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A Spherical Hidden Markov Model for Semantics-Rich Human Mobility Modeling

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Abstract

We study the problem of modeling human mobility from semantic trace data, wherein each GPS record in a trace is associated with a text message that describes the user's activity. Existing methods fall short in unveiling human movement regularities for such data, because they either do not model the text data at all or suffer from text sparsity severely. We propose SHMM, a multi-modal spherical hidden Markov model for semantics-rich human mobility modeling. Under the hidden Markov assumption, SHMM models the generation process of a given trace by jointly considering the observed location, time, and text at each step of the trace. The distinguishing characteristic of SHMM is the text modeling part. We use fixed-size vector representations to encode the semantics of the text messages, and model the generation of the l_2-normalized text embeddings on a unit sphere with the von Mises-Fisher (vMF) distribution. Compared with other alternatives like multi-variate Gaussian, our choice of the vMF distribution not only incurs much fewer parameters, but also better leverages the discriminative power of text embeddings in a directional metric space. The parameter inference for the vMF distribution is non-trivial since it involves functional inversion of ratios of Bessel functions. We theoretically prove, for the first time, that: 1) the classical Expectation-Maximization algorithm is able to work with vMF distributions; and 2) while closed-form solutions are hard to be obtained for the M-step, Newton's method is guaranteed to converge to the optimal solution with quadratic convergence rate. We have performed extensive experiments on both synthetic and real-life data. The results on synthetic data verify our theoretical analysis; while the results on real-life data demonstrate that SHMM learns meaningful semantics-rich mobility models, outperforms state-of-the-art mobility models for next location prediction, and incurs lower training cost.

Introduction

Uncovering human mobility patterns is not only a fundamental task for human behavioral analysis, but also an important building block for urban planning, traffic forecasting, mobile health applications, and location-based recommender systems (Gonzalez, Hidalgo, and Barabasi 2008; Kitamura et al. 2000; Zhang et al. 2016a). Recent years are witnessing an increasing importance of modeling human mobility from semantic trace data, where each record in a trace is associated with a text message that describes the user's activity. With the wide proliferation of mobile devices and the ubiquitous access to the mobile Internet, massive semantic trace data are being collected by various social media services (*e.g.*, Twitter, Instagram, Facebook) and phone carriers on a daily basis (Cheng et al. 2011; Wu et al. 2015; Zhang et al. 2016b; 2017). Meanwhile, raw GPS trajectories can be readily linked with external sources (*e.g.*, map data, land uses) to annotate each record with rich semantic information (Wu et al. 2015).

The wide availability of semantic trace data necessitates a shift in the paradigm of human mobility modeling — it becomes possible to interpret human mobilities in a semantics-rich way. In addition to uncovering the spatiotemporal patterns of human movements, we could move one step further to understand *what* are people's activities at different regions, and *how* and *why* people move from one region to another. Such semantics-rich knowledge not only enables us to understand human mobility in a more interpretable way, but also plays an important role for prediction and decision making in various downstream applications.

Unfortunately, learning semantics-rich human mobility models from semantic trace data is a challenging problem that remains largely unsolved by existing techniques. Traditional mobility modeling techniques (Giannotti et al. 2007; Li et al. 2010; Cho, Myers, and Leskovec 2011; Mathew, Raposo, and Martins 2012) mostly focus on mining pure spatiotemporal regularities and cannot handle the text information in semantic traces. Recently, there have been research efforts that attempt to integrate the text information into the mobility modeling process based on the bag-of-words model (Ying et al. 2011; Wu et al. 2015; Zhang et al. 2016a). Nevertheless, these methods are unable to make the best use of the text information. First, by considering each keyword as an independent dimension, they do not model the correlations between keywords (*e.g.*, 'car', 'taxi' and 'drive') and may fail to correlate semantically similar messages. Further, since the vocabulary size is often large, their performance is limited by the high dimensionality and text sparsity, and meanwhile results in high computational overhead in the model learning process.

To learn semantics-rich mobility models from semantic

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trace data, we propose SHMM, a method that uncovers human mobility regularities by statistically modeling the generation process of the given trace data. SHMM is a multimodal spherical hidden Markov model (HMM). Under the hidden Markov assumption, it jointly models the generation of the observed location, time, and text at each step of an input trace. While the low-dimensional location and time can be modeled with Gaussian distributions, the key challenge is to capture the semantics of the high-dimensional text messages and model textual semantics. To address this challenge, we use fixed-size vector representations to encode the semantics of the text messages, which has been recently shown successful for a wide variety of NLP tasks (Mikolov et al. 2013a; 2013b). With the derived text embeddings, we further model the generation of the l_2-normalized text embeddings on a unit sphere with the von Mises-Fisher (vMF) distribution (Fisher 1953). Compared with other alternatives like multi-variate Gaussian, our choice of the vMF distribution not only incurs much fewer parameters, but also better unleashes the discriminative power of text embeddings in a directional metric space.

