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How Many Properties Do We Need for Gradual Argumentation?

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Abstract

The study of properties of gradual evaluation methods in argumentation has received increasing attention in recent years, with studies devoted to various classes of frameworks/methods leading to conceptually similar but formally distinct properties in different contexts. In this paper we provide a systematic analysis for this research landscape by making three main contributions. First, we identify groups of conceptually related properties in the literature, which can be regarded as based on common patterns and, using these patterns, we evidence that many further properties can be considered. Then, we provide a simplifying and unifying perspective for these properties by showing that they are all implied by the parametric principles of (either strict or non-strict) balance and monotonicity. Finally, we show that (instances of) these principles are satisfied by several quantitative argumentation formalisms in the literature, thus confirming their general validity and their utility to support a compact, yet comprehensive, analysis of properties of gradual argumentation.

1 Introduction

Abstract Argumentation Frameworks (AFs) (Dung 1995) are a well-known formalism to represent and resolve conflicts, expressed as a binary relation of *attack* amongst *arguments*. While AFs have proven useful to study conflict management, some of their underlying assumptions may be rather restrictive in some settings. As a result, several extensions or modifications of AFs and their semantics have been proposed, including the following, all relevant to this paper:

- a *support* relation can be considered alongside (or instead of) the attack relation (e.g. see (Amgoud et al. 2008; Baroni et al. 2015; Amgoud and Ben-Naim 2016b));
- while traditional semantics (Dung 1995) produce simple assessments of argument acceptance, finer *gradual evaluation* methods, based on numerical scales or rankings, can be used (e.g. see (Matt and Toni 2008; Leite and Martins 2011; Gabbay 2012; Amgoud and Ben-Naim 2013; Baroni et al. 2015; Amgoud et al. 2016));
- while arguments are considered a-priori equal in AFs, they can be equipped with an *initial evaluation* (e.g. called 'weighting' in (Amgoud and Ben-Naim 2016b)) reflecting e.g. the authoritativeness of their source or, as in (Leite and Martins 2011), votes they have received.

Given the variety of gradual evaluation methods possible, several works include (e.g. (Matt and Toni 2008; Leite and Martins 2011)) or are completely devoted to (e.g. (Bonzon et al. 2016; Amgoud and Ben-Naim 2016a)) the definition and study of properties that these methods (should) satisfy. This has given rise to a relatively large and varied body of properties, which may share basic intuitions but are presented in technically different contexts. This multiplication of studies, while potentially fruitful, carries the risk of concealing underlying common roots and possible overlappings.

We take a step towards the unification of efforts within this research trend, by analyzing properties in the context of generic *Quantitative Bipolar Argumentation Frameworks* (QBAFs) where each argument has a (possibly empty) set of attackers, a (possibly empty) set of supporters, and an initial evaluation (possibly the same for all arguments) on a chosen *scale*, all contributing to a final argument evaluation, provided by a *strength* function. QBAFs encompass other frameworks as special cases, including AFs, Bipolar Argumentation Frameworks (Amgoud et al. 2008), Social AFs (Leite and Martins 2011), Weighted Argumentation Graphs (Amgoud et al. 2017), Quantitative Argumentation Debate frameworks (Baroni et al. 2015), and Support Argumentation Frameworks (Amgoud and Ben-Naim 2016b).

The paper is organised as follows. After some preliminaries in Section 2, in Section 3 we gather literature properties in groups, each group being based on a basic idea which can give rise to many variants, several of which have not been explicitly considered in the literature. Then in Section 4 we introduce the principles of (strict and non-strict) balance and monotonicity and show that they imply the group properties considered in Section 3 and thus have the potential to support a greatly simplified analysis of actual formalisms. Section 5 confirms this potential since suitable instances of these principles are satisfied by a variety of existing gradual argumentation formalisms. Section 6 concludes.

2 Preliminaries

Let $\mathbb{I}$ be a set equipped with a preorder $\leq$ where, as usual, $a < b$ denotes $a \leq b$ and $b \nleq a$. We allow, but do not impose, that $\mathbb{I}$ contains top ($\top$) and bottom ($\bot$) values; if $\top, \bot \in \mathbb{I}$, $\bot < i < \top$ for all $i \in \mathbb{I} \setminus \{\bot, \top\}$. For example, $\mathbb{I} = [0,1]$ with $\top = 1$, $\bot = 0$, or $\mathbb{I} = (0, \infty)$. A QBAF assigns attackers, supporters and an initial evaluation (base score) in $\mathbb{I}$ to arguments.

Definition 1. A *Quantitative Bipolar Argumentation Framework* (QBAF) is a quadruple $\langle \mathcal{X}, \mathcal{R}^-, \mathcal{R}^+, \tau \rangle$ consisting of a set $\mathcal{X}$ of arguments, a binary (*attack*) relation $\mathcal{R}^-$ on $\mathcal{X}$, a binary (*support*) relation $\mathcal{R}^+$ on $\mathcal{X}$ and a total function $\tau : \mathcal{X} \to \mathbb{I}$; for any $\alpha \in \mathcal{X}$, we call $\tau(\alpha)$ the *base score* of α.

QBAFs can be visualised as graphs, e.g. Figure 1 visualises $\langle \{a,b,c,d\}, \{(c,a),(c,b)\}, \{(d,b)\}, \tau \rangle$ (for any τ).

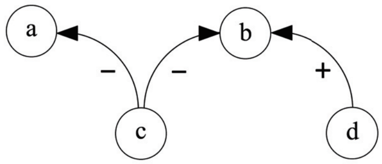

Figure 1: Example QBAF visualised as a graph.

In the remainder of the paper, unless specified otherwise, we assume as given a generic QBAF $\mathcal{Q} = \langle \mathcal{X}, \mathcal{R}^-, \mathcal{R}^+, \tau \rangle$.

