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Learning Robust Search Strategies Using a Bandit-Based Approach

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Abstract

Effective solving of constraint problems often requires choosing good or specific search heuristics. However, choosing or designing a good search heuristic is non-trivial and is often a manual process. In this paper, rather than manually choosing/designing search heuristics, we propose the use of bandit-based learning techniques to automatically select search heuristics. Our approach is online where the solver learns and selects from a set of heuristics during search. The goal is to obtain automatic search heuristics which give robust performance. Preliminary experiments show that our adaptive technique is more robust than the original search heuristics. It can also outperform the original heuristics.

Introduction

Constraint programming (CP) is successfully used in solving combinatorial problems. In CP, problems are modeled as constraint satisfaction problems (CSP), often NP hard. Due to their intractability, CP solvers combine constraint solving with a search strategy to instantiate variables. A good search strategy can significantly reduce the size of the search space giving faster problem solving. Many variable search heuristics, e.g. *ddeg/dom* (Bessiere and Regin 1996; Smith and Grant 1998), *wdeg/dom* (Boussemart et al. 2004), *impact* (Refalo 2004), *activity* (Michel and Hentenryck 2012) and *corr* (Wang, Xia, and Yap 2017), have been proposed.

However, choosing a heuristic from the many existing variable ordering heuristics which works well for a particular problem instance or family is not simple. It may require expertise or experience. A wrong (choice of) heuristic may mean that the solving time is slower by several orders of magnitude, e.g., the *activity* heuristic can be more than 100X faster than *wdeg/dom* for some nurse-rostering problem instances, but can be slower for other benchmarks like radiation therapy (Michel and Hentenryck 2012). By the nature of heuristics, no heuristic always outperforms another. In order to provide an efficient solution to a problem, specific search heuristics may need to be used, requiring considerable effort in choosing/designing the heuristics for good performance. A drawback of an individual heuristic is that it may only make a good decision at a certain solving state of a problem, e.g. some heuristic may only perform well at a certain search depth, but not for the whole solving process.

To address this challenge, we propose automatic and adaptive CSP search heuristics. Our approach is motivated by the multi-armed bandit (MAB) problem (Gittins 1989) in reinforcement learning. We consider the search heuristic choice as selecting dynamically from several candidate variable ordering heuristics during search. Each choice of search heuristic is akin to selecting an action (arm) in the MAB. Failures from a search node are turned into rewards for the choice made which affect subsequent choices. The idea is that learning from the rewards of choosing a particular heuristic will reduce making poor choices and in turn lead to a search heuristic which is more robust.

We adapt two MAB algorithms, Thompson Sampling (Thompson 1933) and UCB1 (upper confidence bound) (Auer, Cesa-Bianchi, and Fischer 2002) with reward functions to select the "best arm", i.e. "variable ordering heuristic". Our variable heuristics learn from information collected during the solving of the particular problem. Thus, learning is *online* and differs from supervised learning approaches which require training examples and an offline training solving phase. We study the performance of the original heuristics and our MAB-based heuristics on a variety of benchmark families. We also compare with a purely stochastic baseline method that randomly selects a candidate heuristic at each node during search. Preliminary experiments show that our proposed adaptive learning techniques are more robust than the original heuristics with less variance for different classes of problems and instances. Our online adaptive heuristics also outperforms the original heuristics on many problem instances.

Making CP solvers black box and robust is highly desirable. This paper is a step in this direction as the solver can determine the search strategy rather than being specified in the constraint model while still giving good performance.

Related work

Adaptive CSP search strategies using machine learning techniques have been studied in CP. One distinguishing factor is whether an offline training phase is used. Portfolio methods employ offline training, using the learned training to select a solving strategy, which could be a search algorithm or a heuristic, when solving a particular problem instance. Some portfolio approaches are CPHydra (O'Mahony et al. 2008) and Proteus (Hurley et al. 2014). In addition, some approaches learn and generate new solving strategies in the offline phase, e.g. (Epstein and Petrovic 2007; Xu, Stern, and Samulowitz 2009).

Online learning approaches differ in that they do not include a static learning phase before solving a problem. For example, the Monte-Carlo tree search based method in (Loth et al. 2013) tries to expand the most promising nodes with online learning. The score function of the value heuristic is learned using a linear regression method in (Chu and Stuckey 2015). Bachiri et al. (Bachiri et al. 2015) propose to learn the rewards of nodes and use the rewards to guide the search to backtrack to certain nodes. A recent work on adaptive search heuristics is the parallel strategies selection (PSS) approach (Palmieri, Régin, and Schaus 2016). PSS first decomposes the CSP into a large number sub-problems. As the sub-problems are independent, parallelism can be readily used. Sampling is used with parallel solving to select a heuristic and the remaining sub-problems are solved in parallel with the heuristic.