In the parameter inference process of our SHMM model, we use the classical Expectation-Maximization (EM) algorithm (Dempster, Laird, and Rubin 1977). However, since the vMF distribution has a complicated mathematical form, literature so far has not yet proved that EM algorithm is able to work with the vMF distribution. We, for the first time, theoretically prove the feasibility of using the EM algorithm on the vMF distribution. Furthermore, while closed-form solutions are hard to be obtained for the M-step, we prove that using Newton's method is guaranteed to converge to the optimal solution with quadratic convergence rate.

To summarize, we make the following contributions:

1. We propose a spherical hidden Markov model for human mobility modeling with semantic trace data. Compared with existing mobility models, our method is novel in that it uses embeddings to encode the semantics of text messages and the von-Mises Fisher distribution to model the generation of text embeddings.
2. We provide rigorous theoretical proof to show that the EM algorithm is able to work with the vMF distribution. Also, while obtaining closed-form solutions for the M-step is intractable, we prove that Newton's method is guaranteed to converge to the optimal solution with quadratic convergence rate. Some other properties of the vMF distribution and modified Bessel functions are also studied.
3. We perform extensive experiments on both synthetic and real-life data. The results on synthetic data verify our theoretical analysis; while the results on real-life data demonstrate that SHMM learns meaningful semantics-rich mobility models, outperforms state-of-the-art mobility models for next location prediction, and incurs lower training cost.

Problem Description

We study the problem of modeling human mobility from semantic trace data. Semantic traces are text-rich GPS traces where each GPS record is associated with a text message that describes the user's activity. We provide the formal definition of a semantic trace as follows.

Definition 1 (Semantic Trace)**.** *The semantic trace of a user u is a time-ordered sequence $S = [x_1, x_2, \dots, x_R]$. Each record x_i is a tuple (t_i, l_i, m_i) where: (1) t_i is the timestamp scalar; (2) l_i is a two-dimensional vector representing the location of the user at time t_i; and (3) m_i is a text message vector describing the activity of the user.*

To capture the semantics of user activities, we use distributed representations for the text messages in our model. Specifically, we first use the CBOW model (Mikolov et al. 2013b) to obtain fixed-size vector representations (*i.e.*, embeddings) for the keywords in the given corpus. The parameters used are: *min-count*=10, *size*=30, *window*=5, *sample*=10^{-4}, *negative*=5. As each text message usually consists of multiple keywords, we compute the TF-IDF weights of the keywords and use the weighted average of keyword embeddings to derive the embedding of the message m_i (Le and Mikolov 2014; Arora, Liang, and Ma 2016).

Now we are ready to formulate our mobility modeling problem. Given the semantic traces for a group of users $D = \{S_1, S_2, \dots, S_G\}$, our work aims to build semantics-rich mobility models for those users. The result mobility model is expected to address two aspects regarding human mobility: (1) **Discovering latent states.** The first aspect is to discover the latent states that govern people's activities. A latent state is an abstraction of *what* the user is doing around *where* during *when*. Examples include *shopping* in the *5th Ave* at *5pm*, and *watching a film* at the *AMC theater* in the *evening*. (2) **Characterizing transition regularity.** The second aspect is to characterize how users move sequentially between the latent states. For example, after shopping in the 5th Ave, what activities will the users do next? We aim to characterize people's transitions among the latent states in a concise and probabilistic way.

The SHMM Model

In this section, we describe SHMM in detail and describe the parameter inference procedure.

Model Description

Consider a sequence of chronologically ordered records $x_1, x_2, \dots, x_R$ of a user u. In SHMM, we adopt the hidden Markov assumption, *i.e.*, assuming each record x_i is generated from a latent state z_i, and the sequence $z_1, z_2, \dots, z_R$ follows a Markov process. The Markov process is parameterized by an initial probability matrix π over the latent states, as well as a matrix A that specifies the transition probabilities among the latent states. When generating x_i from z_i, we assume the location l_i, the timestamp t_i, and the text embedding m_i are generated independently. Therefore, the conditional probability $p(x_i|z_i)$ is given by $p(x_i|z_i) = p(t_i|z_i) \cdot p(l_i|z_i) \cdot p(m_i|z_i)$.

For each record x_i, we assume the following distributions for each component: 1) the timestamp t_i is generated from a univariate Gaussian with mean μ_t and variance σ_t, *i.e.* $p(t_i|z_i) = \mathcal{N}(t_i|\mu_t, \sigma_t)$, where t_i indicates the time in a day; 2) the location l_i is generated from a bivariate Gaussian with mean μ_l and covariance matrix Σ_l, *i.e.* $p(l_i|z_i) = \mathcal{N}(l_i|\mu_l, \Sigma_l)$; 3) the message vector m_i is generated from the von Mises-Fisher (vMF) distribution with mean direction μ and concentration parameter κ, *i.e.* $p(m_i|z_i) = \text{vMF}(m_i|\mu, \kappa)$.

