$ (i.e., preferring proper subsets).

Proposition 8. *If a strict partial order* $\prec$ *that is reasonable inducing sets* $A \prec B$ *for all* $B \subsetneq A$*, then there are sets* A *and* B *such that* $\forall a \in A \forall b \in B, a < b$ *but* $B \prec A$*.*

Hence, $\prec_{glex}$ can not be reasonable including. We observe that a generalization of the leximax order would be reasonable inducing, which is possible as $A \prec B$ for all $A \subsetneq B$ does not contradict reasonable inducing.

We can strengthen reasonable inducing by turning the "for some" into a "for all".

Strong left decomposition If $(\cup_{1 \leq i \leq n} A_i) \prec B$ then for all i, $1 \leq i \leq n$, we have $A_i \prec B$.

[3]In Modgil and Prakken (2018) the property is defined on arguments, we rephrased it to the underlying set comparison.

postulate	*Dem*	*Eli*	*IDem*	*IEli*	*Hoa*	*Smy*	*Dom*	*K*	*F*	*G*	*glex*
Comparability	✓	✓	✓	✓	x	x	✓	✓	✓	✓	x
Reasonable Inducing	✓	✓	x	✓	✓	x	x	✓	x	x	x
Strong Left Decomposition	✓	x	x	✓	✓	x	x	✓	x	x	x
Weak Right Decomposition	x	✓	✓	✓	x	✓	x	✓	x	x	✓
Strong Right Decomposition	x	✓	✓	x	x	✓	x	✓	x	x	✓
Left Extension	✓	✓	✓	x	✓	✓	✓	x	x	x	✓
Strong Left Extension	x	✓	✓	x	x	✓	x	x	x	x	✓
Right Extension	✓	x	✓	✓	✓	✓	✓	x	x	x	x
Strong Right Extension	✓	x	x	✓	✓	x	x	x	x	x	x
Strict Independence	x	x	x	x	x	x	x	x	x	x	✓
Responsiveness	x	x	x	x	x	x	x	x	x	✓	✓
Left Dominance	x	✓	✓	x	x	✓	✓	x	✓	✓	✓
Right Dominance	✓	x	x	✓	✓	x	✓	x	✓	✓	x

Table 1: Satisfaction of postulates

This is a very strong requirement and hence, it is surprising that several of our operators, including $\prec_{Dem}$ satisfy it.

Mirroring the preceding two postulates, the following postulates allow us to decompose the right side of a preference relation.

Weak right decomposition If $A \prec (\cup_{1 \leq i \leq n} B_i)$ then for some i, $1 \leq i \leq n$, we have $A \prec B_i$.

Strong right decomposition If $A \prec (\cup_{1 \leq i \leq n} B_i)$ then for all i, $1 \leq i \leq n$, we have $A \prec B_i$.

These postulates are, in a formal sense, symmetric to the left decomposition postulates.

Proposition 9. *A strict partial order $\prec$ satisfies weak/strong left decomposition if and only if $\prec^{-1}$ satisfies weak/strong right decomposition.*

It is possible to satisfy strong left and strong right decomposition at the same time, as witnessed by Kelly's operator. However, Kelly's operator is already the least restrictive operator with this property.

Proposition 10. *Assume $\prec$ is a strict partial order that satisfies extension, strong right decomposition and strong left decomposition. Then $\prec \subseteq \prec_K$.*

In the other direction, we can look at conditions under which we can add an element on the right or left side of a preference. We consider three conditions. In the weakest case, one can add an element if it is better/worse than all elements on the other side. In the next case, one can add an element if it is better/worse than at least one element on the other side. Finally, we consider the case that one can add an element without any condition.[4] Related postulates to strong left extension (LAMC) and strong right decomposition (RAMC) were studied by Heyninck and Straßer (2019).

Weak left extension If for all $b \in B$ we have $a < b$, then $A \prec B$ implies $A \cup \{a\} \prec B$.

Left extension If there is a $b \in B$ such that $a < b$, then $A \prec B$ implies $A \cup \{a\} \prec B$.