The closest work is Balafrej et al. (Balafrej, Bessiere, and Paparrizou 2015) which proposes a MAB framework to select different levels of propagation during search. They use the UCB1 algorithm to adaptively select the consistency level at each node of the search tree. In their experiments on binary CSPs, they show that learning can find when higher consistencies than arc consistency should be employed during search. Our work differs in that we adapt their MAB framework for the problem of selecting search heuristics dynamically and our experiments are on non-binary CSPs.

Preliminaries

A constraint network $\mathcal{P}$ (CSP) is a pair $(\mathcal{X}, \mathcal{C})$ where $\mathcal{X}$ is a set of n variables $\{x_1, \ldots, x_n\}$ and $\mathcal{C}$ a set of e constraints $\{c_1, \ldots, c_e\}$. $D(x)$ is the domain of $x \in \mathcal{X}$. Each $c \in \mathcal{C}$ involves two components: a scope ($scp(c)$) which is an ordered subset of variables of $\mathcal{X}$; and a relation over the scope ($rel(c)$). Given $scp(c) = \{x_{i_1}, \ldots, x_{i_r}\}$, $rel(c) \subseteq \prod_{j=1}^{r} D(x_{i_j})$ represents the set of satisfying combinations of values for the variables in $scp(c)$. We define *degree* of variable x to be the number of constraints that x belongs to. The *arity* of c is $|scp(c)|$. A binary CSP is of arity 2, while a non-binary CSP has constraints with arity > 2. A *solution* to $\mathcal{P}$ is an assignment to all variables in $\mathcal{X}$ such that every constraint is satisfied.

Constraint solvers typically explore the solution space by instantiating variables in some order. Usually, a variable ordering heuristic defines a score function, instantiating the variable with highest score at each search node. Static variable ordering heuristics compute variable scores before search, thus variable ordering is static. Dynamic ones update scores and tune the variable ordering dynamically during search. In practice, most of the successful variable ordering heuristics are dynamic ones, including *ddeg/dom* (Bessiere and Regin 1996; Smith and Grant 1998), *wdeg/dom* (Boussemart et al. 2004), *impact* (Refalo 2004), and *activity* (Michel and Hentenryck 2012). The *ddeg/dom* and *wdeg/dom* heuristics take the degrees and the current domain sizes of variables as parameters to the score functions. In *ddeg/dom*, a variable's score is the value of its current degree divided by the variable's current domain size. The current degree of a variable is the number of constraints, involving the variable, whose non-instantiated variables are more than one. This is extended to weighted degree in *wdeg/dom*, a variable's score is the values of its weighted degree divided by variable's current domain size. The weighted degree of a variable is the number of accumulated failures of the constraints which the variable belongs to. The *impact* heuristic selects the most influential variable which has made the most search space reduction in the space have been explored. The *activity* heuristic measures activity, i.e. how often a variable's domain is filtered by constraint propagation, selecting the most active.

The multi-armed bandit problem (Gittins 1989) comes from slot machines (one-armed bandit). A player chooses a slot machine from multiple ones (multi-armed bandit) to maximize the total expected payoffs or rewards for a sequence of plays. In MAB, an important consideration is the tradeoff between *exploration* and *exploitation*. An MAB algorithm should exploit the actions with maximal rewards. However, without exploring other actions enough, the algorithm may lose the opportunity for finding better actions. Thus, an MAB algorithm balances between exploration and exploitation. One way is to minimize the cumulative regret. Two of the successful and well-known MAB algorithms are the *Thompson Sampling* (TS) algorithm (Thompson 1933) and the *Upper Confidence Bound algorithm* UCB1 (Auer, Cesa-Bianchi, and Fischer 2002). Thompson sampling is one of the earliest algorithms and easy to implement. In practice, UCB1 is widely used for MAB due to its simplicity. It guarantees a logarithmic increase in regret. We apply these algorithms to our problem because of recent promising results (Balafrej, Bessiere, and Paparrizou 2015; Phillips et al. 2015) and due to their simplicity. They can also be used as a standard baseline (Auer, Cesa-Bianchi, and Fischer 2002; Chapelle and Li 2011).

Multi-armed bandit for adaptive search

We consider the problem of selecting a variable ordering heuristic to pick which variable to explore in the search tree to be analogous to the multi-armed bandit (MAB) problem. We map the automatic selection of variable ordering heuristics as a multi-armed bandit problem as follows. We define a set of $\mathcal{K}$ arms $\{\mathcal{L}_1, \ldots, \mathcal{L}_k\}$ where each arm $\mathcal{L}_i$ corresponds to one candidate heuristic. MAB algorithms aim to maximize the total rewards and take actions based on the reward of each arm. We can determine the rewards during search, thus, a *reward* $R_i(j)$ is associated with each arm $i \in [1, k]$ at each node j in the search tree. While solving CSPs, we would like to explore the solution space more quickly. One strategy is to try to make the search tree smaller. Note that the propagators are fixed during search since the propagators come from the constraints of the problem. Thus, we define the rewards of candidate heuristics taken at each search node to be based on the number of children of the node.