While the Gaussian distribution is suitable for modeling timestamps and locations, it is problematic for modeling text embeddings. The reason is two-fold. First, using Gaussian distributions to model text embeddings would lead to a large co-variance matrix with too many parameters. Second, previous research has demonstrated that the cosine distance better reflects the semantic similarity between text embeddings compared to the Euclidean distance, *i.e.*, the discriminative power of the text embeddings is stronger in a directional metric space. Our choice of the vMF distribution addresses the above two issues. A vMF distribution is defined on the p-dimensional unit sphere, parameterized by two parameters: a mean direction μ and a concentration parameter κ. The mean μ specifies the direction of the semantic focus of the text embeddings, while κ controls how concentrated the text embeddings are around the mean direction. The larger κ is, the more concentrated the text embeddings are around the mean direction. Formally, the probability density function of a vMF distribution for a p-dimensional unit vector m is defined as:

$$f_p(m; \mu, \kappa) = C_p(\kappa) \exp(\kappa \mu^T m),$$

where $||\mu|| = 1$, $\kappa \geq 0$, $C_p(\kappa) = \frac{\kappa^{p/2-1}}{(2\pi)^{p/2} I_{p/2-1}(\kappa)}$, and p is the dimension of the vector. $I_v(\cdot)$ is the modified Bessel function of the first kind at order v and is defined as $I_v(\kappa) = \sum_{q=0}^{\infty} \frac{1}{q!\Gamma(q+v+1)} (\frac{\kappa}{2})^{2q+v}$, and $\Gamma(\cdot)$ is the gamma function.

Parameter Inference

In our SHMM model, the parameters to be estimated are the parameters (π, A) for the hidden states and the distribution parameters $(\mu_t, \sigma_t, \mu_l, \Sigma_l, \mu, \kappa)$. Since we are using standard Gaussian distributions for modeling location and time, the updating rules for all the parameters except κ can be easily derived by the Baum-Welch algorithm (Baum et al. 1970) — an Expectation-Maximization procedure for HMM. The challenge of applying the Baum-Welch algorithm is how to estimate the parameter κ.

