

Budget Feasible Mechanisms Over Graphs

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Abstract

This paper studies the budget-feasible mechanism design over graphs, where a buyer wishes to procure items from sellers, and all participants (the buyer and sellers) can only directly interact with their neighbors during the auction campaign. The problem for the buyer is to use the limited budget to incentivize sellers to propagate auction information to their neighbors, thereby more sellers will be informed of the auction and more item value will be procured. An impossibility result shows that the *large-market* assumption is necessary. We propose efficient budget-feasible diffusion mechanisms for large markets that simultaneously guarantee individual rationality, budget-feasibility, strong budget-balance, incentive-compatibility to report private costs and diffuse auction information. Moreover, the proposed mechanisms achieve logarithmic approximation that the total procured value is within a logarithmic factor of the optimal solution. Compared to most related budget-feasible mechanisms, which do not take the individual interactions among sellers into account, our mechanisms can incentivize sellers to further propagate auction information to other potential sellers. Meanwhile, existing related diffusion mechanisms only focus on seller-centric auctions and fail to satisfy the budget-feasibility of the buyer.

Introduction

Auction theory, as a common paradigm for multi-agent resource allocation (Krishna 2009), has enabled a wide range of applications, *e.g.*, wireless spectrum auctions (Huang, Berry, and Honig 2006) and mobile crowdsourcing markets (Feng et al. 2014). Much effort over past decades focused on designing mechanisms to regulate trading behaviors in markets, *e.g.*, seller-centric auction when a seller sells items to buyers, or buyer-centric/reverse auction when a buyer procures items from sellers. Among the extraordinary progresses, the budget-feasible mechanism design in reverse auctions was initially studied in (Singer 2010), where payments used to regulate behaviors should satisfy the budget constraint. Many following works invest efforts to design *budget-feasible* mechanisms (Chen, Gravin, and Lu 2011; Dobzinski, Papadimitriou, and Singer 2011; Bei et al. 2012; Singer 2012; Singer and Mittal 2013; Zhao, Li, and Ma

2014; Anari, Goel, and Nikzad 2018). All these works assume that sellers are reachable to the buyer and ready to join the auction campaign. However, in reality, many potential sellers will be unaware of the auction information if none of their acquaintances propagates information to them.

In order to propagate auction information to more potential sellers, the buyer can adopt the traditional advertising mechanisms. However, the high agency fee charged by the third party platform might degrade the net revenue of the buyer (Mehta 2000). Against this background, this paper studies the social diffusion mechanisms over a graph. The connections modeled in the graph might represent positive individual interactions, which have been widely adopted for information diffusion (Kempe, Kleinberg, and Tardos 2003), information acquisition (Cebrian et al. 2012), and information search (Watts, Dodds, and Newman 2002). With the local positive interactions, social diffusion mechanisms can bring a volume of potential sellers without any extra cost. Taking the buyer’s budget constraint into account, the following natural question arises: *Can we design a budget-feasible mechanism which guarantees desirable economic properties and stimulates involved sellers to propagate auction information to her neighbors over graphs, while guaranteeing a bounded total payment from the buyer?*

For example, considering the questionnaire, a classic research instrument for the organizer to gather information from respondents especially in the social networks (Sarason et al. 1983), a critical requirement is to get sufficient respondents. However, it is inefficient and costly for the organizer to notify all respondents by herself. A promising way is to incentivize participated respondents to invite potential people, *e.g.*, their followers and friends. There may be costs associated with finishing questionnaires, *e.g.*, participants’ time and privacy. It is common to give participants monetary rewards to motivate the population. Moreover, the organizer usually has a budget and cannot afford unlimited monetary rewards. Therefore, such scenario calls for viable budget feasible diffusion mechanisms.

In the budget-feasible mechanism design literature, although there exists a general rule characterizing the truthful bidding behavior in single-parameter domains (Myerson 1981), there is no general rule on how to incentivize sellers to bid truthfully while at the same time to propagate auction information to their neighbors. On the other hand, the

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well-known VCG mechanism (Vickrey 1961; Clarke 1971; Groves 1973) fails in budget-feasible mechanisms as an unlimited payment may be incurred. Last but not least, the VCG mechanism aims to maximize the total procured value with a budget, which is NP-hard.

In the information diffusion mechanism design literature, all critical participants whose invitations increase social welfare will be rewarded (Li et al. 2017; Zhao et al. 2018; Li et al. 2019; Kawasaki et al. 2020; Zhang, Zhao, and Chen 2020). Existing diffusion mechanisms mainly focus on seller-centric auctions where the seller incentivizes buyers to diffuse the information and decides the final winners. While in budget-feasible mechanism design, as a fundamental difference from the seller-centric auction, the payment used to elicit truthful behaviour should satisfy the buyer’s budget constraint. The payment scheme used to elicit truthfulness or propagate the auction information in seller-centric auctions cannot meet the budget-feasibility requirement in reversed auctions as an unbounded payment may be incurred¹. Recently, (Shi et al. 2019) consider the budget constraint in information diffusion mechanisms, but they only focus on rewarding agents to spread information via social connections, not on auction problem of procuring items from sellers.

Our contributions can be summarized as follows.

- We first model a budget-feasible diffusion mechanism over graphs where the buyer with a budget wants sellers to propagate auction information to their neighbors, which builds the bridge between budget-feasible mechanisms and information diffusion mechanisms.
- We show that without the large market assumption, no budget-feasible diffusion mechanisms can achieve an approximation ratio better than n (the number of sellers).
- By incorporating the large market assumption, we overcome the non-approximability and design budget-feasible diffusion mechanisms that guarantee desirable theoretical properties including budget feasibility, computational efficiency, individual rationality, truthfulness, strong budget-balance, and logarithmic approximation.