Strong left extension $A \prec B$ implies $A \cup \{a\} \prec B$.

Weak right extension If for all $a \in A$ we have $a < b$, then $A \prec B$ implies $A \prec B \cup \{b\}$.

Right extension If there is a $a \in A$ such that $a < b$, then $A \prec B$ implies $A \prec B \cup \{b\}$.

Strong right extension $A \prec B$ implies $A \prec B \cup \{b\}$.

All operators that we consider satisfy weak left as well as weak right extension. It turns out that Kelly's operator is the most restrictive operator that satisfies the extension rule, weak left and weak right extension.

Proposition 11. *Assume $\prec$ is a strict partial order that satisfies the extension rule, weak left and weak right extension Then $\prec_K \subseteq \prec$.*

We observe that Proposition 10 and 11 together characterize Kelly's operator.

The "left" versions of the preceding postulates have an argumentative flavour, since adding a defeasible element can be construed as weakening an argument; however scenarios are feasible where one does not prescribe such a behavior, and our postulates indicate potentially fitting operators.

We observe that strong left and strong right extension are incompatible under very weak conditions.

Proposition 12. *No strict partial order satisfies extension, strong left extension, and strong right extension.*

On the other hand, it is possible to satisfy strong right extension together with left extension, as witnessed by the Democratic operator. However, it could be argued that the behavior of the Elitist operator (strong left extension and weak right extension), Inverse Democratic and Smyth operator (strong left extension and right extension) are more desirable in certain argumentative contexts.

Next, we observe that it is not possible to satisfy strong extension, comparability, and strong decomposition postulates on the same side.

Proposition 13. *No strict partial order satisfies extension, comparability, strong right decomposition and strong right extension at the same time.*

[4]Many other cases are possible, for example adding elements that are incomparable to all elements on the other side. However, for the operators that we studied the given conditions turned out to be the most interesting ones.

Now, we consider postulates from the social choice literature. First, we consider strict independence, a monotonicity postulate that is desirable in many contexts (Barberà, Bossert, and Pattanaik 2004) and the responsiveness postulate often considered in fair allocation (Aziz et al. 2015).

Strict independence If $A \prec B$ and $x \notin A \cup B$, then $A \cup \{x\} \prec B \cup \{x\}$.

Responsiveness If $x \in A$, $y \notin A$ and $x < y$, then $A \prec (A \setminus \{x\}) \cup \{y\}$.

We observe the following:

Proposition 14. *Any strict partial order that satisfies extension and strict independence also satisfies responsiveness.*

Especially responsiveness can be appealing in the argumentation contexts: replacing a defeasible element with a more preferred one results in higher lifted preference. However, both postulates are hard to combine with the previously discussed postulates.

Proposition 15. *Let $\prec$ be a strict partial order that is reasonable inducing and satisfies extension and comparability. Then it is not possible that $\prec$ satisfies strong left extension together with either strict independence or responsiveness.*

Proposition 16. *No strict partial order satisfies extension, comparability and strong left decomposition together with either strict independence or responsiveness.*

Finally, we consider two dominance postulates, often called Gärdenfors's principle after Peter Gärdenfors who introduced a version of this postulate in Gärdenfors (1976).

Left dominance Let A be a non-empty set and $x \in X$ an element. If $x < y$ for all $y \in A$, then $A \cup \{x\} \prec A$.

Right dominance Let A be a non-empty set and $x \in X$ an element. If $y < x$ for all $y \in A$, then $A \prec A \cup \{x\}$.

In the social choice literature both postulates are often considered together as just dominance. A well known impossibility result by Barberà and Pattanaik (1984) tells us, that both dominance axioms together are incompatible with strict independence.

Proposition 17 (Barberà and Pattanaik 1984)**.** *There is no strict partial order that satisfies strict independence, left dominance and right dominance at the same time.*

On the other hand, we observe that strong right extension is incompatible with left dominance and vice versa.

Proposition 18. *No strict partial order satisfies strong right extension and left dominance. Similarly, no strict partial order satisfies strong left extension and right dominance.*

We already characterized Kelly's operator (Proposition 10 and 11). As it turns out, the postulates that we consider also suffice to characterize the Democratic as well as the Elitist operator (and therefore also their mirrored versions).