QBAFs capture several existing formalisms in the literature as special cases. Let a QBAF $\mathcal{Q}$ be referred to as:

aQBAFf if $\forall \alpha \in \mathcal{X}, \mathcal{R}^+(\alpha) = \emptyset \wedge \tau(\alpha) = \top$
aQBAF if $\forall \alpha \in \mathcal{X}, \mathcal{R}^+(\alpha) = \emptyset$
sQBAF if $\forall \alpha \in \mathcal{X}, \mathcal{R}^-(\alpha) = \emptyset$
QBAFf if $\forall \alpha \in \mathcal{X}, \tau(\alpha) = i$ for some $i \in \mathbb{I}$

Then, for the purposes of our analysis in this paper, aQBAFfs (i.e. QBAFs with empty support and *fixed* base score $\top$ for all arguments) correspond to AFs (Dung 1995), aQBAFs (i.e. QBAFs with empty support) correspond to Social AFs (Leite and Martins 2011) (where base scores are determined by votes) and to Weighted Argumentation Graphs (Amgoud et al. 2017), sQBAFs (i.e. QBAFs with empty attack) correspond to Support Argumentation Frameworks (Amgoud and Ben-Naim 2016b), and QBAFfs (i.e. QBAFs with a *fixed* base score for all arguments) correspond to Bipolar Argumentation Frameworks, as in (Amgoud et al. 2008). Quantitative Argumentation Debate (QuAD) frameworks (Baroni et al. 2015) can be seen as generic QBAFs.

Arguments in a QBAF have a final evaluation (strength):

Definition 2. For any $\alpha \in \mathcal{X}$, the *strength* of α is given by $\sigma(\alpha)$ where $\sigma : \mathcal{X} \to \mathbb{I}$ is a total function. For any $A \subseteq \mathcal{X}$, we refer to the multiset $\{\sigma(\beta) | \beta \in A\}$ as $\sigma(A)$.[1]

If $\bot \in \mathbb{I}$, it may or not play a role in evaluating arguments. Thus, we consider two alternative notions of set equivalence:

Definition 3. Let $*$ be either $\sigma\bot$ or $\sigma\not\bot$. For $Z \subseteq \mathcal{X}$, let Z^* denote $\sigma(Z)$ if $* = \sigma\bot$ and $\sigma(Z) \setminus \{\sigma(z) \in \sigma(Z) | \sigma(z) = \bot\}$ if $* = \sigma\not\bot$. Then, for $A, B \subseteq \mathcal{X}$, A is *$*$-strength equivalent to* B, denoted $A =_* B$, iff $A^* = B^*$; A is *at least as $*$-strong as* B, denoted $A \geq_* B$, iff there exists an injective mapping f from B^* to A^* such that $\forall \alpha \in B^*$, $\sigma(f(\alpha)) \geq \sigma(\alpha)$; A is *$*$-stronger than* B, denoted $A >_* B$, iff $A \geq_* B$ and $B \not\geq_* A$.

[1] In this paper we use the same notation for sets and multisets.

(Matt and Toni 2008)	aQBAFf	$[0,1]$
(Amgoud and Ben-Naim 2013)	aQBAFf	ranking
(Thimm and Kern-Isberner 2014)	aQBAFf	ranking
(Amgoud and Ben-Naim 2016a)	aQBAFf	[0,1]
(Bonzon et al. 2016)	aQBAFf	pre-order
(Amgoud et al. 2016)	aQBAFf	$[1, \infty)$
(Leite and Martins 2011)	aQBAF	$\mathbb{I} \supseteq \{\bot, \top\}$ (ordered set)
(Amgoud et al. 2017)	aQBAF	$[0,1]$
(Amgoud and Ben-Naim 2016b)	sQBAF	$[0,1]$
(Amgoud et al. 2008)	QBAFf	$[-1,1]$
(Baroni et al. 2015)	QBAF	$[0,1]$
(Rago et al. 2016)	QBAF	$[0,1]$

Table 1: Choices of QBAFs (second column) and $\mathbb{I}$ (third column) for the literature considered (first column).

Note that Definition 3 for $* = \sigma\bot$ reformulates the notion of *group comparison* in (Amgoud and Ben-Naim 2013; Bonzon et al. 2016). In the remainder $*$ is either $\sigma\bot$ or $\sigma\not\bot$.

Definition 4. For any $\alpha \in \mathcal{X}$, the set of *attackers* of α is $\mathcal{R}^-(\alpha) = \{\beta \in \mathcal{X} | (\beta, \alpha) \in \mathcal{R}^-\}$ and the set of *supporters* of α is $\mathcal{R}^+(\alpha) = \{\beta \in \mathcal{X} | (\beta, \alpha) \in \mathcal{R}^+\}$. Also, $\mathcal{R}^-_*(\alpha) = \mathcal{R}^-(\alpha)$ if $* = \sigma\bot$ and $\mathcal{R}^-_*(\alpha) = \mathcal{R}^-(\alpha) \setminus \{\beta \in \mathcal{R}^-(\alpha) | \sigma(\beta) = \bot\}$ if $* = \sigma\not\bot$. Similarly, $\mathcal{R}^+_*(\alpha) = \mathcal{R}^+(\alpha)$ if $* = \sigma\bot$ and $\mathcal{R}^+_*(\alpha) = \mathcal{R}^+(\alpha) \setminus \{\beta \in \mathcal{R}^+(\alpha) | \sigma(\beta) = \bot\}$ if $* = \sigma\not\bot$.

Thus, we use $\mathcal{R}^-_*(\alpha)$ ($\mathcal{R}^+_*(\alpha)$) to denote the set of attackers (supporters, resp.) of an argument α including or excluding, depending on the choice of $*$, those with bottom strength. Note that $\mathcal{R}^-_*(\alpha) = \mathcal{R}^-_*(\beta)$ implies $\mathcal{R}^-(\alpha) =_* \mathcal{R}^-(\beta)$ but not vice versa (similarly for support). The notations $\mathcal{R}^-_*, \mathcal{R}^+_*$ and $=_*$ will be needed to capture properties from the literature.