Preliminaries

The Model

We consider an undirected graph consisting of a buyer a and a set of n sellers $S = \{s_1, s_2, \dots, s_n\}$ where buyer a wants to procure items with a budget B and each seller has a single item to sell. Seller s_i has a *cost* c_i and a *value* v_i when selling her item to the buyer. In this paper, we consider the *incomplete information* case where each seller’s cost c_i is privately known by herself. Following previous budget-feasible mechanisms (Singer 2010; Dobzinski, Papadimitriou, and Singer 2011), we assume that each item’s value v_j is common knowledge to the buyer ranging in $[1, V]$. The objective is for the buyer to maximize the total value of items procured within the budget. Let $A = \{a, s_1, \dots, s_n\}$ denote the set of all participants including the buyer and all sellers. Each

¹A detailed comparison to existing mechanisms can be found in Section: The Inefficiency of Existing Mechanisms.

seller $s_i \in S$ has a set of neighbors $n_i \subseteq A \setminus \{s_i\}$ with whom s_i can exchange information directly. Specifically, the buyer a ’s neighbors are $n_a (n_a \neq \emptyset)$. A seller can participate in the auction if and only if one of her neighbors is informed of the auction information and further propagates the information to her. Initially, only n_a know that the buyer a wants to procure items. According to the types of items, we mainly consider two cases: the homogeneous item setting where sellers hold identical items, and the heterogeneous item setting where sellers hold diverse items with different values.

We focus on the strategic scenario where sellers behave strategically to maximize their own utilities. Each seller reports cost c'_i of her item that may be different from the real cost c_i . Additionally, seller s_i may reject to further propagate the auction information to her neighbors or report false neighbor set $n'_i \subset n_i$. Let $t_i = (c_i, n_i)$ denote the *truthful bid* that s_i truthfully declares her cost and propagates the auction information to all her neighbors, while $b_i = (c'_i, n'_i)$ denotes the bid when s_i participates in the auction. Let $\mathbf{b} = \{b_1, \dots, b_n\}$ denote the bid profile of all sellers and \mathbf{b}_{-i} denote the bid profile of all sellers except the seller s_i . The bid profile \mathbf{b} can also be represented by $\mathbf{b} = (b_i, \mathbf{b}_{-i})$.

Large Market Assumption: Informally speaking, a market is large if the cost of a single item is very small compared to the buyer’s budget B , i.e., $c_i \ll B$. This assumption is used in other large-market problems (Anari, Goel, and Nikzad 2014) with application in crowdsourcing markets, where there is a requester who has a limited budget (e.g., \$500). The requester publishes a large quantity of micro-tasks, like image labeling, content generation, and validating recommendation engines. After finishing a micro-task, the worker will get a tiny reward (\$0.2) for cost compensation. To improve the efficiency of crowdsourcing, the requester can estimate the quantity of workers for better micro-task pricing. Therefore, this paper also assumes that there is an upper bound on the market size, i.e., $|S| \leq N_{max}$, which is known by the buyer. We will show that the large market assumption is necessary by proposing an impossibility result at the beginning of Section .

The Mechanism: An *auction mechanism* $\mathcal{M} = (X, P)$ consists of an allocation policy $X = \{x_i\}_{i \in S}$ deciding winners and a payment policy $P = \{p_i\}_{i \in S}$ deciding how much payment is paid to each seller. If s_i is selected as a winner, $x_i(\mathbf{b}) = 1$, otherwise, $x_i(\mathbf{b}) = 0$. Let S_w denote the set of winners. Given s_i ’s bid b_i , bid profile \mathbf{b} and mechanism $\mathcal{M} = (X, P)$, we define s_i ’s utility as the difference between the payment she receives and her real cost, i.e., $u_i(b_i, \mathbf{b}, \mathcal{M}) = p_i - x_i \cdot c_i$. The utility of the buyer is the total value of items procured, i.e., $u_a(\mathbf{b}, \mathcal{M}) = \sum_{1 \leq i \leq n} x_i \cdot v_i$. If no ambiguity arises, $u_i(b_i, \mathbf{b}, \mathcal{M})$ and $u_a(\mathbf{b}, \mathcal{M})$ will be written as $u_i(b_i)$ and u_a , respectively.

A mechanism \mathcal{M} is individual rational if the utility of each seller is non-negative when she truthfully reports her private information.

Definition 1. A mechanism $\mathcal{M} = (X, P)$ is *individual rational (IR)* if $u_i(t_i, (t_i, \mathbf{b}_{-i}), \mathcal{M}) \geq 0, \forall s_i \in S$.

We also consider the incentive compatibility or truthfulness property. Different from traditional graph-ignorant

budget-feasible mechanism problems in (Singer 2010) where truthfulness means that for each seller, truthfully reporting her cost is a dominant strategy. In our model, we also need to consider sellers' incentives to propagate the auction information to their neighbors. Thus, we also require the designed mechanism to guarantee that for each seller, reporting her real cost and diffusing the auction information to all her neighbors is a dominant strategy.

Definition 2. A mechanism $\mathcal{M} = (X, P)$ is incentive compatible (IC) or truthful if $u_i(t_i, (t_i, \mathbf{b}_{-i}), \mathcal{M}) \geq u_i(b_i, (b_i, \mathbf{b}'_{-i}), \mathcal{M})$.

Note that \mathbf{b}_{-i} is replaced by \mathbf{b}'_{-i} on the right side of the inequality. The reason is that if bid b_i does not propagate the auction information to all her neighbors, then some sellers, who can get the information under bid t_i , may not receive the information under bid b_i . Thus, under b_i , the feasible bid profile of sellers except s_i is changed from \mathbf{b}_{-i} to \mathbf{b}'_{-i} .

Given the payment policy, we define the budget feasibility and strong budget-balance.

Definition 3. A mechanism $\mathcal{M} = (X, P)$ is budget-feasible if the total payment paid to sellers does not exceed the budget, i.e., $\sum_{1 \leq i \leq n} p_i \leq B$.