Proposition 19. *Let $\prec$ be a strict partial order that is reasonable inducing and satisfies extension and strong right decomposition. Then $\prec \subseteq \prec_{Eli}$.*

Proposition 20. *Let $\prec$ be a strict partial order that satisfies extension, strong left extension and weak right extension. Then $\prec_{Eli} \subseteq \prec$.*

Switching left and right gives a characterization of $\prec_{IEli}$.

Proposition 21. *Let $\prec$ be a strict partial order that satisfies comparability and strong left decomposition. Then $\prec \subseteq \prec_{Dem}$.*

Proposition 22. *Let $\prec$ be a strict partial order that satisfies extension, strong right extension and left extension. Then $\prec_{Dem} \subseteq \prec$.*

Switching left and right gives a characterization of $\prec_{IDem}$.

6 Related Work and Conclusions

The idea of ranking sets of objects based on a ranking of the objects can be traced back at least to antiquity where humans ordered words lexicographically (Daly 1967). In the social choice community, interest in rankings sets of object was sparked by investigating manipulability, e.g., by the famous result by Gibbard (1973) and Satterthwaite (1975). The axiomatic approach to the order lifting problem was mainly studied in utility theory. First works were published from the fifties onward (Kraft, Pratt, and Seidenberg 1959; Kim and Roush 1980), including the seminal results by Kannai and Peleg (1984) (see also Barberà, Bossert, and Pattanaik (2004)). However, a usual assumption is that the underlying order is total; which seems not plausibly to assume in argumentation theory. Lifted orders are also required in fair allocation, when one allocates according to ordinal preferences (Bouveret, Endriss, and Lang 2010; Aziz et al. 2015); in this context responsiveness is often the only assumption.

There are several works in the argumentation literature studying preferences (Beirlaen et al. 2018): in abstract argumentation (e.g., Amgoud and Vesic (2011, 2014)) and in structured argumentation (e.g., Modgil and Prakken (2018); Young, Modgil, and Rodrigues (2016); Čyras and Toni (2016); Wakaki (2017)). General axiomatic studies (Amgoud 2014; Dung 2016; Dung and Thang 2018; Liao et al. 2016) and studies of rationality (Caminada 2018) were presented, as well. Properties of liftings were studied by Dyrkolbotn, Pedersen, and Broersen (2018), Heyninck and Straßer (2019), and D'Agostino and Modgil (2020), but, to our knowledge, there is no study like ours in argumentation that collects a wide variety of operators and postulates, and studies implications, impossibility and characterization results.

In particular, we show that there are two important families of operators of increasing strength, from Elitist to Smyth and from Inverse Elitist to Hoare, while all other operators branch off differently from Fishburn's operator. Moreover, Kelly's operator turns out the best choice when only unequivocal preferences should be taken into account. We observe that the set of postulates satisfied by Elitist seems particularly desirable for applications in argumentation, but its extensions Inverse Democratic and Smyth are not reasonable inducing and hence less useful in ASPIC$^+$. Finally, Smyth and the generalized lexicographic order are sensible candidates for operators that are not solely comparison-based, for example if small sets are valued highly.

For future work, both studying further postulates as well as impact of applying lifting operators on argumentative acceptance in structured argumentation appear intriguing.

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References

Amgoud, L. 2014. Postulates for logic-based argumentation systems. *International Journal of Approximate Reasoning* 55(9): 2028–2048.

Amgoud, L.; and Vesic, S. 2011. Two Roles of Preferences in Argumentation Frameworks. In Liu, W., ed., *Proc. ECSQARU*, volume 6717 of *Lecture Notes in Computer Science*, 86–97. Springer.

Amgoud, L.; and Vesic, S. 2014. Rich preference-based argumentation frameworks. *International Journal of Approximate Reasoning* 55(2): 585–606.

Atkinson, K.; Baroni, P.; Giacomin, M.; Hunter, A.; Prakken, H.; Reed, C.; Simari, G. R.; Thimm, M.; and Villata, S. 2017. Towards Artificial Argumentation. *AI Magazine* 38(3): 25–36.