The definition of our group properties and principles will also be parametric w.r.t. an operator over $\mathbb{I}$, $\ll$, expressing a form of strict comparison between values in $\mathbb{I}$. The basic requirement for this operator is that $< \subseteq \ll \subseteq \leq$ and, naturally, $m \gg n$ iff $n \ll m$. Throughout the paper, we will consider the following instances of $\ll$, in addition to $\ll = <$ and $\ll = \leq$:

- $\ll = <_\times$, where $m <_\times n$ iff $m < n$ or $m = n = \times$, where $\times$ is one of $\bot$, $\top$, or some other specified element of $\mathbb{I}$.

3 Grouping Literature Properties

In this section we show that 29 *literature properties* (Ps) introduced and analysed separately[2] for one gradual argumentation formalism or another, can be seen as instances of or are directly implied by 11 *group properties* (GPs) that we introduce for *generic QBAFs* in terms of parameters $\mathbb{I}$, *, $\ll$. Table 1 summarises the concrete choices of $\mathbb{I}$[3] and the appropriate special type of QBAF for the Ps that we consider (we will give the other choices of parameters in Table 2 later).

Our analysis in this section results in a principled catalogue of Ps, collated into groups, as well as, as a side-effect, the (implicit) definition of several other properties,

[2] Some properties were introduced referring to a generic notion of strength, others were proven for specific strength proposals.

[3] The ranges in Table 1 can be identified by inspection of the papers; rankings/orders can be naturally mapped onto $\mathbb{I}$.

variants of Ps, which, to the best of our knowledge, have not been considered yet. To give a simple illustrative example, consider the property of *maximality*, stating for attack-only frameworks that an argument with no attackers has strength equal to its base score (Amgoud et al. 2017), and the property of *minimality*, stating for support-only frameworks that an argument with no supporters has strength equal to its base score (Amgoud and Ben-Naim 2016b). Clearly, they share the same basic idea that if an argument is not the dialectical target of any arguments, then its strength coincides with its base score. Thus these two Ps can be grouped (intuitively by conjoining them) giving a further property for QBAFs. Further, the basic idea of this GP admits variants concerning whether arguments with bottom strength should count as effective attackers/supporters, i.e. whether the sets whose emptiness has to be checked are those of attackers/supporters *tout court* or with non-bottom strength. Each variant of the basic idea is a reasonable property on its own but not all variants have been given in the literature.

In this section we give formally our GPs in turn, each preceded by a short informal description of the underlying basic idea and followed by formal results on Ps[4] that are instances of or directly implied by the GP. For ease of reference and to disambiguate some homonymies, we number Ps. Tables 1 and 2 summarise the choices of parameters for capturing Ps as (implied by) instances of our GPs. Table 2 also indicates with hyphens instances of GPs without an explicit counterpart in the literature. Thus, each hyphen corresponds to a new property, instance of one of our GPs. As an illustration, the first row of Table 2 specifies that P1 (i.e. *maximality*, discussed earlier) is the instance of GP1 (defined later) for aQBAF(f)s with $* = \sigma\perp$, but no corresponding instance of GP1 for aQBAF(f)s with $* = \sigma\not\perp$, let us call it $P1_{\sigma\not\perp}$, has been considered. Note that in Table 2 hyphens clearly prevail over numbers. Also, where several Ps are present in a single cell of a GP (e.g. P8, P12 in GP6) in principle the same number of properties could be present in other cells of the same GP and one hyphen in these cells actually counts for more than one "missing" property. Thus, overall, before entering into details, we remark that our analysis identifies a significantly wider spectrum of possible properties than the literature so far. We will support this claim later by exemplifying additional properties that our analysis identifies.

Basic Idea 1. *The strength of an argument differs from its base score only if the argument is the dialectical target of other arguments, i.e. the strength is equal to the base score if the argument is not the dialectical target of other arguments.*

GP1. If $\mathcal{R}^-_*(\alpha) = \emptyset$ and $\mathcal{R}^+_*(\alpha) = \emptyset$ then $\sigma(\alpha) = \tau(\alpha)$.

Proposition 1. The following are instances of GP1[5]:
P1. *Maximality* (Amgoud and Ben-Naim 2016a; Amgoud et al. 2017), (Leite and Martins 2011, Prop. 11) - For any $\alpha \in \mathcal{X}$, if $\mathcal{R}^-(\alpha) = \emptyset$ then $\sigma(\alpha) = \tau(\alpha)$.
P2. *Minimality* (Amgoud and Ben-Naim 2016b) - For any $\alpha \in \mathcal{X}$, if $\mathcal{R}^+(\alpha) = \emptyset$ then $\sigma(\alpha) = \tau(\alpha)$.
P3. *Equation 4* (Baroni et al. 2015) - For any $\alpha \in \mathcal{X}$, if $\nexists \beta \in \mathcal{R}^-(\alpha)$ such that $\sigma(\beta) > \perp$ and $\nexists \gamma \in \mathcal{R}^+(\alpha)$ such that $\sigma(\gamma) > \perp$ then $\sigma(\alpha) = \tau(\alpha)$.