MAB-based adaptive search framework

In this paper, we propose a generic MAB-based search framework and adapt two specific MAB algorithms, Thompson Sampling and Upper Confidence Bound 1, to this framework. Our MAB search framework adapts MAB algorithm for the problem of dynamic selection of heuristics during backtracking search in CSP solving as follows:

1. (initialize) Initialize data structures of the MAB algorithm before search starts.
2. (chooseArm) For each unexplored search node, first use the MAB algorithm to select a heuristic (an arm) and bind the selected heuristic to the node. Then use the selected heuristic to instantiate variables at the node.
3. (update) Once search backtracks from a child node to its parent, which indicates that the child node is fully explored, update the mean rewards of the heuristic bound to the child node.

The main question is how to define the rewards for the MAB. Our aim is to speed up solving by reducing the size of the explored search tree. As we select a heuristic at each search node, we will define the reward of the heuristic to be based on observations of the node. We propose to use the number of children of each search node to estimate the exploration of each search node since search cost is usually correlated with the number of choices from a node. Thus, we set the reward of the heuristic taken at a certain search node to be based on the number of children of that node.

We emphasize that at each search node, only one heuristic is taken. Thus, the MAB selection of the choice of heuristic is performed only once at a node. Some of the underlying heuristics used (in arm selection) may the scores of variables in some accumulated fashion during search, e.g. *wdeg/dom* counts the accumulated number of failures. So the scores of variables for all the relevant heuristics need to be maintained at each search node during search. When search backtracks from a child node j to its parent node, we compute the rewards of the heuristic taken at node j and update the rewards of the particular heuristic and other parameters depending of the particular MAB algorithm during the execution of the update step.

Thompson adaptive search

Thompson sampling (TS) is an MAB algorithm which maintains a beta distribution for the reward of each arm (Thompson 1933; Chapelle and Li 2011), where arms are pulled randomly according to their probabilities of being optimal.

The idea is that the reward is based on the number of direct choices made from a variable selected at node j in the search tree. The rationale is to make the reward more position dependent compared with sub-tree size which varies depending on position. However, we employ the usual 2-way branching for search (i.e. left branch ($x = a$), right branch ($x \neq a$)) so the node degree is not the desired number of children. We define the (*effective*) *number of children* of node j, $C(j)$ as follows. When node j fails, $C(j)$ is the number of left branches along the right most failed path in the sub-tree from node j.

Algorithm 1: Thompson Sampling

```
  procedure initialize()
  begin
      for i ∈ {1...k} do
          R_i^ts ← 0, R_best^ts[i] ← 0
          α_i ← 1, β_i ← 1
  procedure chooseArm()
  begin
      for i ∈ {1...k} do
          // sample from the
             distribution
          ρ[i] ~ Beta(α_i, β_i)
      return arm i s.t. ρ[i] = max{ρ[1], ..., ρ[k]}
  procedure update(i, r)
  begin
      R_i^ts ← r
1     if R_i^ts ≥ R_best^ts[i] then
          R_best^ts[i] ← R_i^ts
          α_i ← α_i + 1
      else
          β_i ← β_i + 1
```

To make larger rewards better, we take the inverse value of $C(j)$ to be the reward $R^{ts}(j)$ of the heuristic at node j:

$$R^{ts}(j) = 1/C(j) \tag{1}$$

Algorithm 1 is the TS algorithm applied to our MAB adaptive search framework, with a simple an efficient implementation. The functions initialize(), chooseArm(), update() correspond to the three steps in the framework.

In initialize(), we initialize the two parameters α_i and β_i for each MAB selector to be 1, so the beta distribution starts as a uniform distribution. The mean rewards R_i^{ts} and best rewards $R_{best}^{ts}[i]$ of each arm i are initialized to 0.

We call chooseArm() to select the heuristic before exploring a search node. In chooseArm(), we draw a sample from each arm's beta distribution and choose the arm with maximum sample value. Once the arm is selected, the algorithm applies the selected heuristic to instantiate variables and explore the search node.

When a backtrack happens, we compute the reward r for arm i, and update the beta distribution of arm i in function update(i, r). We compare an arm's current reward with its best reward seen so far (line 1). If r improves or equals the current best reward $R_{best}^{ts}[i]$ of arm i, i.e. arm i explored the fewest number of children at current node, we consider as a success trial and increase α_i in the $Beta$ distribution by 1, otherwise it is a failed trial, increasing β_i by 1.