Due to the complicated form of the vMF distribution, it is intractable to derive closed-form solutions for κ in the M-step of the Baum-Welch algorithm. However, we found that one can use Newton's method to find an approximate solution of κ that asymptotically converges to the optimal value. Below we first present our method for updating κ based on Banerjee's work (Banerjee et al. 2005) and Newton's method, and then show our theoretical analysis that our update rule achieves quadratic convergence rate. In the M-step of the Baum-Welch algorithm, we estimate the κ value with the following iterative procedure:

```
$\bar{r} \leftarrow \frac{||\sum_{i=1}^{N} m_i||}{N}$
$\kappa \leftarrow \frac{\bar{r}p - \bar{r}^3}{1 - \bar{r}^2}$
repeat
    $\kappa \leftarrow \kappa - \frac{A_p(\kappa) - \bar{r}}{1 - A_p^2(\kappa) - \frac{p-1}{\kappa} A_p(\kappa)}$
until convergence
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where m_i is a l_2-normalized p-dimensional text embedding, N is the total number of text embeddings belonging to the current state, and $A_p(\kappa) = \frac{I_{p/2}(\kappa)}{I_{p/2-1}(\kappa)}$.

Theoretical Analysis

Due to the complicated mathematical form of the vMF distribution, no existing literature has proved that the vMF distribution can work under the EM framework. In this section, we theoretically prove that:

1. The EM algorithm is able to work with the vMF distribution, because there exists a unique κ such that the Q-function in the M-step can be maximized.
2. While closed-form solutions for κ are hard to be obtained, one can use Newton's method for obtaining an approximate solution, which is guaranteed to converge to the optimal κ for M-step with quadratic convergence rate.

Theorem 1. *There exists a unique κ that maximizes the Q-function in the M-step of the EM algorithm.*

Proof. To maximize the Q-function, it is equivalent to solve $A_p(\kappa) = \bar{r}$ where $\bar{r} = \frac{||\sum_{i=1}^{N} m_i||}{N}$ (Banerjee et al. 2005). Based on this result, we have the following claims:

1. Claim 1: $0 < \bar{r} \leq 1$.
   Proof. It is obvious that $\bar{r} > 0$. Since $m_{i1}^2 + m_{i2}^2 + \ldots + m_{ip}^2 = 1$, we have $||\sum_{i=1}^{N} m_i||^2 = (m_{11} + ... + m_{N1})^2 + ... + (m_{1p} + ... + m_{Np})^2 \leq N(m_{11}^2 + ... + m_{N1}^2 + ... + m_{1p}^2 + ... + m_{Np}^2) = N^2$. Hence, $\bar{r} = \frac{||\sum_{i=1}^{N} m_i||}{N} \leq 1$.
2. Claim 2: $\lim_{\kappa \to 0} A_p(\kappa) = 0$, $\lim_{\kappa \to \infty} A_p(\kappa) = 1$
   Proof. The first equation is given by Lemma 2.1 in (Segura 2011). With Corrolary 1 in (Balachandran, Viles, and Kolaczyk 2013), we have $\exp(-\frac{p}{2\kappa}) \leq A_p(\kappa) \leq \exp(-\alpha_0 \frac{p-1}{2\kappa})$, where $\alpha_0 = -log(\sqrt{2} - 1)$, if $p \leq 2\kappa$. Hence, we have $\lim_{\kappa \to \infty} \exp(-\frac{p}{2\kappa}) \leq \lim_{\kappa \to \infty} A_p(\kappa) \leq \lim_{\kappa \to \infty} \exp(-\alpha_0 \frac{p-1}{2\kappa})$. Therefore, $\lim_{\kappa \to \infty} A_p(\kappa) = 1$.
3. Claim 3: $A_p(\kappa)$ is a continuous function if κ is real-valued and positive.
   Proof. By the definition of the modified Bessel function and its recurrence relation (Equation 9.6.1 and 9.6.26 in (Abramowitz and Stegun 1964)), we can get $A_p'(\kappa) = 1 - A_p^2(\kappa) - \frac{p-1}{\kappa} A_p(\kappa)$. Since $A_p(\kappa)$ is differentiable when κ is real-valued and positive, $A_p(\kappa)$ is continuous at all real-valued positive κ.

By the intermediate value theorem and the above claims, we have that there exists a solution for $A_p(\kappa) = \bar{r}$. Since we have $A'_p(\kappa) > 0$ for all positive κ (Equation 15 in (Amos 1974)), there exists a unique κ such that $A_p(\kappa) = \bar{r}$ and the Q-function is maximized. This completes the proof. □

So far, we have proved that there exists one unique κ to maximize the Q-function. This implies the EM algorithm is able to work with vMF distributions. However, it is non-trivial to find the optimal κ since it involves calculating the ratios of modified Bessel functions. Next, we show that the solution can be found by Newton's Method.

Lemma 1. *$A_p(\kappa)$ is a strictly concave function, when $\kappa > 0$ and $p \geq 2$.*

Proof. It is shown in Theorem 3 in (Simpson and Spector 1984) that $\frac{\kappa}{A_p(\kappa)}$ is strictly convex for $\kappa \geq 0$, $p \geq 2$. Thus, $\frac{-\kappa A''_p(\kappa)A^2_p(\kappa) - 2A_p(\kappa)A'_p(\kappa)(A_p(\kappa) - \kappa A'_p(\kappa))}{A^4_p(\kappa)} > 0$. Since $A_p(\kappa) > 0$, we get $A''_p(\kappa) < \frac{2A'_p(\kappa)}{\kappa A_p(\kappa)}(\kappa A'_p(\kappa) - A_p(\kappa))$. Now we have: **1)** $A'_p(\kappa) > 0$ and $\kappa A'_p(\kappa) - A_p(\kappa) < 0$, (Equation 15 in (Amos 1974)); **2)** $A_p(\kappa) = \frac{I_{p/2}(\kappa)}{I_{p/2-1}(\kappa)} > 0$, since $I_v(\kappa)$ is positive when $v \geq 0$, and $\kappa > 0$ (Equation 9.6.1 in (Abramowitz and Stegun 1964)). Therefore, $A''_p(\kappa) < 0$ and it is strictly concave. This completes the proof of Lemma 1. □