<table>
<tr><td></td><td></td><td colspan="3">$\sigma\perp$</td><td colspan="3">$\sigma\not\perp$</td></tr>
<tr><td></td><td></td><td>$<$</td><td>$<_\times$</td><td>$\leq$</td><td>$<$</td><td>$<_\times$</td><td>$\leq$</td></tr>
<tr><td rowspan="3">GP1</td><td>aQBAF(f)</td><td colspan="3">**P1**</td><td colspan="3">-</td></tr>
<tr><td>sQBAF</td><td colspan="3">**P2**</td><td colspan="3">-</td></tr>
<tr><td>QBAF(f)</td><td colspan="3">-</td><td colspan="3">**P3**</td></tr>
<tr><td rowspan="2">GP2</td><td>aQBAF(f)</td><td>-</td><td>-</td><td>-</td><td>-</td><td>P4</td><td>-</td></tr>
<tr><td>QBAF(f)</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td rowspan="2">GP3</td><td>sQBAF</td><td>-</td><td>-</td><td>-</td><td>-</td><td>P5</td><td>-</td></tr>
<tr><td>QBAF(f)</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td rowspan="2">GP4</td><td>aQBAF(f)</td><td colspan="3">-</td><td colspan="3">**P6**</td></tr>
<tr><td>QBAF(f)</td><td colspan="3">-</td><td colspan="3">-</td></tr>
<tr><td rowspan="2">GP5</td><td>sQBAF</td><td colspan="3">-</td><td colspan="3">**P7**</td></tr>
<tr><td>QBAF(f)</td><td colspan="3">-</td><td colspan="3">-</td></tr>
<tr><td rowspan="3">GP6</td><td>aQBAF(f)</td><td colspan="3">**P8**, P12</td><td colspan="3">**P9**</td></tr>
<tr><td>sQBAF</td><td colspan="3">**P10**</td><td colspan="3">**P11**</td></tr>
<tr><td>QBAF(f)</td><td colspan="3">-</td><td colspan="3">-</td></tr>
<tr><td rowspan="2">GP7</td><td>aQBAF(f)</td><td>P14</td><td>-</td><td>P15</td><td>-</td><td>P16, P17</td><td>-</td></tr>
<tr><td>QBAF(f)</td><td>-</td><td>-</td><td>**P13**</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td rowspan="2">GP8</td><td>sQBAF</td><td>-</td><td>-</td><td>-</td><td>-</td><td>P19</td><td>-</td></tr>
<tr><td>QBAF(f)</td><td>-</td><td>-</td><td>**P18**</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td rowspan="3">GP9</td><td>aQBAF(f)</td><td>-</td><td>P20</td><td>-</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td>sQBAF</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td>QBAF(f)</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td rowspan="2">GP10</td><td>aQBAF(f)</td><td>**P21**</td><td>P22, P23</td><td>-</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td>QBAF(f)</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td rowspan="2">GP11</td><td>sQBAF</td><td>-</td><td>P24, P25</td><td>-</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td>QBAF(f)</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td></tr>
</table>

Table 2: GPs and their instances (in bold) or implied instances (in normal font) in the literature. GP1, GP4-GP6 do not use $\ll$. GP2, GP4, GP7, GP10 have no sQBAF variants and GP3, GP5, GP8, GP11 have no aQBAF(f) variants.

The hyphens in Table 2 in the rows for GP1 correspond to the following novel properties, interesting in their own right:

- $P1_{\sigma\not\perp}$ - For any $\alpha \in \mathcal{X}$, if $\nexists \beta \in \mathcal{R}^-(\alpha)$ such that $\sigma(\beta) > \perp$ then $\sigma(\alpha) = \tau(\alpha)$.
- $P2_{\sigma\not\perp}$ - For any $\alpha \in \mathcal{X}$, if $\nexists \beta \in \mathcal{R}^+(\alpha)$ such that $\sigma(\beta) > \perp$ then $\sigma(\alpha) = \tau(\alpha)$.
- $P3_{\sigma\perp}$ - For any $\alpha \in \mathcal{X}$, if $\mathcal{R}^-(\alpha) = \emptyset$ and $\mathcal{R}^+(\alpha) = \emptyset$ then $\sigma(\alpha) = \tau(\alpha)$.

Here, $P1_{\sigma\not\perp}$ and $P2_{\sigma\not\perp}$ are the instance of GP1 for aQBAF(f)s and sQBAFs, resp., with $* = \sigma\not\perp$, and $P3_{\sigma\perp}$ is the instance of GP1 for QBAF(f)s with $* = \sigma\perp$.

Basic Idea 2. *In the absence of supporters, if there is at least an attacker then the strength of an argument is lower than its base score.*

GP2. If $\mathcal{R}^-_*(\alpha) \neq \emptyset$ and $\mathcal{R}^+_*(\alpha) = \emptyset$ then $\sigma(\alpha) \ll \tau(\alpha)$.

[4] Throughout the paper proofs are omitted for lack of space.

[5] Each P is expressed equivalently using our notation. Here and throughout, instantiation amounts to the choices of QBAF and $\mathbb{I}$ in Table 1 and of $*$ and $\ll$, if applicable, in Table 2, except for $<_\times$, specified in text. By showing that Ps are instances of GPs we show that the former are equivalent to (suitable instances of) the latter.

Proposition 2. GP2 implies the following for $<_\times = <_\perp$[6]:
P4. *Weakening* (Amgoud and Ben-Naim 2016a; Amgoud et al. 2017) - For any $\alpha \in \mathcal{X}$, if $\tau(\alpha) > \perp$ and $\exists\beta \in \mathcal{R}^-(\alpha)$ such that $\sigma(\beta) > \perp$ then $\sigma(\alpha) < \tau(\alpha)$.

Basic Idea 3. *In the absence of attackers, if there is at least a supporter then the strength of an argument is greater than its base score.*

GP3. If $\mathcal{R}^-_*(\alpha) = \emptyset$ and $\mathcal{R}^+_*(\alpha) \neq \emptyset$ then $\tau(\alpha) \ll \sigma(\alpha)$.

Proposition 3. GP3 implies the following for $<_\times = <_\top$:
P5. *Strengthening* (Amgoud and Ben-Naim 2016b) - For any $\alpha \in \mathcal{X}$, if $\tau(\alpha) < \top$ and $\exists\beta \in \mathcal{R}^+(\alpha)$ such that $\sigma(\beta) > \perp$ then $\sigma(\alpha) > \tau(\alpha)$.