Definition 4. A mechanism $\mathcal{M} = (X, P)$ is strong budget-balanced if the total payment charged from the buyer equals the total payment paid to sellers.

The objective is to maximize the utility of the auctioneer/buyer, i.e., $\max u_a(\mathbf{b}, \mathcal{M})$.

Definition 5. A mechanism $\mathcal{M} = (X, P)$ is α -approximation if the ratio between the optimal solution where mechanism knows sellers' true private information and the solution by mechanism \mathcal{M} is no greater than α .

The Inefficiency of Existing Mechanisms

There are two main directions for the truthful auction design problem. One is to apply Vickrey-Clarke-Groves (VCG) mechanisms. The other is to design truthful mechanisms via Myerson's theorem (Myerson 1981) to guarantee two properties, the monotonicity and the threshold payment. Next, we show the inefficiencies of these two existing classes of mechanisms in the budget-feasible diffusion problem.

The inefficiency of VCG mechanisms: (Li et al. 2017) show that the classical VCG mechanism can incentivize auction information diffusion in seller-centric auctions and further improve upon VCG mechanism on the revenue efficiency. However, such kind of methods totally ignore the budget-feasibility requirement and thus cannot be applied to deal with the key challenge in budget-feasible mechanism design², as an unbounded payment may be incurred. For instance, given the graph in Fig. 1, we consider a linear graph with $n > 2$ sellers and a buyer a located at the left endpoint of the line. The cost of seller s_{n-1} is 5ϵ while the costs of the remaining sellers are ϵ . All sellers have identical value $v_i = 1$. In addition, the budget of the buyer is $B = (n+2)\epsilon$. The VCG mechanism will choose all sellers except s_{n-1} as

²Actually, VCG would also fail in the computational efficiency as computing the optimal item allocation is NP-hard (Singer 2010).

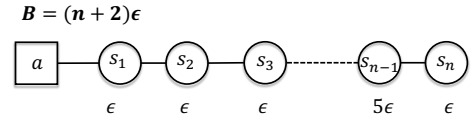


Figure 1: An example showing the inefficiency of existing mechanisms.

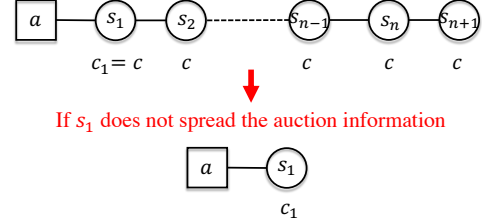


Figure 2: An example for the proof of Theorem 1.

winners since the total cost is within budget B , and pay each winner 4ϵ , which exceeds the budget $B = (n+2)\epsilon$.

The inefficiency of graph-ignorant budget-feasible mechanisms: In the traditional budget-feasible mechanisms (Singer 2010; Chen, Gravin, and Lu 2011), which are based on Myerson's theorem, the proportional share allocation rule is widely used to generate budget-feasible allocations and elicit the truthfulness.

Proportional share allocation rule

- Sort sellers according to their non-decreasing costs relative to their values.
- Allocate seller s_i to winner set S_w if $\frac{c_i}{v_i} \leq \frac{B}{\sum_{1 \leq h \leq i} v_h}$.

Assume that the last winner is s_k , then the payment for each winner $s_i \in S_w$ will be $v_i \cdot \min\{\frac{c_{k+1}}{v_{k+1}}, \frac{B}{\sum_{1 \leq h \leq k} v_h}\}$. Applying the mechanism in (Singer 2010) to the graph in Fig. 1, all sellers with cost ϵ will be selected as winners and the payment for each winner is $\min\{5\epsilon, \frac{B}{n-1}\} = \frac{B}{n-1}$. Seller s_n can reject to propagate the auction information to her neighbors and then her payment now is $\frac{B}{3}$ which implies that she can get more utility by manipulating the diffusion strategy. Thus, such kind of mechanisms cannot incentivize sellers to propagate the auction information to their neighbors.

Mechanism Design

In this section, we investigate budget-feasible diffusion mechanisms. We start from the homogeneous item setting, where sellers' items have identical values, and then extend it to the heterogeneous item setting. Before introducing our mechanisms, we first show an impossibility result without the large market assumption as follows,

Theorem 1. Without the large market assumption, no truthful budget-feasible diffusion mechanisms can achieve an approximation ratio better than n .

Proof. Assume that there exists a budget-feasible diffusion mechanism \mathcal{M}' which can achieve $(n-\varepsilon)$ -approximation ratio, for any $\varepsilon > 0$. Suppose that sellers have identical values $v_i = 1$. As shown in Fig. 2, we consider an example where there are $n+1$ sellers each with cost $c = \frac{B}{n}$ in the linear network. We have $0 < c_1 = c \leq B$. It is clear that the optimal solution can procure n items from sellers and each winner is paid $\frac{B}{n}$. If s_1 rejects to propagate the auction information to her neighbor s_2 , no sellers after s_1 can get the auction information. Thus, seller s_1 must be a winner with payment B to ensure truthfulness (as s_1 can bid any false cost no greater than B without the large market assumption). The utility of s_1 now is $u_1 = B - c_1$. Recall that a seller obtains the maximum utility when she reports real cost and propagates the auction information to her neighbors in budget-feasible diffusion mechanism \mathcal{M}' . Thus, s_1 should achieve at least $B - c_1$ utility to incentivize her to propagate the auction information to her neighbors. On one hand, if s_1 is selected as a winner when she propagates the auction information, her payment should be B to obtain at least $B - c_1$ utility. \mathcal{M}' cannot procure one more item from other sellers due to the budget constraint. On the other hand, if s_1 is a loser, her payment should be no smaller than the utility when she propagates the information, which is $B - c_1$. Thus, \mathcal{M}' can procure at most one item from other sellers. For both cases, Mechanism \mathcal{M}' can only procure at most one item from sellers, which contradicts with the assumption that \mathcal{M}' achieves $(n - \varepsilon)$ -approximation. \square

Mechanism for the Homogeneous Item Setting

In this subsection, we introduce the budget-feasible diffusion mechanism for the homogeneous item setting, called Mechanism BDM-H. The high level idea is as follows. We first generate a seller sequence from the graph according to each seller's distance to the buyer. Considering the budget constraint, we divide the mechanism into multiple phases, in each of which we set a target number of winners and a fixed payment for each winner. In addition, to achieve better utility on the buyer's side, we aim to find more winners with lower costs in the later phase by setting a larger target number of winners and a lower fixed payment. The detailed design is presented as follows.