Aziz, H.; Gaspers, S.; Mackenzie, S.; and Walsh, T. 2015. Fair assignment of indivisible objects under ordinal preferences. *Artificial Intelligence* 227: 71–92.

Barberà, S. 2011. Strategyproof social choice. In *Handbook of Social Choice and Welfare*, volume 2, 731–831. Elsevier.

Barberà, S.; Bossert, W.; and Pattanaik, P. K. 2004. Ranking Sets of Objects. In Barberà, S.; Hammond, P. J.; and Seidl, C., eds., *Handbook of Utility Theory*, 893–977. Springer.

Barberà, S.; and Pattanaik, P. K. 1984. Extending an order on a set to the power set: some remarks on Kannai and Peleg's approach. *Journal of Economic Theory* 32(1): 185–191.

Baroni, P.; Caminada, M.; and Giacomin, M. 2011. An introduction to argumentation semantics. *Knowledge Engineering Review* 26(4): 365–410.

Baroni, P.; Gabbay, D.; Giacomin, M.; and van der Torre, L., eds. 2018. *Handbook of Formal Argumentation*. College Publications.

Beirlaen, M.; Heyninck, J.; Pardo, P.; and Straßer, C. 2018. Argument Strength in Formal Argumentation. *IfCoLog Journal of Logics and their Applications* 5(3): 629–676.

Bench-Capon, T. J. M. 2003. Persuasion in Practical Argument Using Value-based Argumentation Frameworks. *Journal of Logic and Computation* 13(3): 429–448.

Bench-Capon, T. J. M.; and Dunne, P. E. 2007. Argumentation in artificial intelligence. *Artificial Intelligence* 171(10-15): 619–641.

Benferhat, S.; Lagrue, S.; and Papini, O. 2004. Reasoning with partially ordered information in a possibilistic logic framework. *Fuzzy Sets and Systems* 144(1): 25—41.

Besnard, P.; and Hunter, A. 2008. *Elements of Argumentation*. MIT Press.

Bondarenko, A.; Dung, P. M.; Kowalski, R. A.; and Toni, F. 1997. An Abstract, Argumentation-Theoretic Approach to Default Reasoning. *Artificial Intelligence* 93: 63–101.

Bouveret, S.; Endriss, U.; and Lang, J. 2010. Fair Division under Ordinal Preferences: Computing Envy-Free Allocations of Indivisible Goods. In Coelho, H.; Studer, R.; and Wooldridge, M. J., eds., *Proc. ECAI*, volume 215 of *Frontiers in Artificial Intelligence and Applications*, 387–392. IOS Press.

Brandt, F.; and Brill, M. 2011. Necessary and sufficient conditions for the strategyproofness of irresolute social choice functions. In Apt, K. R., ed., *Proc. TARK*, 136–142. ACM.

Brandt, F.; Brill, M.; and Harrenstein, P. 2016. Tournament Solutions. In Brandt, F.; Conitzer, V.; Endriss, U.; Lang, J.; and Procaccia, A. D., eds., *Handbook of Computational Social Choice*, 57–84. Cambridge University Press.

Brandt, F.; Conitzer, V.; Endriss, U.; Lang, J.; and Procaccia, A. D., eds. 2016. *Handbook of Computational Social Choice*. Cambridge University Press.

Brewka, G.; Truszczynski, M.; and Woltran, S. 2010. Representing Preferences Among Sets. In Fox, M.; and Poole, D., eds., *Proc. AAAI*, 273–278. AAAI Press.

Caminada, M. 2018. Rationality Postulates: Applying Argumentation Theory for Non-monotonic Reasoning. In Baroni, P.; Gabbay, D.; Giacomin, M.; and van der Torre, L., eds., *Handbook of Formal Argumentation*. College Publications.

Čyras, K.; and Toni, F. 2016. ABA+: Assumption-Based Argumentation with Preferences. In Baral, C.; Delgrande, J. P.; and Wolter, F., eds., *Proc. KR*, 553–556. AAAI Press.