Basic Idea 4. *If the strength of an argument is lower than its base score then the argument has at least one attacker.*

GP4. If $\sigma(\alpha) < \tau(\alpha)$ then $\mathcal{R}^-_*(\alpha) \neq \emptyset$.

Proposition 4. The following are instances of GP4:
P6. *Weakening Soundness* (Amgoud and Ben-Naim 2016a; Amgoud et al. 2017) - For any $\alpha \in \mathcal{X}$ with $\tau(\alpha) > \perp$, if $\sigma(\alpha) < \tau(\alpha)$ then $\exists\beta \in \mathcal{R}^-(\alpha)$ with $\sigma(\beta) > \perp$.

Basic Idea 5. *If the strength of an argument is higher than its base score then the argument has at least one supporter.*

GP5. If $\sigma(\alpha) > \tau(\alpha)$ then $\mathcal{R}^+_*(\alpha) \neq \emptyset$.

Proposition 5. The following are instances of GP5:
P7. *Strengthening Soundness* (Amgoud and Ben-Naim 2016b) - For any $\alpha \in \mathcal{X}$, if $\sigma(\alpha) > \tau(\alpha)$ then $\exists\beta \in \mathcal{R}^+(\alpha)$ such that $\sigma(\beta) > \perp$.

Basic Idea 6. *Arguments with equal conditions in terms of attackers, supporters and base score have the same strength.*

GP6. If $\mathcal{R}^-_*(\alpha) = \mathcal{R}^-_*(\beta)$, $\mathcal{R}^+_*(\alpha) = \mathcal{R}^+_*(\beta)$ and $\tau(\alpha) = \tau(\beta)$ then $\sigma(\alpha) = \sigma(\beta)$.

Proposition 6. The following are instances of GP6:
P8. *Equivalence* (Amgoud and Ben-Naim 2016a; Amgoud et al. 2017) - For any $\alpha, \beta \in \mathcal{X}$, if $\tau(\alpha) = \tau(\beta)$ and there exists a bijective function f from $\mathcal{R}^-(\alpha)$ to $\mathcal{R}^-(\beta)$ such that $\forall\gamma \in \mathcal{R}^-(\alpha)$, $\sigma(\gamma) = \sigma(f(\gamma))$ then $\sigma(\alpha) = \sigma(\beta)$.
P9. *Neutrality* (Amgoud and Ben-Naim 2016a; Amgoud et al. 2017) - For any $\alpha, \beta \in \mathcal{X}$, if $\tau(\alpha) = \tau(\beta)$, $\mathcal{R}^-(\alpha) = \mathcal{R}^-(\beta)\backslash\{\gamma\}$, $\gamma \in \mathcal{R}^-(\beta)$ and $\sigma(\gamma) = \perp$ then $\sigma(\beta) = \sigma(\alpha)$.
P10. *Equivalence* (Amgoud and Ben-Naim 2016b) - For any $\alpha, \beta \in \mathcal{X}$, if $\tau(\alpha) = \tau(\beta)$ and $\sigma(\mathcal{R}^+(\alpha)) = \sigma(\mathcal{R}^+(\beta))$ then $\sigma(\alpha) = \sigma(\beta)$.
P11. *Dummy* (Amgoud and Ben-Naim 2016b) - For any $\alpha, \beta \in \mathcal{X}$, if $\tau(\alpha) = \tau(\beta)$, $\mathcal{R}^+(\alpha) = \mathcal{R}^+(\beta)\backslash\{\gamma\}$ and $\gamma \in \mathcal{R}^+(\beta)$ with $\sigma(\gamma) = \perp$ then $\sigma(\beta) = \sigma(\alpha)$.

Proposition 7. GP6 implies the following:
P12. *Non-Attacked Equivalence* (Bonzon et al. 2016) - For any $\alpha, \beta \in \mathcal{X}$, if $\mathcal{R}^-(\alpha) = \emptyset$ and $\mathcal{R}^-(\beta) = \emptyset$ then $\sigma(\alpha) = \sigma(\beta)$.

As mentioned earlier, Table 2 gives yet more properties than those indicated for the hyphens. For example, the $\sigma\not\perp$ version of P12 is noticeably different from P9 (with which it would share a cell) and interesting in its own right:

- $P12_{\sigma\not\perp}$ - For any $\alpha, \beta \in \mathcal{X}$, if $\mathcal{R}^-_{\sigma\not\perp}(\alpha) = \emptyset$ and $\mathcal{R}^-_{\sigma\not\perp}(\beta) = \emptyset$ then $\sigma(\alpha) = \sigma(\beta)$.

Basic Idea 7. *A strictly larger set of attackers determines a lower strength.*

GP7. If $\mathcal{R}^-_*(\alpha) \subsetneq \mathcal{R}^-_*(\beta)$ and $\mathcal{R}^+_*(\alpha) = \mathcal{R}^+_*(\beta)$ and $\tau(\alpha) = \tau(\beta)$ then $\sigma(\beta) \ll \sigma(\alpha)$.

Proposition 8. The following are instances of GP7:
P13. *Equation 2* (Baroni et al. 2015), (Rago et al. 2016, Prop. 12) - For any $\alpha, \beta \in \mathcal{X}$, such that $\tau(\alpha) = \tau(\beta)$ and $\mathcal{R}^+(\alpha) = \mathcal{R}^+(\beta)$, if $\mathcal{R}^-(\alpha) \subsetneq \mathcal{R}^-(\beta)$ then $\sigma(\alpha) \geq \sigma(\beta)$.