UCB1 adaptive search

UCB1 (Auer, Cesa-Bianchi, and Fischer 2002) is designed to give an expected logarithmic growth of regret. In UCB1, the MAB selector pulls the arm, arm i, which maximizes the value of $\rho(i)$ according to the following function:

$$\rho(i) = R_i + \sqrt{2log(m)/m_i} \tag{2}$$

In Equation 2, R_i is the mean of the past reward of arm i, m_i is the number of past trials of arm i and m is the total number of trials that have been done. So the first term R_i promotes the arm gaining more rewards in the past, while the second term is for exploration by encouraging the arms which have been less frequently applied.

Typically, each constraint in a CSP can be thought of as mapping to a propagator in the solver and each propagator has a certain level of consistency, e.g., generalised arc consistency, bounds consistency, etc. Since the size of search tree, the number of explored nodes, can dominate the solving time of solving a CSP solutions where the consistency level of propagation is fixed, we define a reward function which depends on the ability of the heuristic (arm) in reducing the search space. The reward $R_i(j)$ for arm i at search node j is defined as:

$$R_i(j) = 1 - C(j)/max_{m=1..j}(C(m)) \tag{3}$$

Our reward is inspired by (Balafrej, Bessiere, and Paparrizou 2015) but uses the number of effective children of a node versus CPU time of sub-tree and a uniform selector.

In our framework for UCB1, the mean reward R_i is initialized as 0 in the initialize() procedure. Before backtracking, all candidate heuristics are selected in a round robin fashion, because rewards are only updated when a backtrack happens. This setting follows the second term of UCB Equation (2). There is also the possibility of customizing the initial mean rewards of different arms to make the selection biased towards some heuristic in cases where certain heuristics may be known to give good results for certain problems. Before exploring a search node, the MAB arm-select procedure chooseArm() selects an arm which maximizes $\rho(i)$ in Equation (2), then the chosen heuristic from the arm is used to explore the tree node. When search backtracks from node j, the rewards of the heuristic i used at j is computed using Equation (3) as in procedure update() to update the mean reward of arm i.

Dynamic UCB1 and TS search

The TS and UCB1 algorithms are meant for when the distribution of rewards during search are fixed, i.e. a stationary probability distribution. We can take the view that rewards could vary over time during search, thus, we propose to apply a non-stationary form of TS and UCB1, which consider the rewards of the most recent K search nodes dynamically. A non-stationary form of the UCB algorithm, sliding-window UCB, was proposed in (Garvier and Moulines 2011). We also apply the sliding-window form of UCB to TS. This gives us two dynamic adaptive heuristic variants with window size K: UCB1-K and TS-K.

In our UCB1-K (TS-K) algorithm during search, we first check the number of explored search nodes. When this number is less and equal than K, UCB1-K (TS-K) is the same as UCB1 (TS); and when the number is greater than K, we only take the most recent K search nodes into consideration. Specifically, for UCB-K, the value of R_i and m_i in Equation (2) are based on the recent K nodes, and m equals K. Similarly, for TS-K, the value of $R^{ts}_{best}[i]$, α_i, and β_i are also from the recent K nodes. Note that R_i and m_i of UCB1-K can be updated in constant time, while $R^{ts}_{best}[i]$ needs a priority queue to find the minimum within logarithmic time in the worst-case. Thus the overhead of TS-K is more that of the UCB1-K.

Experiments

We present experiments to evaluate our MAB inspired search heuristics. Our aim is to investigate if the MAB-based heuristics are more robust than the original ones. We also want to investigate the overall performance of all the tested heuristics. We compare our MAB-based methods with the candidate variable search heuristics used as choices with the MAB algorithms. Thus, the candidate search heuristics are one baseline. We also compare with another straightforward baseline stochastic strategy, *random-arm*, which chooses one heuristic from the candidate ones randomly at each search node. Note that random-arm is different from a pure random heuristic which instantiates variables randomly. In the TS and UCB1-based methods, we employ only a single MAB selector for the whole search tree. An alternative is to have multiple MAB selectors for each search level. Preliminary experiments on MAB with multiple search level selectors did not show them to be superior to a single selector. We have omitted the results due to lack of space. In the evaluation, the TS and random-arm methods are stochastic, while UCB1 and the four baseline heuristics are deterministic.

We evaluate our search heuristics on a variety of structured and unstructured problems, to investigate the search behavior across a range of problems. The benchmarks are from the CSP solver competition (http://www.cril.univ-artois.fr/CSC09). We use 363 problem instances from 15 problem series.[1] The structured problems are: traveling salesman (TSP), costas array, resource constrained project scheduling (RCPSP), balanced incomplete block designs (BIBD), all-interval, golomb ruler, crossword, FPGA, ssa and modified-renault. The unstructured problems are the hard random ones: rand3-20-20, rand3-20-20-fcd, rand8-20-5, dagrand, and half. The benchmark CSPs are all non-binary (but can have some binary constraints) and chosen to have diverse constraints, including extensional (table), intensional and also global constraints. The experiments were run on a 3.40GHz Intel i7-4770 machine.