Generate seller sequence: We first construct a seller sequence according to bid profile \mathbf{b} . We calculate each seller's shortest distance to the buyer and sort sellers in the non-decreasing order of the shortest distance with arbitrary tie-breaking rule, denoted as a sequence $\mathcal{O} = \langle s_1, s_2, \dots, s_n \rangle$. Fig. 3 shows an example of seller sequence generation over a graph. We use circles to represent sellers and the rectangle to represent the buyer. The number in each circle is her cost, and the pair consisting of a capital letter and a number indicates her index and the shortest distance. The number in the rectangle is the buyer's budget. By sorting all sellers' shortest distances, we can get the seller sequence $\mathcal{O} = \langle \mathbf{B}, \mathbf{C}, \mathbf{E}, \mathbf{D}, \mathbf{F}, \mathbf{G}, \mathbf{I}, \mathbf{J}, \mathbf{L}, \mathbf{H}, \mathbf{M}, \mathbf{K} \rangle$.

Phase Division: We divide the winner selection process into $L = \lceil \log_2 N_{max} \rceil$ phases, each of which is allocated with a partial budget $\hat{B} = \frac{B}{L}$. In the l -th phase, we set a fixed

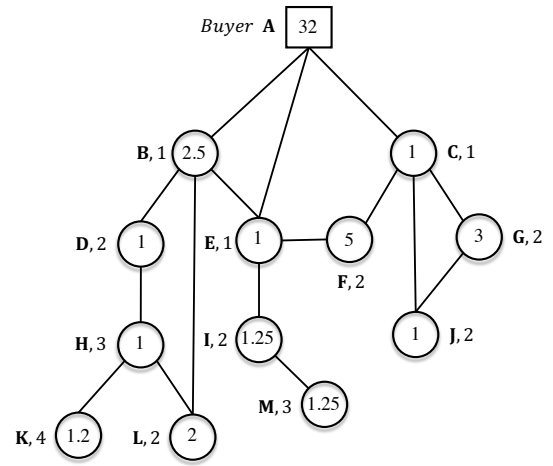


Figure 3: An example of a graph. We have $\langle \mathbf{B}, \mathbf{C}, \mathbf{E} \rangle$ with distance 1, $\langle \mathbf{D}, \mathbf{F}, \mathbf{G}, \mathbf{I}, \mathbf{J}, \mathbf{L} \rangle$ with distance 2, $\langle \mathbf{H}, \mathbf{M} \rangle$ with distance 3 and $\langle \mathbf{K} \rangle$ with distance 4.

payment τ_p^l and a target number of winners τ_w^l . Specifically, we have $\tau_p^l \cdot \tau_w^l = \hat{B}$ which can ensure that the partial budget in each phase is not exceeded.

Winner selection and payment scheme: Now, we start from the first phase, *i.e.*, $l = 1$, by setting $\tau_p^l = \frac{\hat{B}}{2^{l-1}}$ and $\tau_w^l = 2^{l-1}$. In addition, we use τ_w^l to count the number of sellers with costs no greater than τ_p^l in this phase, which is initialized to be zero. Assume that we are now considering seller s_i in the l -th phase. If $c'_i > \tau_p^l$, we continue to test the next seller s_{i+1} . If $c'_i \leq \tau_p^l$ and $\tau_w^l < \tau_w^l - 1$, the value of τ_w^l increases by one and we go to the next seller s_{i+1} . If $c'_i \leq \tau_p^l$ and $\tau_w^l = \tau_w^l - 1$, the number of sellers with costs no greater than τ_p^l in the l -th phase reaches 2^{l-1} after considering seller s_i . This seller is also denoted as *key seller* \mathcal{H}_l . We use \mathcal{H} and $I(\mathcal{H}_l)$ to denote the set of all key sellers and \mathcal{H}_l 's position in the seller sequence \mathcal{O} , respectively. Once the value of τ_w^l reaches 2^{l-1} , sellers with costs no greater than τ_p^l in the l -th phase will all be selected as winners and added into winner set S_w , *i.e.*, $s_i \in S_w$ if $i \in (I(\mathcal{H}_{k-1}), I(\mathcal{H}_k)]$ and $c'_i \leq \tau_p^l$. In addition, the payment to each of these winners in the l -th phase is set with $p_i = \tau_p^l = \frac{\hat{B}}{2^{l-1}}$.

Then, we move to the $(l+1)$ -th phase where the fixed payment will be reduced to half of the fixed payment of the previous phase, *i.e.*, $\tau_p^{l+1} = \frac{1}{2}\tau_p^l = \frac{\hat{B}}{2^l}$, and we will try to select twice as many winners as in the previous phase, *i.e.*, $\tau_w^{l+1} = 2\tau_w^l = 2^l$. In addition, we set the value of τ_w^l to be zero and continue to test the next seller s_{i+1} . We repeat this process until no sufficient winners can be selected, that is, after k phases, we cannot further find $\tau_w^{k+1} = 2^k$ winners whose bids are no greater than $\tau_p^{k+1} = \frac{\hat{B}}{2^k}$, *i.e.*,

$$\{h | c'_h \leq \tau_p^{k+1}, h > I(\mathcal{H}_k)\} < \tau_w^{k+1}. \quad (1)$$