Proposition 9. GP7 implies the following:
P14. *Void Precedence* (Amgoud and Ben-Naim 2013; Bonzon et al. 2016), (Amgoud et al. 2016, Th. 5) - For any $\alpha, \beta \in \mathcal{X}$, if $\mathcal{R}^-(\alpha) = \emptyset$ and $\mathcal{R}^-(\beta) \neq \emptyset$ then $\sigma(\alpha) > \sigma(\beta)$.
P15. *Weak Void Precedence* (Thimm and Kern-Isberner 2014) - For any $\alpha, \beta \in \mathcal{X}$, if $\mathcal{R}^-(\alpha) = \emptyset$ then $\sigma(\alpha) \geq \sigma(\beta)$.
P16. *Triggering* (Amgoud and Ben-Naim 2016a) - for $<_\times = <_\perp$ - For any $\alpha, \beta \in \mathcal{X}$, if $\sigma(\alpha) > \perp$, $\forall\gamma \in \mathcal{R}^-(\alpha)$, $\sigma(\gamma) = \perp$, and $\mathcal{R}^-(\alpha) = \mathcal{R}^-(\beta)\backslash\{\delta\}$ for some $\delta \in \mathcal{R}^-(\beta)$ with $\sigma(\delta) > \perp$ then $\sigma(\beta) < \sigma(\alpha)$.
P17. *Counting* (Amgoud et al. 2017)[7] - for $<_\times = <_\perp$ - For any $\alpha, \beta \in \mathcal{X}$, if $\tau(\alpha) = \tau(\beta)$, $\sigma(\alpha) > \perp$ and $\mathcal{R}^-(\alpha) = \mathcal{R}^-(\beta)\backslash\{\gamma\}$ for some $\gamma \in \mathcal{R}^-(\beta)$ with $\sigma(\gamma) > \perp$ then $\sigma(\beta) < \sigma(\alpha)$.

Basic Idea 8. *A strictly larger set of supporters determines a higher strength.*

GP8. If $\mathcal{R}^-_*(\alpha) = \mathcal{R}^-_*(\beta)$ and $\mathcal{R}^+_*(\alpha) \subsetneq \mathcal{R}^+_*(\beta)$ and $\tau(\alpha) = \tau(\beta)$ then $\sigma(\alpha) \ll \sigma(\beta)$.

Proposition 10. The following are instances of GP8:
P18. *Equation 3* (Baroni et al. 2015), (Rago et al. 2016, Prop. 13) - For any $\alpha, \beta \in \mathcal{X}$ such that $\tau(\alpha) = \tau(\beta)$ and $\mathcal{R}^-(\alpha) = \mathcal{R}^-(\beta)$, if $\mathcal{R}^+(\alpha) \subset \mathcal{R}^+(\beta)$ then $\sigma(\alpha) \leq \sigma(\beta)$.

Proposition 11. GP8 implies the following:
P19. *Counting* (Amgoud and Ben-Naim 2016b) - for $<_\times = <_\top$ - For any $\alpha, \beta \in \mathcal{X}$, if $\tau(\alpha) = \tau(\beta)$, $\mathcal{R}^+(\alpha) = \mathcal{R}^+(\beta)\backslash\{\gamma\}$ for some $\gamma \in \mathcal{R}^+(\beta)$ with $\sigma(\gamma) > \perp$ and $\sigma(\alpha) < \top$, then $\sigma(\beta) > \sigma(\alpha)$.

Basic Idea 9. *A higher base score gives a higher strength.*

GP9. If $\mathcal{R}^-_*(\alpha) = \mathcal{R}^-_*(\beta)$, $\mathcal{R}^+_*(\alpha) = \mathcal{R}^+_*(\beta)$ and $\tau(\alpha) < \tau(\beta)$ then $\sigma(\alpha) \ll \sigma(\beta)$.

Proposition 12. GP9 implies the following:
P20. *Proportionality* (Amgoud et al. 2017) - for $<_\times = <_\perp$ - For any $\alpha, \beta \in \mathcal{X}$, if $\mathcal{R}^-(\alpha) = \mathcal{R}^-(\beta)$, $\tau(\beta) > \tau(\alpha)$ and either $\sigma(\alpha) > \perp$ or $\sigma(\beta) > \perp$, then $\sigma(\beta) > \sigma(\alpha)$.

Basic Idea 10. *A weaker set of attackers determines a higher strength.*

GP10. If $\mathcal{R}^-(\alpha) <_* \mathcal{R}^-(\beta)$, $\mathcal{R}^+_*(\alpha) = \mathcal{R}^+_*(\beta)$ and $\tau(\alpha) = \tau(\beta)$ then $\sigma(\beta) \ll \sigma(\alpha)$.

Proposition 13. The following are instances of GP10:
P21. *Strict Counter-Transitivity* (Amgoud and Ben-Naim 2013; Bonzon et al. 2016), (Amgoud et al. 2016, Th. 8) - For any $\alpha, \beta \in \mathcal{X}$, if $\mathcal{R}^-(\alpha) <_{\sigma\perp} \mathcal{R}^-(\beta)$, then $\sigma(\alpha) > \sigma(\beta)$.

[6]Throughout, implication results assume instantiation of parameters as in Tables 1,2 and, if applicable, choice of $<_\times$ as given.

[7]We ignore here the more restrictive form of Counting given earlier in (Amgoud and Ben-Naim 2016a).

Proposition 14. GP10 implies the following for $<_\times = <_\perp$:
P22. *Boundedness* (Amgoud and Ben-Naim 2016a) - For any $\alpha, \beta \in \mathcal{X}$ with $\gamma \in \mathcal{R}^-(\alpha)$ and $\delta \in \mathcal{R}^-(\beta)$ where $\sigma(\delta) > \sigma(\gamma)$, if $\mathcal{R}^-(\beta) \setminus \{\delta\} = \mathcal{R}^-(\alpha) \setminus \{\gamma\}$ and $\sigma(\alpha) = \perp$ then $\sigma(\beta) = \perp$.
P23. *Reinforcement* (Amgoud et al. 2017)[8] - For any $\alpha, \beta \in \mathcal{X}$, with $\tau(\alpha) = \tau(\beta)$, either $\sigma(\alpha) > \perp$ or $\sigma(\beta) > \perp$, $\gamma \in \mathcal{R}^-(\alpha)$ and $\delta \in \mathcal{R}^-(\beta)$ where $\sigma(\delta) > \sigma(\gamma)$, if $\mathcal{R}^-(\beta) \setminus \{\delta\} = \mathcal{R}^-(\alpha) \setminus \{\gamma\}$ then $\sigma(\beta) < \sigma(\alpha)$.