We use the AbsCon solver (https://www.cril.univ-artois.fr/~lecoutre/software.html) for its versatility as a blackbox solver: many propagation algorithms and heuristics are implemented and selectable. We focus on search heuristics and their relative effectiveness in the experiments, thus the consistency levels and propagators for constraints are the AbsCon defaults.[2] We employ a full initialization of variable impact and activity at the root node of the search tree using

[1] Instances that are solved with no search or those where all heuristics timeout are ignored. Note that applying SAC at the root node can solve some problems without search.

[2] The default consistency is Generalized Arc Consistency.

		ddeg/dom	wdeg/dom	impact	activity	UCB1	UCB1-100	UCB1-500	TS	TS-100	TS-500	random-arm
#solved instance		311	311	314	311	323	318	316	324	322	**328**	317
runtime	mean	44.3	43.3	13.9	6.4	**3.5**	7.9	7.2	15.4	8.3	5.1	12.6
ratios	standard dev.	332.1	331.5	31.3	25.4	**9.1**	40.2	34.9	96.6	34.9	12.2	53.5
to	geomean	2.3	2.5	4.8	2.3	**2.1**	2.4	2.3	3.1	2.9	2.7	3.5
VBS	maximum	4607.8	4619.1	298.7	308.4	**120.7**	553.9	452.7	1348.6	416.5	136.6	516.9

Table 1: Overall results for all search heuristics.

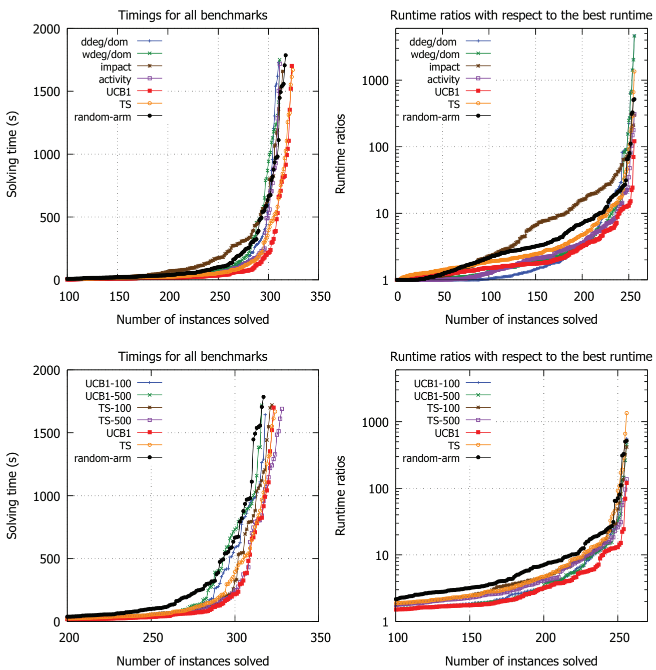


Figure 1: Distribution of runtime and runtime ratios to VBS.

the singleton arc consistency (Prosser, Stergiou, and Walsh 2000) propagator SAC3 (Lecoutre and Cardon 2005) in AbsCon which is needed to initialise the activity and impact heuristics. In order to ensure that search starts from the same state for different solving strategies, we apply the SAC3 propagation at the root node for all methods. The overhead of this initialization is negligible. CPU time is limited to 1800 seconds per instance and memory to 8GB.

In our experiments, we employ the four well-known and commonly used variable ordering heuristics (discussed previously): $ddeg/dom$, $wdeg/dom$, *impact*, and *activity*. We use these heuristics as the candidates (arms) of the MAB methods and also for random-arm. As we focus on investigating variable heuristics, we use the same lexicographic value heuristic (*lexico*) for all cases.

Overall results

To investigate robustness, we can measure performance with respect to the best runtime (Virtual Best Solver (VBS)) per instance as the runtime ratio to VBS, i.e. runtime/(VBS runtime). In order to compute the runtime ratios of all heuristics, we ignore an instance if there is one evaluated heuristic which cannot solve the instance within the timeout. Thus, the runtime ratios are computed based on 256 (out of 363) instances that are solved by all heuristics.