Then, sellers after \mathcal{H}_k are all losers. The payments for all losers are zero, *i.e.*, $p_i = 0$ if $s_i \notin S_w$. Note that we have at least one key seller, *i.e.*, $k \geq 1$ because we can find at

Algorithm 1: Mechanism BDM-H($B, \mathbf{b}', S, N_{max}$)

Input: $B, \mathbf{b}', S, N_{max}$.
Output: P, S_w

- 1 $P \leftarrow 0, X \leftarrow 0, S_w \leftarrow \emptyset, \mathcal{H} \leftarrow \emptyset$;
- 2 $L \leftarrow \lceil \log_2^{N_{max}} \rceil, \hat{B} \leftarrow \frac{B}{L}$;
- 3 Calculate the shortest distance for each seller and sort all sellers in the non-decreasing order of the distance, *i.e.*, $\mathcal{O} = \langle s_1, s_2, \dots, s_n \rangle$;
- 4 $i \leftarrow 1, l \leftarrow 1$;
- 5 **while** $l \leq L$ **do**
- 6 $\tau_p^l \leftarrow \frac{\hat{B}}{2^{l-1}}, \tau_w^l \leftarrow 2^{l-1}, \tau_w^c \leftarrow 0$;
- 7 **while** $i \leq n, \tau_w^c < \tau_w^l$ **do**
- 8 **if** $c'_i \leq \tau_p^l$ **then**
- 9 $\tau_w^c \leftarrow \tau_w^c + 1$;
- 10 **end**
- 11 $i \leftarrow i + 1$;
- 12 **end**
- 13 **if** $\tau_w^c = \tau_w^l$ **then**
- 14 $\mathcal{H}_l \leftarrow s_{i-1}, \mathcal{H} \leftarrow \mathcal{H} \cup \{\mathcal{H}_l\}$;
- 15 **for** $I(\mathcal{H}_{l-1}) < h \leq I(\mathcal{H}_l)$ **and** $c'_h \leq \frac{\hat{B}}{2^{l-1}}$ **do**
- 16 $p_h \leftarrow \frac{\hat{B}}{2^{l-1}}, x_h \leftarrow 1, S_w \leftarrow S_w \cup \{s_h\}$;
- 17 **end**
- 18 $l \leftarrow l + 1$
- 19 **else**
- 20 **break**;
- 21 **end**
- 22 **end**
- 23 **return** P, S_w

least one seller with cost no greater than \hat{B} in the first phase according to the large market assumption.

Running example: We now show a running example of Mechanism BDM-H using the graph in Fig. 3. Assume that all sellers report real private information. The buyer's budget is $B = 32$ and the maximum number of sellers is $N_{max} = 16$. Thus, we have $L = \log_2 16 = 4$ and $\hat{B} = 8$. We select winners by the sequence \mathcal{O} as shown in Fig. 4. We start from the first phase which tries to find $\tau_w^1 = 2^0$ sellers with costs no greater than $\tau_p^1 = \frac{\hat{B}}{2^0} = 8$. It is clear that the first seller with cost 2.5 marked by red box is selected as a winner and her payment is $\tau_p^1 = 8$. In addition, she is also the first key seller \mathcal{H}_1 . Then, we consider the next phase to find $\tau_w^2 = 2^1$ sellers with costs no greater than $\tau_p^2 = \frac{\hat{B}}{2^1} = 4$. In this phase, seller **C** and **E** marked by the blue box are winners each paid $\tau_p^2 = 4$. And seller **E** is the second key seller \mathcal{H}_2 . Similarly, we can see sellers **D, I, J, L** are winners each with payment $\tau_p^3 = 2$ in the third phase. Last, in the fourth phase, we cannot find $\tau_w^4 = 8$ sellers with costs no greater than $\tau_p^4 = \frac{\hat{B}}{2^3} = 1$. Thus, we finally have the winner set $S_w = \{\mathbf{B}, \mathbf{C}, \mathbf{E}, \mathbf{D}, \mathbf{I}, \mathbf{J}, \mathbf{L}\}$ and three key sellers $\mathcal{H} = \{\mathbf{B}, \mathbf{E}, \mathbf{L}\}$.

Next, we analyze the theoretical performance of BDM-H.

Theorem 2. *Mechanism BDM-H guarantees individual rationality, strong budget-balance, budget feasibility and com-*

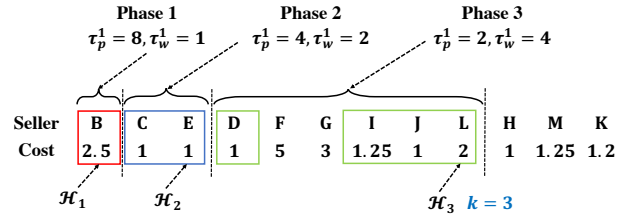


Figure 4: Running example of Mechanism BDM-H.

putational efficiency.

Before analyzing the truthfulness of Mechanism BDM-H, we first introduce a useful lemma.

Lemma 1. *The order of sellers before s_i (including s_i herself) in seller sequence \mathcal{O} will not change if s_i misreports her neighbors $n'_i \subset n_i$.*

Proof. Let $\rho(s_i), d(s_i)$ denote the shortest path and the corresponding distance from the buyer to seller s_i , respectively. Note that the shortest distance of seller s_j before s_i in seller sequence \mathcal{O} is no greater than $d(s_i)$, *i.e.*, $d(s_j) \leq d(s_i)$. We can find that s_i cannot be on the path $\rho(s_j)$, otherwise, we can find a path from the buyer to s_i shorter than $\rho(s_j)$, which leads to a contradiction. After s_i misreports her neighbors $n'_i \subset n_i$, the number of paths from the buyer to s_j will not increase and the path $\rho(s_j)$ will still exist since s_i is not on path $\rho(s_j)$. Thus, seller s_j ' shortest distance to the buyer is still $d(s_j)$. This implies that the order of sellers before s_i will not change. Furthermore, sellers on path $\rho(s_i)$ are all before s_i and will still propagate the auction information to s_i which means the position of s_i will also not change after misreporting her neighbors. This finishes the proof. \square