Building on Lemma 1, we proceed to show that Newton's Method is guaranteed to converge to the solution for $A_p(\kappa) = \bar{r}$.

Theorem 2. *Newton's method is guaranteed to converge to the solution for $A_p(\kappa) = \bar{r}$.*

Proof. Assume $\kappa = r$ is the solution, *i.e.* $A_p(r) - \bar{r} = 0$. Let's start with a point κ_0 and $0 < \kappa_0 \leq r$. We define $e_n = \kappa_n - r$. By Newton's updating rule, we have

$$e_{n+1} = e_n - \frac{A_p(\kappa_n) - \bar{r}}{A'_p(\kappa_n)} \tag{1}$$

By Taylor's Theorem, we have $A_p(r) = A_p(\kappa_n - e_n) = A_p(\kappa_n) - e_n A'_p(\kappa_n) + \frac{1}{2}e_n^2 A''_p(\xi_n) = \bar{r}$, where $\kappa_n \leq \xi_n \leq r$. Then we have

$$e_{n+1} = \frac{e_n A'_p(\kappa_n) - A_p(\kappa_n) + \bar{r}}{A'_p(\kappa_n)} = \frac{1}{2}\frac{A''_p(\xi_n)}{A'_p(\kappa_n)}e_n^2. \tag{2}$$

Therefore, $e_n \leq 0$ for all n since $A''_p(\xi_n) < 0$ and $A'_p(\kappa_n) > 0$. That implies $\kappa_n \leq r$ and $A_p(\kappa_n) \leq \bar{r}$ for all n. Therefore, from Equation (1), we have $e_{n+1} \geq e_n$. Thus, the sequence $e_0, e_1, \ldots, e_n$ is an increasing sequence and bounded by 0, and therefore, it must have a limit e^*. Accordingly, the sequence $\kappa_0, \kappa_1, \ldots, \kappa_n$ must have a limit κ^*. From Equation (1), we have $\lim\limits_{n\to\infty} e_{n+1} = \lim\limits_{n\to\infty}(e_n - \frac{A_p(\kappa_n) - \bar{r}}{A'_p(\kappa_n)})$, and thus $A_p(\kappa^*) = \bar{r}$ and $\kappa^* = r$. Therefore, if we choose any positive starting point κ_0, such that $\kappa_0 \leq r$, Newton's method is guaranteed to converge to the solution. □

We have shown that Newton's method is guaranteed to converge to the solution for $A_p(\kappa) = \bar{r}$. The update rule is simply given by $\kappa_{n+1} = \kappa_n - \frac{A_p(\kappa_n) - \bar{r}}{A'_p(\kappa_n)}$, where $A'_p(\kappa_n) = 1 - A^2_p(\kappa_n) - \frac{p-1}{\kappa_n}A_p(\kappa_n)$. Next, we show the convergence rate of Newton's method.

Lemma 2. *The rate of convergence for calculating $A_p(\kappa) = \bar{r}$ using Newton's Method is quadratic and $e_{n+1} \approx Ce_n^2$ with $C \in (-1, 0)$.*

Proof. Equation (2) shows the convergence rate is quadratic. Then, we have $e_{n+1} = \frac{1}{2}\frac{A''_p(\xi_n)}{A'_p(\kappa_n)}e_n^2 \approx \frac{1}{2}\frac{A''_p(\kappa_n)}{A'_p(\kappa_n)}e_n^2$ and

$$\begin{aligned}\frac{A''_p(\kappa_n)}{A'_p(\kappa_n)} &= \frac{(-2A_p(\kappa_n) - \frac{p-1}{\kappa_n})A'_p(\kappa_n) + \frac{p-1}{\kappa_n^2}A_p(\kappa_n)}{A'_p(\kappa_n)} \\ &> -2A_p(\kappa_n) - \frac{p-1}{\kappa_n} + \frac{p-1}{\kappa_n^2}\kappa_n = -2A_p(\kappa_n)\end{aligned}$$

where the last inequality uses Equation 15 in (Amos 1974). Therefore, $\frac{A''_p(\kappa_n)}{A'_p(\kappa_n)} \in (-2A_p(\kappa_n), 0)$. We have shown that $\lim\limits_{\kappa\to 0} A_p(\kappa) = 0$, $\lim\limits_{\kappa\to\infty} A_p(\kappa) = 1$ and $A'_p(\kappa) > 0$ in Theorem 1, and hence $A_p(\kappa_n) \in (0, 1)$. Therefore, $e_{n+1} \approx Ce_n^2$, where $C \in (-1, 0)$. □

Experiments

In this section, we empirically evaluate the performance of SHMM. We implemented SHMM and the baseline methods in JAVA and conducted all the experiments on a computer with 2.9 GHz Intel Core i7 CPU and 16GB memory.

Experimental Setup

Data We use both synthetic and real-life datasets to evaluate SHMM. The real-life datasets are semantic traces from Twitter users collected by Zhang *et al.* (Zhang et al. 2016a). The first dataset (LA) consists of million-scale geo-tagged tweets created by Los Angeles users from 2014.08.01 to 2014.11.30. Following the preprocessing steps in (Zhang et al. 2016a), we first group the tweets by user ID to obtain the location history for each user. Since two consecutive records in a raw location history can be large, we further segment the location history into dense semantic traces with a time threshold $\Delta t = 6\ h$, such that the time gap between any two consecutive records is no larger than 6 hours. After preprocessing, we obtain approximately 30 thousand semantic traces. The second dataset (NY) consists of the geo-tagged tweets in New York City during 2014.08.01 and 2014.11.30. We preprocess the NY data in a similar manner and obtain approximately 42 thousand trajectories in total.

We also generate multiple synthetic data to verify the theoretical analysis of our SHMM model. For generating points from the vMF distribution, we use the code from Chen *et al.* (Chen et al. 2015). Given the synthetic points, we apply our proposed Newton's method for estimating the parameters of the SHMM model, and evaluate the approximation errors and convergence speed.

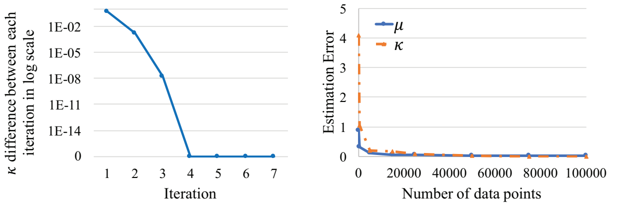