Table 1 gives the overall statistics (arithmetic and geometric mean, standard deviation, maximum) of all search heuristics using their runtime ratios. We see that UCB1 is the most robust with the smallest standard deviation and maximum ratio with respect to the VBS runtime. UCB1 also has the smallest mean ratio of 3.5 to VBS. The maximum ratio of UCB1 is 120.7, which is about 38X smaller than *ddeg/dom* and *wdeg/dom*. This shows that the baseline heuristics can give highly variable results highlighting the importance of robust heuristics. We see that TS-500 solves the most problems, slightly more than UCB1, but has higher means and standard deviation.

The graphs in Fig. 1 show the overall runtime distribution. The top two graphs are for individual heuristics and non-dynamic MAB-search while the bottom two graphs are for the dynamic MAB-search methods. The graphs on the the left are based on all 363 instances as they use solving time while the graphs on the right use the runtime ratio to VBS and are based on 256 instances solved by all search heuristics. Each point (x, y) in the left graphs shows that the technique solves x instances within y seconds while each point (x, y) in the right graphs shows that the technique solves x instances within y times of the VBS runtime. From Table 1, we saw that UCB1 had good robustness, the runtime distribution in Fig. 1 show that UCB1 (red line) also has the best overall result for the majority of instances. We highlight that the MAB methods have higher overheads as they also include the overhead of maintaining the rewards of the heuristic taken at each search node, as well as the variable scores of the unselected heuristics, whereas the underlying heuristics do not have this overhead.

The runtime of the TS method is also robust. In both of the graphs in Fig. 1, the lines for TS are closer to the best search strategies compared with the worst ones. We also see a surprising result. The simple random-arm heuristic is not the worst strategy, which might not be expected a priori, and can beat some of the original baseline heuristics. We observe that the random-arm method choosing among the baseline heuristics results in some robustness but as it does not have any exploitation, it has worse overall performance.

Robustness by benchmark series

The graphs in Fig. 2 present in detail the runtime distribution of four specific problem series showing that the MAB methods are robust–especially the UCB1 strategy. Dynamic

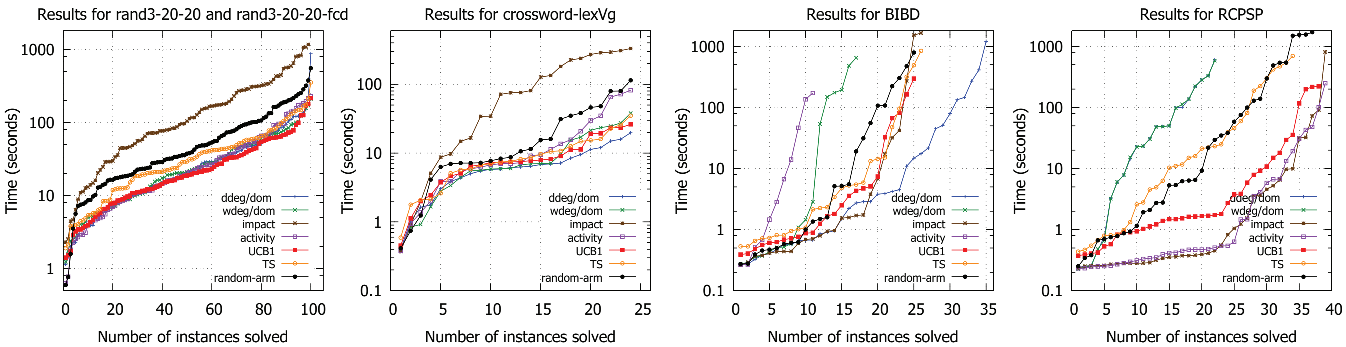


Figure 2: Runtime distribution for benchmark series rand3-20-20-fcd, dagrand, crossword-lexVg, RCPSP, BIBD and FPGA. All FPGA instances timeout under the $ddeg/dom$ heuristic, thus there are no points in the FPGA graph for $ddeg/dom$.

MAB methods are not given to avoid cluttering the graph as they were not as good overall as UCB1 on these series. Fig. 2 illustrates that the baseline heuristics while being good on some series are not robust. For example, if we ignore the UCB1 method, $ddeg/dom$ and the $wdeg/dom$ are the best two variable heuristics for rand3-20-20 and rand3-20-20-fcd, and the worst is the $impact$ heuristic. However for RCPSP, the result is just the opposite that $impact$ is much better than $ddeg/dom$ and $wdeg/dom$. We also see large differences between different heuristics for other problems. For example, in BIBD, the $ddeg/dom$ heuristic is faster than $activity$ by several orders of magnitude, but for the FPGA problem, $ddeg/dom$ does not solve any instance (graph for FPGA is not given for space reasons). This highlights the importance of having a robust heuristic.