Theorem 3. *Mechanism BDM-H guarantees incentive-compatibility/truthfulness.*

Proof. To achieve more utility, seller s_i may propagate auction information to a subset of her neighbors $n'_i \subset n_i$ and report a false cost $c'_i \neq c_i$. It is clear that seller s_i 's position in sequence \mathcal{O} will not change by Lemma 1 even when she chooses to propagate auction information to a subset of neighbors n'_i . Thus, s_i , regardless of which subset of neighbors she reports, will still be considered at the same phase as when she had reported the true set of neighbors. Assume that s_i is considered in the l -th phase. When she is being considered under reporting false neighbor set, the value of target seller number τ_w^l , the fixed payment τ_p^l and the current number of specified sellers τ_w^c are the same as when she reports real cost and propagates auction information to all her neighbors. Thus, it is easy to find that s_i cannot achieve more utility by diffusing information to a subset of neighbors. Thus, it remains to discuss the case that a seller bids a false cost when she propagates the information to all her neighbors. Based on the position of s_i in the sequence \mathcal{O} , we consider the following cases:

Case 1: Seller s_i where $I(\mathcal{H}_{l-1}) < i \leq I(\mathcal{H}_l)$ and $l \leq k$: Recall that BDM-H selects 2^{l-1} winners with bids no greater than $\frac{\hat{B}}{2^{l-1}}$ from sellers between \mathcal{H}_{l-1} and \mathcal{H}_l , and

pays each winner $\frac{\hat{B}}{2^{l-1}}$. We further consider two sub-cases, whether s_i is a winner or not when she reports the true cost and propagates the information to all her neighbors.

1) Seller s_i is a winner, i.e., $c_i \leq \frac{\hat{B}}{2^{l-1}}$ and $p_i = \frac{\hat{B}}{2^{l-1}}$: Seller s_i would become a loser by bidding a higher cost $c'_i > \frac{\hat{B}}{2^l}$, leading to zero utility. If s_i bids a cost lower than $\frac{\hat{B}}{2^{l-1}}$, she is still a winner with the same payment $\frac{\hat{B}}{2^{l-1}}$ which implies her utility will not increase. Thus, winners cannot get more utilities by bidding false costs.

2) Seller s_i is a loser, i.e., $c_i > \frac{\hat{B}}{2^{l-1}}$ and $p_i = 0$: Seller s_i is still a loser by bidding a higher cost $c'_i > \frac{\hat{B}}{2^{l-1}}$. If she bids a lower cost $c'_i \leq \frac{\hat{B}}{2^{l-1}}$, she will be selected as a winner and the payment is $\frac{\hat{B}}{2^{l-1}}$ which leads to negative utility. Thus, losers also cannot get more utilities by bidding false costs.

Case 2: Seller s_i where $i > I(\mathcal{H}_k)$: Recall that all these sellers are losers. Since \mathcal{H}_k is the last key seller, there are no 2^k sellers with bids no greater than $\frac{\hat{B}}{2^k}$. Given that seller s_i reports real neighbor set n_i , if $c_i \leq \frac{\hat{B}}{2^k}$, s_i cannot be a winner whatever she bids. If $c_i > \frac{\hat{B}}{2^k}$, she either is still a loser or is a winner by bidding a lower cost $c'_i \leq \frac{\hat{B}}{2^k}$ with payment $\frac{\hat{B}}{2^k}$ which leads to negative utility. Thus, any seller $s_{i| i > I(\mathcal{H}_k)}$ cannot get more utility by bidding a false cost.

Thus, sellers have no incentive to bid false costs. Therefore, Mechanism BDM-H guarantees the truthfulness. \square

Theorem 4. Mechanism BDM-H achieves $2(\lceil \log_2 N_{max} \rceil + 1)$ -approximation.

Proof. Let \mathbf{S}' and \mathbf{S}'' denote the set of sellers before \mathcal{H}_k (including \mathcal{H}_k herself) and sellers after \mathcal{H}_k , respectively. Meanwhile, we use $\mathbf{S}''_{<k}$ and $\mathbf{S}''_{>k}$ to denote the set of sellers in \mathbf{S}'' whose costs satisfy $c_i \leq \frac{\hat{B}}{2^k}$ and $c_i > \frac{\hat{B}}{2^k}$, respectively. Since \mathcal{H}_k is the last key seller, there are at most $2^k - 1$ sellers whose costs are no greater than $\frac{\hat{B}}{2^k}$, i.e., $|\mathbf{S}''_{<k}| \leq 2^k - 1$, while the costs of the remaining sellers after \mathcal{H}_k are greater than $\frac{\hat{B}}{2^k}$. Denote by OPT_h and ALG_h the optimal solution and the solution of Mechanism BDM-H for the homogeneous item setting, respectively. According to the sellers' positions in the sequence \mathcal{O} , we consider two cases:

1) Sellers in $\mathbf{S}' = \{s_i | i \leq I(\mathcal{H}_k)\}$: It is clear that the number of winners in \mathbf{S}' is

$$ALG_h = \sum_{1 \leq h \leq k} 2^{h-1} = 2^k - 1. \quad (2)$$

For the optimal solution, the best case is that costs of winners in \mathbf{S}' are zero while losers' costs are higher than $\frac{\hat{B}}{2^{k-1}}$, i.e., $c_i = 0, \forall i \in \mathbf{S}' \cap S_w$ and $c_i > \frac{\hat{B}}{2^{k-1}}, \forall i \in \mathbf{S}' \setminus S_w$. Thus, OPT_h might procure all winners in \mathbf{S}' with payment zero.