Baseline Methods We compare our SHMM model with the following baseline methods:

1. LAW (Brockmann, Hufnagel, and Geisel 2006) is a widely used mobility model based on the Lévy flight law with long-tailed distributions.
2. HMM (Mathew, Raposo, and Martins 2012) uses HMM to model the spatial locations in the trace data for mobility modeling.
3. ST-HMM is an extension of HMM that models both spatial and temporal information in the trace data.
4. GMove (Zhang et al. 2016a) is the state-of-the-art mobility model for semantic traces. It differs from SHMM in the text modeling part. It uses the bag-of-words model to represent text messages and uses multinomial distributions to generate the observed messages.
5. GHMM is an adaption of SHMM. It uses independent Gaussians to model text vectors instead of the vMF distribution.

Evaluation Protocol We use next location prediction as a downstream task for evaluating the quality of the SHMM model. Given a semantic trace dataset, we randomly select 70% traces for model training and use the rest 30% for testing. For a test trajectory $[x_1, x_2, ..., x_R]$, we assume the first $R-1$ locations $[x_1, x_2, ..., x_{R-1}]$ are observed and attempt to recover the last record x_R. Specifically, we first form a candidate pool by mixing x_R with other records whose creating time and distances are close to x_R. [1] After the candidate pool is formed, we use the SHMM model to select the top-K most likely visited records and see whether the ground-truth appears in the top-K list. We use the prediction accuracy $@K$ to measure the performance of different models, *i.e.*, the percentage of test traces for which the ground-truth record is recovered by the top-K list.

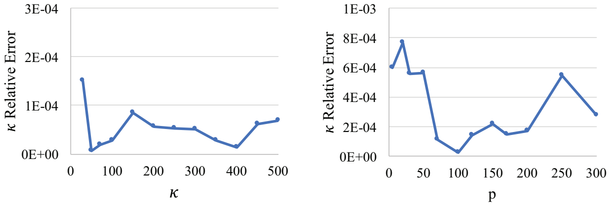
Results on Synthetic Data

Figure 1 shows the performance of our used Newton's method for estimating the parameters of a vMF distribution on synthetic data. As shown in Figure 1 (a), our estimation method converges extremely fast for estimating κ, achieving approximation errors smaller than 10^{-13} after three iterations. Figure 1 (b) shows the μ and κ estimation performance on synthetic datasets with different sizes. We can see the approximation error tends to become smaller on synthetic data sets with more samples. This is expected, as a small number of samples may lead to biased estimations of the true parameter values. The results in Figure 2 shows the estimation performance for different κ and dimension p (the number of samples is 100,000). Generally, we do not observe obvious patterns showing how the approximation errors change with different κ and p, but the relative approximation errors are quite small under different κ and p values.



(a) κ convergence plot (b) vMF parameter estimation error with $p = 100$ and $\kappa = 100$

Figure 1: The convergence of the Newton's method.



(a) varying κ with $p = 100$ (b) varying p with $\kappa = 100$

Figure 2: Estimation error for the concentration parameter κ

Results on Real-Life Data

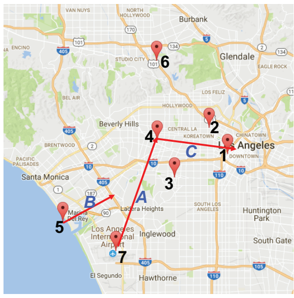
Visualization of the Mobility Models In this set of experiments, we set the number of states to 50 on LA and NY to obtain mobility models. After parameter inference, each state is characterized by: (1) a two-dimensional Gaussian distribution for the spatial location; (2) a one-dimensional Gaussian distribution for the time; and (3) a 30-dimensional vMF distribution for the semantics.

Figure 3 visualizes a number of representative states and some frequent transitions among them. We plot the mean location of some states as well as the top-10 keywords from the vocabulary whose embeddings are the closest to the vMF mean directions. Most of the top-10 keywords for the same state carry consistent and clear semantics. For example, for the BASEBALL state on the LA dataset: *homeruns* is a specific baseball term; *dodgers* is the baseball team in LA; *giants* is the baseball team in San Francisco; *162* indicates that there are 162 games for each team in the Major League Baseball (MLB) season; and the rest six keywords are all related to baseball too. We have examined the center locations of the states, and found that the geographical locations well match the semantic meanings of different states.

Another interesting finding is that in LA dataset, the mean direction of the *General Sports* state lies in-between the *Basketball* state and the *Baseball* state in the embedding space. Also, the concentration parameter κ for *General Sports* state is lower than *Basketball* and *Baseball* state. Such phenomenon intuitively makes sense since *General Sports* is a broader topic and the semantics of the tweets are more scattered.

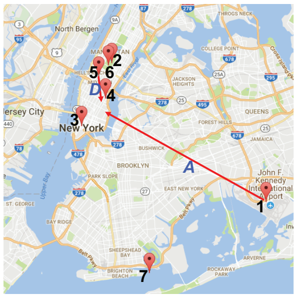
We have also observed some interesting state transitions. As shown in Figure 3, the following transitions receive high probabilities in the SHMM model: (A) moving from air-