Basic Idea 11. *A stronger set of supporters determines a higher strength.*

GP11. If $\mathcal{R}^-_*(\alpha) = \mathcal{R}^-_*(\beta)$, $\mathcal{R}^+(\alpha) >_* \mathcal{R}^+(\beta)$ and $\tau(\alpha) = \tau(\beta)$ then $\sigma(\beta) \ll \sigma(\alpha)$.

Proposition 15. GP11 implies the following, for $<_\times = <_\top$:
P24. *Boundedness* (Amgoud and Ben-Naim 2016b) - For any $\alpha, \beta \in \mathcal{X}$ with $\gamma \in \mathcal{R}^+(\alpha)$, $\delta \in \mathcal{R}^+(\beta)$ and $\sigma(\delta) > \sigma(\gamma)$, if $\tau(\alpha) = \tau(\beta)$, $\mathcal{R}^+(\beta) \setminus \{\delta\} = \mathcal{R}^+(\alpha) \setminus \{\gamma\}$ and $\sigma(\alpha) = \top$ then $\sigma(\beta) = \top$.
P25. *Reinforcement* (Amgoud and Ben-Naim 2016b) - For any $\alpha, \beta \in \mathcal{X}$, with $\gamma \in \mathcal{R}^+(\alpha)$ and $\delta \in \mathcal{R}^+(\beta)$ where $\sigma(\delta) > \sigma(\gamma) > \perp$, if $\tau(\alpha) = \tau(\beta)$, $\mathcal{R}^+(\beta) \setminus \{\delta\} = \mathcal{R}^+(\alpha) \setminus \{\gamma\}$ and $\sigma(\alpha) < \top$ then $\sigma(\beta) > \sigma(\alpha)$.

Other Ps are implied by combinations of GPs, as illustrated by the following theorem:

Theorem 1.

- GP6 and GP7 with $* = \sigma\perp$ and $\ll = \leq$ imply:
  P26. *Monotony* (Amgoud and Ben-Naim 2016a), (Matt and Toni 2008, Prop. 6) - For any $\alpha, \beta \in \mathcal{X}$, if $\tau(\alpha) = \tau(\beta)$ and $\mathcal{R}^-(\alpha) \subseteq \mathcal{R}^-(\beta)$ then $\sigma(\alpha) \geq \sigma(\beta)$.
- GP6 and GP8 with $* = \sigma\perp$ and $\ll = \leq$ imply:
  P27. *Monotony* (Amgoud and Ben-Naim 2016b) - For any $\alpha, \beta \in \mathcal{X}$, if $\tau(\alpha) = \tau(\beta)$ and $\mathcal{R}^+(\alpha) \subseteq \mathcal{R}^+(\beta)$ then $\sigma(\alpha) \leq \sigma(\beta)$.
- GP6 and GP11 with $* = \sigma\perp$ and $\ll = \leq$ imply:
  P28. *Theorem 7* in (Amgoud et al. 2016) - For any $\alpha, \beta \in \mathcal{X}$, if $\tau(\alpha) = \tau(\beta)$ and there exists an injective mapping f from $\mathcal{R}^-(\alpha)$ to $\mathcal{R}^-(\beta)$ such that $\forall \gamma \in \mathcal{R}^-(\alpha)$, $\sigma(f(\gamma)) \geq \sigma(\gamma)$ then $\sigma(\beta) \leq \sigma(\alpha)$.
  P29. *Counter-Transitivity* (Amgoud and Ben-Naim 2013; Bonzon et al. 2016) - For any $\alpha, \beta \in \mathcal{X}$, if $\tau(\alpha) = \tau(\beta)$ and $\mathcal{R}^-(\alpha) \leq_{\sigma\perp} \mathcal{R}^-(\beta)$, then $\sigma(\alpha) \geq \sigma(\beta)$.

4 (Strict) Balance/Monotonicity Principles

GPs capitalise on similarities across argumentation frameworks and instantiation of parameters to synthesise various Ps, while pointing to several new ones, but are still numerous. In this section we define more synthetic *principles* of (strict) *balance* (Section 4.1) and *monotonicity* (Section 4.2), in terms of the same parameters as for GPs, and show that they imply all the GPs (by showing that if the implied GPs' premises hold, then so do the implying principle's, and that the principle's conclusions imply the conclusions of the GPs). As a by-product, any Ps equivalent to or implied by the GPs, as shown in Section 3, are also implied by these principles.

[8] We ignore here the more restrictive form of Reinforcement given earlier in (Amgoud and Ben-Naim 2016a).

4.1 (Strict) Balance

The intuition behind our first principle is that a difference between strength and base score of an argument must correspond to some imbalance between the strengths of its attackers and supporters, e.g. from a dialectical viewpoint, because the reasons against an argument are stronger than the reasons for it. Thus, an argument with attackers $*$-strength equivalent to its supporters has a strength equal to its base score, while an argument with attackers $*$-stronger than ($*$-weaker than) its supporters has a strength not greater (not less, resp.) than its base score and cannot attain the top (bottom, resp.) value. (Strict) Balance, admitting multiple instances for different choices of $*$ and $\ll$, expresses this:

Principle 1. A strength function σ is:
• *balanced* iff for any $\alpha \in \mathcal{X}$:
1. If $\mathcal{R}^-(\alpha) =_* \mathcal{R}^+(\alpha)$ then $\sigma(\alpha) = \tau(\alpha)$.
2. If $\mathcal{R}^-(\alpha) >_* \mathcal{R}^+(\alpha)$ then $\sigma(\alpha) \ll \tau(\alpha)$.
3. If $\mathcal{R}^-(\alpha) <_* \mathcal{R}^+(\alpha)$ then $\sigma(\alpha) \gg \tau(\alpha)$.
• *strictly balanced* iff σ is balanced and for any $\alpha \in \mathcal{X}$:
4. If $\sigma(\alpha) < \tau(\alpha)$ then $\mathcal{R}^-(\alpha) >_* \mathcal{R}^+(\alpha)$.
5. If $\sigma(\alpha) > \tau(\alpha)$ then $\mathcal{R}^-(\alpha) <_* \mathcal{R}^+(\alpha)$.