D'Agostino, M.; and Modgil, S. 2020. A Fully Rational Account of Structured Argumentation Under Resource Bounds. In Bessiere, C., ed., *Proc. IJCAI*, 1841–1847. ijcai.org.

Daly, L. W. L. W. 1967. *Contributions to a History of Alphabetization in Antiquity and Middle Ages*. Bruxelles : Latomus.

Dung, P. M. 2016. An axiomatic analysis of structured argumentation with priorities. *Artificial Intelligence* 231: 107–150.

Dung, P. M.; and Thang, P. M. 2018. Fundamental properties of attack relations in structured argumentation with priorities. *Artificial Intelligence* 255: 1–42.

Dyrkolbotn, S. K.; Pedersen, T.; and Broersen, J. M. 2018. On Elitist Lifting and Consistency in Structured Argumentation. *IfCoLog Journal of Logics and their Applications* 5(3): 709–746.

Fishburn, P. C. 1972. Even-chance lotteries in social choice theory. *Theory and Decision* 3: 18–40.

Fishburn, P. C. 1974. Lexicographic Orders, Utilities and Decision Rules: A Survey. *Management Science* 20(11): 1442–1471.

García, A. J.; and Simari, G. R. 2004. Defeasible Logic Programming: An Argumentative Approach. *Theory and Practice of Logic Programming* 4(1-2): 95–138.

Gärdenfors, P. 1976. Manipulation of social choice functions. *Journal of Economic Theory* 13(2): 217–228.

Gibbard, A. 1973. Manipulation of Voting Schemes: A General Result. *Econometrica* 41(4): 587–601.

Heyninck, J.; and Straßer, C. 2019. A Fully Rational Argumentation System for Preordered Defeasible Rules. In Elkind, E.; Veloso, M.; Agmon, N.; and Taylor, M. E., eds., *Proc. AAMAS*, 1704–1712. International Foundation for Autonomous Agents and Multiagent Systems.

Kannai, Y.; and Peleg, B. 1984. A note on the extension of an order on a set to the power set. *Journal of Economic Theory* 32(1): 172–175.

Kelly, J. 1977. Strategy-proofness and social choice functions without single-valuedness. *Econometrica* 45(2): 439–446.

Kim, K. H.; and Roush, F. W. 1980. Preferences on subsets. *Journal of Mathematical Psychology* 21(3): 279 – 282.

Kraft, C. H.; Pratt, J. W.; and Seidenberg, A. 1959. Intuitive Probability on Finite Sets. *The Annals of Mathematical Statistics* 30(2): 408–419.

Liao, B.; Oren, N.; van der Torre, L. W. N.; and Villata, S. 2016. Prioritized Norms and Defaults in Formal Argumentation. In Roy, O.; Tamminga, A. M.; and Willer, M., eds., *Proc. DEON*, 139–154. College Publications.

Maly, J.; Truszczynski, M.; and Woltran, S. 2019. Preference Orders on Families of Sets - When Can Impossibility Results Be Avoided? *Journal of Artificial Intelligence Research* 66: 1147–1197.

Modgil, S.; and Prakken, H. 2013. A general account of argumentation with preferences. *Artificial Intelligence* 195: 361–397.

Modgil, S.; and Prakken, H. 2018. Abstract Rule-Based Argumentation. In Baroni, P.; Gabbay, D.; Giacomin, M.; and van der Torre, L., eds., *Handbook of Formal Argumentation*, chapter 6, 287–364. College Publications.

Satterthwaite, M. A. 1975. Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory* 10(2): 187 – 217.

Wakaki, T. 2017. Assumption-Based Argumentation Equipped with Preferences and its Application to Decision Making, Practical Reasoning, and Epistemic Reasoning. *Computational Intelligence* 33(4): 706–736.

Young, A. P.; Modgil, S.; and Rodrigues, O. 2016. Prioritised Default Logic as Rational Argumentation. In Jonker, C. M.; Marsella, S.; Thangarajah, J.; and Tuyls, K., eds., *Proc. AAMAS*, 626–634. ACM.