We compare the runtime of different methods in pairs in Fig. 3. Fig. 3(a) compares the runtime of UCB1 with individual heuristics. The points located on the top and right boundaries are instances which timeout on the individual heuristics and UCB1 respectively. We can see that there are more points above the $x = y$ line including timeout points, indicating that UCB1 is faster than the compared heuristic. Furthermore, we see that the points in the upper portion are further away from the $x = y$ line than the points in the bottom portion, e.g. most UCB1 points are within the 10x (dotted) line while $wdeg/dom$ have many points outside the 10x line. This shows that when $wdeg/dom$ is slower than UCB1, it is much slower; but when UCB1 is slower, the slowdown is lesser. We see a similar trend in the other graphs.

Similarly, Fig. 3(b) gives the runtime of UCB1 compared with the dynamic MAB methods UCB1-100 and TS-500, and also UCB1 and TS compared with the random-arm. The graphs of "UCB1 vs. UCB1-100" and "UCB1 vs. TS-500" show that UCB1 is better than the dynamic UCB1-100, but close to TS-500. The graphs of UCB1 and TS versus random-arm show that learning is effective. Note that UCB1 is deterministic while TS and random-arm are stochastic.

The frequency of candidate heuristics

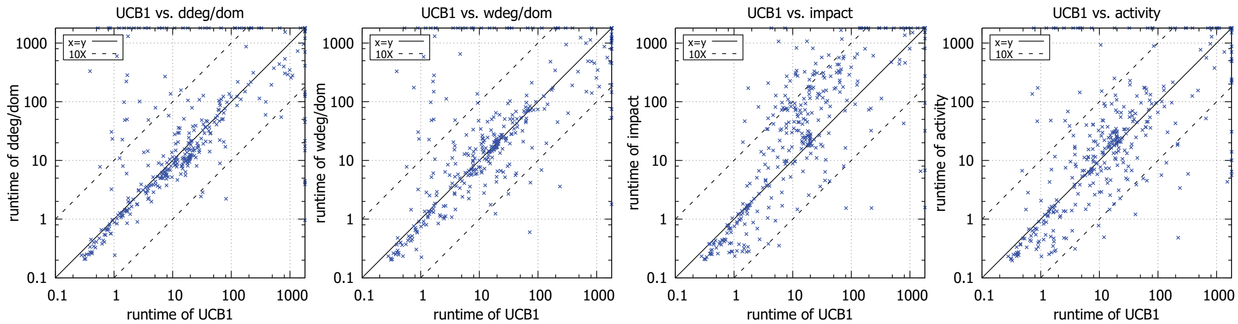
We investigate the frequency of candidate heuristics of MAB search and its correlation with the performance of the candidate heuristic on various problems. Fig. 4 gives the mean frequency of application of each heuristic when solving a benchmark series by UCB1 and TS. We see that UCB1 and TS can automatically differentiate between different heuristics. A correlation can be seen between the performance of individual heuristic and its application frequency in the MAB-based method. For example, in rand3-20-20 and rand3-20-20-fcd, the worst heuristic as shown in Fig. 2, is $impact$ which is used the least frequently. However as our MAB-based methods are online, such a correlation is not always the case, e.g. for BIBD $activity$ is the worst heuristic but is the most frequent heuristic applied in UCB1 and TS. We can also see that the frequency of heuristics used in the MAB-based algorithms vary significantly which suggests some complex interaction with the search process.

Discussion on PSS

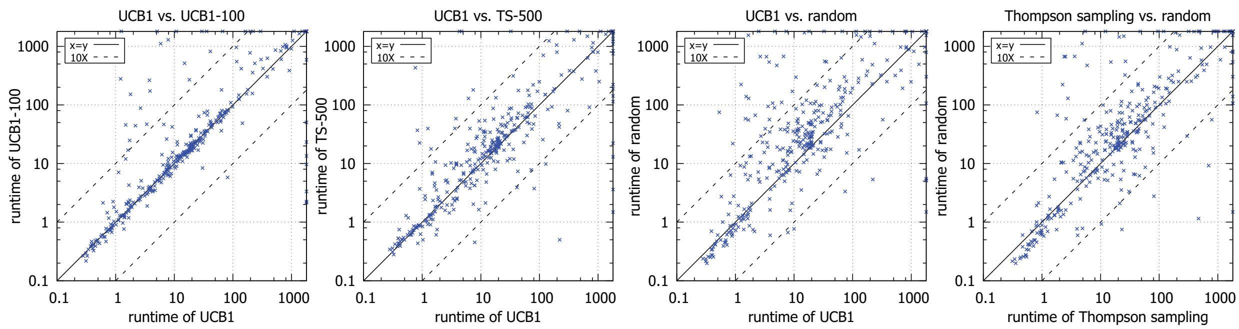
Recently, parallel strategies selection (PSS) was shown to be a promising approach for selecting search heuristics (Palmieri, Régin, and Schaus 2016). PSS is quite different from online or supervised learning based methods. Firstly, it exploits that a CSP can be decomposed into a large number of sub-problems which are independent and hence parallelism can be easily exploited. Secondly, it uses a statistical sampling approach, sampling sub-problems to choose the heuristic.