[1] To select negative records, we set the distance threshold to 3.5 kilometers on LA and 2.0 kilometers on NY; and we set the time threshold to 300 seconds on both LA and NY.



State 1: **Basketball**	State 2: **Baseball**	State 3: **General Sports**	State 4: **Restaurants/ Food**	State 5: **Beach**	State 6: **Universal Studios**	State 7: **Airport**
lakers	homeruns	cfbplayoff	osteriamamma	monica	universal	international
grizzlies	dodgers	oilers	island's	beachselfie	waterworld	angeles
celtics	giants	2-May	hotwings	zoltar	hhn2014	los
nba	162	jets	jicama	beach	hollywoodhorrornights	airport
bryant	sfvspit	preseason	flore	fishbarmb	universalcitywalk	lax
hornets	freebaseball	scoreboard	edamame	piersantamonica	theboxtrolls	int'n
boozer	lincecum	athleticism	paninicafe	venice	universalstudio	toastbakerycafe
byronscott_4	kcroyals	basketball	cmcafela	beachbum	studios	riyadh
warriors	morse	hockey's	comfortfood	stamonica	thepurge	cdg
lin	zackgreinke	overtime	crabcake	baywatch	universalhorrornights	dn

(a) LA dataset state keywords and transitions



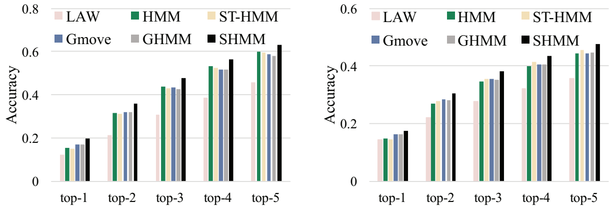
State 1: **Flight / Airport**	State 2: **Museum**	State 3: **Job/ Career/ Company**	State 4: **TV/ News/ Post/ Reporter**	State 5: **Broadway/ Music**	State 6: **Park/ Natural resource**	State 7: **Beach**
kennedy	museum	job	rt_america	motownthemusical	park	playlandmotel
international	museumofthemovingimage	careerarc	michaelmeans49	off-broadway	central	sandcastle
hkg	chuckjones	unify	ebolahearing	musical	parklife	rockaways
lingus	momapoprally	relati	gretchencarlson	broadway	reservoir	beach
jfk	silentfilm	newyorkus	unalienable	cinderellaonbroadway	mothernature	beachday
londonbound	musuem	scottrade	says	sondheim	royaltenenbaums	q53
icelandair	moma_live	sohallc	prisons	theatre	adventuring	riss
laguaria	nightatthemuseum	kwikset	thomasduncan	disgracedbway	centralparkwest	beachlife
yyz	moma_85	sybase	idfspokesperson	disenchanted	picnics	thunderboltnyc
nyc-usa	exhibits	caremark	seangchappellem	nederlander	race4thecure	rockawaybeach

(b) NY dataset state keywords and transitions

Figure 3: The visualization of the mobility models. The digits (1-7) indicate different state locations and the letters (A-D) indicate frequent human movements.

ports to restaurants; (B) enjoying beach activities at the Venice beach, and then moving around for other leisure activities; (C) going to concerts after having food; (D) watching shows at Broadway and then having other sightseeing activities in NYC Downtown. These high-probability transitions match people's movements in the real world well.

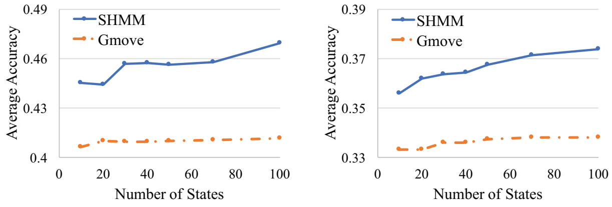
Performance for Next Location Prediction Figure 4 shows the performance of next location prediction for different mobility models. It can be seen that our SHMM model outperforms the state-of-the-art GMove model by 3.2% on average. The performance difference shows that the text embedding can better capture the semantics of text messages and reduce text sparsity. Also, the vMF distribution unleashes the discriminative power of text embeddings in a directional metric space.



(a) LA (b) NY

Figure 4: The accuracies of top-K location prediction.

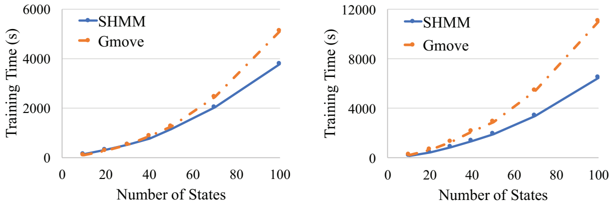
Effects of Parameters In Figure 5, we study the performance of SHMM and GMove when the number of states varies. We find that the performance of both models generally increases with the number of states. One major reason is that the semantics of people's activities are separated at finer granularities when the number of states is large. For example, we can see from the LA data set that *General Sports*, *Basketball* and *Baseball* are three separated topics. If the number of states is not large enough, these states may be clustered as one single topic. On the other hand, one caveat for choosing the number of states is that a large number of states could incur high computational overhead, and also harm the interpretability of the result model because the same semantics may be split into duplicate ones.



(a) LA (b) NY

Figure 5: Prediction accuracy v.s. the number of states.

Efficiency Comparison Finally, we compare the modeling training time between SHMM and GMove when the number of states varies. Generally speaking, the training time of both models increases quadratically with the number of states. The training time of SHMM is obviously smaller than that of GMove, *e.g.*, when the number of states is 100, training SHMM is 25.7% faster on LA and 40.9% faster on NY, and the speedups are even larger when the number of states or the data size increases. This is because SHMM models low-dimensional text embeddings instead of high-dimensional bag-of-words, and thus involves much fewer parameters. In addition, the estimation of the parameters for the vMF distribution is cheap and converges fast.



(a) LA