2) Sellers in $\mathbf{S}'' = \{s_i | i > I(\mathcal{H}_k)\}$: For the best case, the costs of sellers in $\mathbf{S}''_{<k}$ and $\mathbf{S}''_{>k}$ are zero and $\frac{\hat{B}}{2^k}$, respectively. Within the budget B , the optimal solution OPT_h can procure at most $2^k - 1$ value from $\mathbf{S}''_{<k}$ with cost zero and $B/\frac{\hat{B}}{2^k} = 2^k \cdot L$ value from $\mathbf{S}''_{>k}$ by paying each seller $\frac{\hat{B}}{2^k}$.

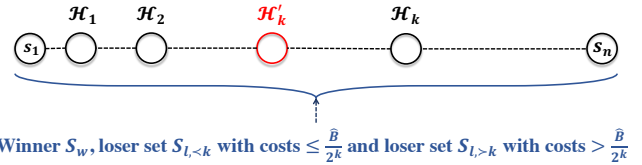


Figure 5: Example of position of \mathcal{H}'_k in the sequence \mathcal{O} .

By combining these two cases, the optimal solution can procure at most $OPT_h \leq 2(2^k - 1) + 2^k L = 2^{k+1} - 2 + 2^k L$. Thus, $\frac{OPT_h}{ALG_h} \leq 2 + \frac{2^k \cdot L}{2^k - 1} \leq 2(\lceil \log_2 N_{max} \rceil + 1)$. \square

Recall that we use the upper bound N_{max} of the market size (denoted by $N = |S|$) as the input, where $N \leq N_{max}$. In the real scenario, an estimated upper bound may be inaccurate. We thus show that, even when N_{max} is not that accurate, the outputs of the mechanism has a small gap when running over the input N_{max} and real number N . Let ALG_h and ALG'_h denote the solutions of BDM-H with the input number N_{max} and N of participants, respectively.

Theorem 5. The gap between ALG_h and ALG'_h is within $(8\alpha - 1)$ where $\alpha = \frac{\lceil \log_2 N_{max} \rceil}{\lceil \log_2 N \rceil}$.

Proof. We have $L' = \lceil \log_2 N \rceil$ and $\hat{B}' = \frac{B}{L'}$. It is easy to find that $L' \leq L$ and $\hat{B}' \geq \hat{B}$. Let S'_w denote the winner set in solution ALG'_h . Assume that there are k key sellers in solution ALG_h . If $L' < k \leq L$, the number of key sellers of ALG'_h must be less than solution ALG_h which implies that ALG'_h procures less value of items than solution ALG_h .

If $k \leq L'$, due to $\hat{B}' \geq \hat{B}$, there are at least k key sellers in ALG'_h and the k -th key seller \mathcal{H}'_k must be on the left side of \mathcal{H}_k , i.e., $I(\mathcal{H}'_k) \leq I(\mathcal{H}_k)$ as shown in Fig. 5. For ALG_h , let S_l and S_w denote the loser and winner set, respectively. The costs of losers between \mathcal{H}_{l-1} and \mathcal{H}_l are greater than $\frac{\hat{B}}{2^{l-1}}$ ($l \leq k$). Thus, the costs of losers before \mathcal{H}_k are all greater than $\frac{\hat{B}}{2^{k-1}}$. Note that all sellers after \mathcal{H}_k are also losers. Among these losers, there are at most $2^k - 1$ sellers with costs no greater than $\frac{\hat{B}}{2^k}$ while the costs of the remaining losers are greater than $\frac{\hat{B}}{2^k}$, since ALG_h only has k key sellers. Then, losers in ALG_h can be divided into two groups: losers with costs no greater than $\frac{\hat{B}}{2^k}$ denoted by $S_{l,<k}$, and the remaining losers with costs greater than $\frac{\hat{B}}{2^k}$, denoted by $S_{l,>k}$. All sellers consist of three groups: S_w , $S_{l,<k}$, $S_{l,>k}$, and we have $|S_w| = 2^k - 1$, $|S_{l,<k}| \leq 2^k - 1$.

Now, we analyze the maximum value ALG'_h can procure from sellers after \mathcal{H}'_k . Besides the sellers in S_w and $S_{l,<k}$, ALG'_h can procure items from $S_{l,>k}$ when the fixed payment is higher than $\frac{\hat{B}}{2^k}$. Assume that there are at most x phases whose fixed payments are higher than $\frac{\hat{B}}{2^k}$ after the k -th phase, i.e., $x = \operatorname{argmax}_{h \geq 1} \{ \frac{\hat{B}'}{2^{k+h-1}} \geq \frac{\hat{B}}{2^k} \}$ which implies $x = \lceil \log_2 \alpha \rceil + 1$ where $\alpha = \lceil \log_2 N_{max} \rceil / \lceil \log_2 N \rceil$. Then, the fixed payment in the $(k + x + 1)$ -th phase will

Algorithm 2: Mechanism BDM-G($B, \mathbf{b}', S, N_{max}$)

Input: $B, \mathbf{b}', S, N_{max}$.**Output:** P, S_w

- 1 $\beta \leftarrow \lceil \log_2 V \rceil$;
 - 2 Divide all sellers into β groups according to (3);
 - 3 **for** each group G_j **do**
 - 4 $(P_j, S_w^j) = \mathbf{BDM-H}(B, \mathbf{b}, G_j, N_{max})$;
 - 5 **end**
 - 6 With probability $\frac{1}{\beta}$, **return** (P_j, S_w^j) ;
-

be lower than $\frac{\hat{B}}{2^k}$, and no sellers in $S_{l, > k}$ can be selected as winners in ALG'_h after the $(k+x)$ -th phase. Thus, the only possible sellers to be selected as winners in ALG'_h after the $(k+x)$ -th phase are from $S_w \cup S_{l, < k}$. Since the target number of winners after the $(k+x)$ -th phase is at least $2^{k+x} \geq 2^k$, sellers in $S_w \cup S_{l, < k}$ can only make the number of phases increase by at most one since they contain at most $2(2^k - 1)$ sellers. Thus, there are at most $k+x+1$ phases in ALG'_h , and we have $ALG'_h \leq \sum_{l \leq k+x+1} 2^{l-1} = 2^{k+x+1} - 1$. Therefore, we have $\frac{ALG'_h}{ALG_h} \leq \frac{2^{k+x+1} - 1}{2^k - 1} \leq 8\alpha - 1$. \square

Mechanism for the Heterogeneous Item Setting

Now we extend the result to the general setting where sellers hold heterogeneous items. The main challenge is that sellers with valuable items should get higher payments which makes it more difficult to ensure budget-feasibility.