As PSS exploits a large parallelism factor from independent parallelism, it is not comparable with sequential methods. Most works on search heuristics are sequential as is this paper. However, PSS was shown to work well, hence, we also investigate PSS although it is not learning-based. We implemented a form of PSS, sequential PSS (sPSS), which is PSS with a parallelism factor of one. This allows decomposition and sampling strategies to be compared independently of the parallelism.

For space reasons, we summarize the results. We found on our benchmarks that sPSS is much slower than the MAB method especially on unsatisfiable problems. This is because all the sampled subproblems and the remainder subproblems have to be solved and there is no super-linear speedup from parallelism. Preliminary experiments show that the total number of explored search nodes of all subproblems of sPSS can be much more than that of MAB heuristics,



(a) UCB1 vs. individual candidate heuristics



(b) Comparison of MAB search strategies

Figure 3: Runtime comparison of MAB methods, individual candidate heuristics, and random heuristic on all instances.

e.g. for rand3-20-20-1, the total search nodes of sPSS is 507K while the search nodes of MAB-UCB1 is 73K. This makes the sPSS much slower than MAB and also other individual search heuristics. We also found that sometimes the decomposed subproblems are too easy, with few search nodes, which makes the solver initialization overhead more significant in the sequential case. For example, the mean number of search nodes of each subproblem of the unsatisfiable instance ruler-34-9-a4 is 8.4, although the total number of search nodes of all subproblems is close to that of UCB1. As such the total runtime of sPSS is 116.8s, compared with 6.9s of UCB1. We also found that the performance of sPSS approach can depend greatly on the decomposition, which suggests that sPSS not as robust as our MAB methods.

We caution that sPSS is not PSS and comparing sequential versus parallel algorithms is tricky. One results illustrate the expected behavior that sequential solving of sub-problems independently can fall into the unlucky cases that satisfiable subproblems are only selected late in execution. As such to benefit from the PSS approach, one should have sufficient parallelism, consistent with (Palmieri, Régin, and Schaus 2016). We also note that our sPSS implementation is only preliminary and can possibly be more efficient.

Conclusion

We propose a bandit-based approach which applies various variable heuristics automatically during CSP solving. Unlike independent heuristics, which explores the search space only based on a single approach (e.g. score function), our method considers several individual heuristics together and learns to apply better ones dynamically during search in an online fashion. Experiments show that our MAB methods are more robust than the original heuristics and can also give better performance.

Search heuristics for CSPs has been investigated extensively, e.g. utilizing the failures of constraints in $wdeg/dom$, or measuring the *activity* of variables during propagation. However the combination of various heuristics deserves more study as the solving can benefit from applying different heuristics according to a different status of the problem during search. Our MAB-based learning methods shows promise in this direction. Making a CP solver automatic and as "black box" as possible is highly desirable. Our experiments show that an automatic search strategy within the solver can be robust with good performance on many problems. In contrast, the common practice for performance requires the model or constraint program to provide a good search strategy. However, manual search heuristic selection may require expert knowledge with extensive tuning effort.

It would be interesting to combine online search heuristic selection with propagator selection. To be general, the solver should have non-binary propagators of different consistencies which may be interesting for global constraints.

Acknowledgments

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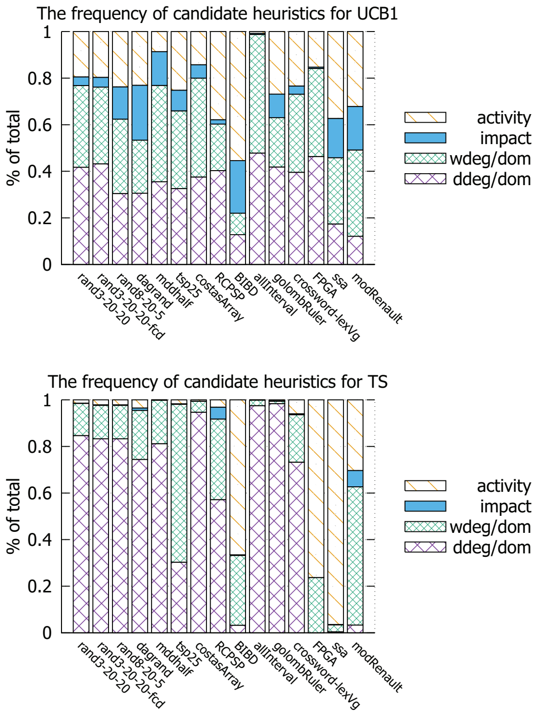

Figure 4: Frequency of candidate heuristics applied in the MAB-methods.

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