(b) NY

Figure 6: Running time v.s. the number of states.

Related Work

Human mobility modeling. Classic human mobility modeling methods focus on mining the spatiotemporal regularities underlying human movements. Generally, existing mobility modeling methods can be divided into two categories: pattern-based methods and model-based methods. Pattern-based methods aims at discovering specific mobility patterns that occur regularly. Different mobility patterns have been introduced to capture people's movement regularities, such as frequent sequential patterns (Giannotti et al. 2007), periodic patterns (Li et al. 2010), and co-location patterns (Kalnis, Mamoulis, and Bakiras 2005). Model-based methods use statistical models to characterize the human mobility, and learn the parameters of the designed model from the observed trace data. Mathew *et al.* (Mathew, Raposo, and Martins 2012) use the hidden Markov model to capture the sequential transition regularities of human mobility; Brockmann *et al.* (Brockmann, Hufnagel, and Geisel 2006) proposed that human mobility can be modeled by a continuous-time random-walk model with long-tail distribution; Cho *et al.* (Cho, Myers, and Leskovec 2011) introduce periodic mobility models to discover the periodicity underlying human movements.

While the above mobility modeling methods focus on spatiotemporal regularities without considering text data, recent years are witnessing growing interest in modeling human mobility from semantic trace data (Ying et al. 2011; Wu et al. 2015; Zhang et al. 2016a; 2014). Among these works, the state-of-the-art GMove model (Zhang et al. 2016a) is the most relevant to our model. Both GMove and SHMM use hidden Markov models to model the generation process of the observed semantic trace data. However, SHMM is different from GMove in that it encodes the semantics of user activities with text embeddings, and uses the vMF distribution to model the text embeddings in the HMM model. As such, the SHMM involves much fewer parameters and well unleashes the discriminative power of text embeddings in a directional metric space.

It is worth mentioning that, there are quite a number of works that uses human trace data for the location prediction problem (Wang et al. 2015; Liu and *et al.* 2016). Typically, they extract features that are important for predicting which place the user tends to visit next based on discriminative models such as recurrent neural networks. While we use location prediction as an evaluation task in our experiments, our work is quite different from these works. Instead of optimizing the performance of location prediction, our focus is to learn interpretable models that reveals the regularities underlying human movements. Besides location prediction, our learned mobility models can be used for many other downstream tasks as well.

vMF-based learning. There are some existing works that utilize vMF distribution for different learning tasks. Dhillon *et al.* (Dhillon and Sra 2003) and Banerjee *et al.* (Banerjee et al. 2005) are two pioneering works that use the vMF distributions to handle directional data, which demonstrate inspiring results for text categorization and gene expression analysis. Besides, Gopal and Yang (Gopal and Yang 2014) recently applied vMF distributions for clustering analysis, and proposed variational inference procedures for estimating the parameters of the vMF clustering model. Batmanghelich *et al.* (Batmanghelich et al. 2016) proposed a spherical topic model based on the vMF distribution, which accounts for word semantic regularities in language and has been demonstrated to be superior than multi-variate Gaussian distributions. However, there are no previous works that integrate the vMF distribution with HMMs for semantic trace data. To the best of our knowledge, we are the first to demonstrate that the vMF distribution can work well with HMM for directional data with theoretical guarantees.

Conclusion

We proposed a spherical hidden Markov model for learning interpretable human mobility model from semantic trace data. Our model uses text embeddings to capture the semantics of text messages and integrate the vMF distribution into the hidden Markov model for generating such text embeddings. We have theoretically proved that the Expectation-Maximization algorithm is able to work with vMF distribution, and that the Newton's method can be applied for efficiently solving the M-step with quadratic convergence rate. Our experiments on synthetic data simulations verify our theoretical analysis. Furthermore, by applying our model to real-life semantic trace datasets, we are able to obtain highly interpretable mobility models, which intuitively make sense and outperform baseline models for downstream tasks like location prediction.

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