We design a budget-feasible diffusion mechanism BDM-G for the heterogeneous item setting. The idea is that we try to transform the heterogeneous item setting problem into multiple homogeneous item setting problems. Intuitively, we divide all sellers into multiple groups so that each group of sellers have similar values, and thus we can consider each group of sellers as owning items of homogeneous value. The mechanism proposed in the homogeneous item setting can then be used to deal with each group of sellers.

We first divide all the sellers into β groups, denoted by $\mathcal{G} = \{G_1, G_2, \dots, G_\beta\}$, by their values of items where $\beta = \lceil \log_2 V \rceil$ as follows,

$$\mathbb{G}(s_i) = \begin{cases} G_j, & \text{if } v_i \in (2^{j-1}, 2^j], 1 \leq j \leq \beta \\ G_1, & \text{otherwise} \end{cases} \quad (3)$$

where $\mathbb{G}(s_i)$ is the group seller s_i allocated to. Then, we view that each seller in the same group owns the same value. Thus, we call Mechanism BDM-H for the homogeneous setting to deal with the sellers in the same group. Denote by S_w^j and P_j the output winners and the corresponding payment returned by Mechanism BDM-H on group G_j , i.e., $(P_j, S_w^j) = \mathbf{BDM-H}(B, \mathbf{b}, G_j, N_{max})$. Last, we sample one of the outputs from all groups with probability $\frac{1}{\beta}$ as the final solution.

Next we analyze the theoretical performance of BDM-G.

Theorem 6. *Mechanism BDM-G guarantees individual rationality, computational efficiency, strong budget balance, budget feasibility and incentive compatibility.*

Theorem 7. *Mechanism BDM-G achieves $(3 + 4L)\beta$ -approximation in expectation where $L = \lceil \log_2 N_{max} \rceil$ and $\beta = \lceil \log_2 V \rceil$.*

Proof. Denote by OPT_g and ALG_g the optimal solution and the solution of Mechanism BDM-G, respectively. In group G_l , assume that there are k_l key sellers. Thus, we have $2^{k_l} - 1$ winners in group G_l similar as in Equation (2). After the last key seller, we have at most $2^{k_l} - 1$ items with costs no greater than $\frac{\hat{B}}{2^{k_l}}$ and the remaining sellers are with costs greater than $\frac{\hat{B}}{2^{k_l}}$ according to Equation (1). The expected total value procured by Mechanism BDM-G is

$$ALG_g = \frac{1}{\beta} \sum_{l \leq \beta} \sum_{j \in S_w^l} v_j \geq \frac{1}{\beta} \sum_{l \leq \beta} 2^{l-1} \cdot (2^{k_l} - 1). \quad (4)$$

Thus, we have

$$2\beta ALG_g \geq \sum_{l \leq \beta} 2^l \cdot (2^{k_l} - 1). \quad (5)$$

On one hand, in group G_l , OPT_g can procure $2^{k_l} - 1$ items from winners in ALG_g with cost zero, and $2^{k_l} - 1$ items whose costs are no greater than $\frac{\hat{B}}{2^{k_l}}$. On the other hand, with budget B , OPT_g can procure at most $B / \frac{\hat{B}}{2^{k_l}} = 2^{k_l} L$ items from sellers whose costs are greater than $\frac{\hat{B}}{2^{k_l}}$ in G_l . Then, OPT_g can procure at most $2^l \cdot 2^{k_l} L$ value from G_l with budget B . Thus, OPT_g can procure at most

$$\begin{aligned} OPT_g &\leq \sum_{l \leq \beta} \sum_{j \in S_w^l} v_j + \sum_{l \leq \beta} 2^l (2^{k_l} - 1) + \sum_{l \leq \beta} 2^l \cdot 2^{k_l} L \\ &\leq 3\beta ALG_g + L\beta \cdot \frac{\sum_{l \leq \beta} 2^l \cdot 2^{k_l}}{\beta} \\ &= 3\beta ALG_g + L\beta \cdot \frac{\sum_{l \leq \beta} (2^l \cdot 2^{k_l} - 2^l) + 2^l}{\beta} \\ &= (3\beta + 2L\beta) \cdot ALG_g + L \cdot \sum_{l \leq \beta} 2^l \\ &\leq (3\beta + 2L\beta) ALG_g + 2L\beta ALG_g = (3 + 4L)\beta \cdot ALG_g. \end{aligned}$$

where the second and third inequalities are due to (5). Therefore, we have $\frac{OPT_g}{ALG_g} \leq (3 + 4L)\beta$. \square

Conclusion

In this paper, we consider the budget-feasible diffusion mechanisms over graphs that can incentivize sellers to further propagate auction information to the potential sellers and satisfy the buyer's budget constraint, simultaneously. For the homogeneous item setting, we propose Mechanism BDM-H, which selects winners by phases based on a generated seller sequence and controls the total payment at each phase by setting a target number of winners and a fixed payment for each winner. We then extend the result to the heterogeneous item setting by proposing Mechanism BDM-G, which transforms the heterogeneous item setting into multiple homogeneous item setting problems by putting sellers with similar values into the same group. The designed mechanisms can guarantee desirable properties like individual rationality, budget-feasibility, strong budget-balance, incentive-compatibility and logarithmic approximation.

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