For illustration, consider the QBAF given in Figure 1 with $\tau(a) = \tau(b)$, $\sigma(c) = \perp$ and $\sigma(d) > \perp$. Let σ be balanced and $\ll = <$. If $* = \sigma\perp$, by Definition 3, $\mathcal{R}^-(a) >_* \mathcal{R}^+(a)$ and so, by Point 2 of Principle 1, $\sigma(a) \ll \tau(a)$, meaning $\sigma(a) < \tau(a)$. However, if $* = \sigma\not\perp$, by Definition 3, $\mathcal{R}^-(a) =_* \mathcal{R}^+(a)$ and so, by Point 1 of Principle 1, $\sigma(a) = \tau(a)$. Let instead $\ll = <_\top$. Then, for both choices of $*$, $\mathcal{R}^-(b) <_* \mathcal{R}^+(b)$ and so, by Point 3 of Principle 1, $\sigma(b) \gg \tau(b)$, meaning $\sigma(b) > \tau(b)$ or b's strength is saturated, i.e. $\sigma(b) = \tau(b) = \top$.

Proposition 16. If σ is balanced for some $*, \ll$ then it satisfies GP1 to GP3 for the same $*, \ll$; if σ is strictly balanced for some $*, \ll$ then it satisfies GP1 to GP5 for the same $*, \ll$.

Note that the reverse of Proposition 16 does not hold. For example, if a is an argument in a QBAF such that $\mathcal{R}^-_*(a) = \mathcal{R}^+_*(a) \neq \emptyset$ but $\sigma(a) \neq \tau(a)$ then GP1–GP5 may hold while σ is not (strictly) balanced.

4.2 (Strict) Monotonicity

Our second principle requires that the strength of an argument depends monotonically on its base score and on the strengths of its attackers and supporters. From a dialectical viewpoint, the strength of an argument depends exclusively on its intrinsic strength, the reasons for it and the reasons against it, and any strengthening or weakening of these will strength or weaken the argument. To define this principle formally, we first define the notion of *shaping triple* of an argument, as follows.

Definition 5. For any $\alpha \in \mathcal{X}$, the *shaping triple* of α is $(\tau(\alpha), \mathcal{R}^+(\alpha), \mathcal{R}^-(\alpha))$, denoted $\mathcal{ST}(\alpha)$.

We define partial orders over shaping triples, parametric w.r.t. $*$, based on the ordering of their elements. Intuitively, the orders rank the ability of the triples to boost arguments.

Definition 6. Given $\alpha, \beta \in \mathcal{X}$, $\mathcal{ST}(\beta)$ is said to be:

- *as $*$-boosting as* $\mathcal{ST}(\alpha)$, denoted $\mathcal{ST}(\alpha) \simeq_* \mathcal{ST}(\beta)$, iff $\tau(\alpha) = \tau(\beta)$, $\mathcal{R}^+(\alpha) =_* \mathcal{R}^+(\beta)$, and $\mathcal{R}^-(\beta) =_* \mathcal{R}^-(\alpha)$;
- *at least as $*$-boosting as* $\mathcal{ST}(\alpha)$, denoted $\mathcal{ST}(\alpha) \leq_* \mathcal{ST}(\beta)$, iff $\tau(\alpha) \leq \tau(\beta)$, $\mathcal{R}^+(\alpha) \leq_* \mathcal{R}^+(\beta)$, and $\mathcal{R}^-(\beta) \leq_* \mathcal{R}^-(\alpha)$.
- *strictly more $*$-boosting than* $\mathcal{ST}(\alpha)$, denoted $\mathcal{ST}(\alpha) <_* \mathcal{ST}(\beta)$, iff $\mathcal{ST}(\alpha) \leq_* \mathcal{ST}(\beta)$ and $\mathcal{ST}(\beta) \nleq_* \mathcal{ST}(\alpha)$.

In the remainder of Section 4, whenever we refer to $\alpha, \beta \in \mathcal{X}$, we assume that, unless stated otherwise, $\tau(\alpha) = \tau(\beta)$, $\mathcal{R}^+(\alpha) =_* \mathcal{R}^+(\beta)$ and $\mathcal{R}^-(\beta) =_* \mathcal{R}^-(\alpha)$.

(Strict) Monotonicity is defined by comparing shaping triples using the orders in Definition 6. Both are parametric w.r.t. $*$, and strict monotonicity is also parametric w.r.t. $\ll$.

Principle 2. A strength function σ is:

- *monotonic* iff for any $\alpha, \beta \in \mathcal{X}$, if $\mathcal{ST}(\alpha) \simeq_* \mathcal{ST}(\beta)$ then $\sigma(\alpha) = \sigma(\beta)$ and if $\mathcal{ST}(\alpha) \leq_* \mathcal{ST}(\beta)$ then $\sigma(\alpha) \leq \sigma(\beta)$;
- *strictly monotonic* iff σ is monotonic and for any $\alpha, \beta \in \mathcal{X}$, if $\mathcal{ST}(\alpha) <_* \mathcal{ST}(\beta)$ then $\sigma(\alpha) \ll \sigma(\beta)$.

For illustration, consider the QBAF in Figure 1 with $\tau(a) = \tau(b)$ and $\sigma(d) = \perp$. For a strictly monotonic σ:

- if $* = \sigma\perp$, by Definitions 3 and 6, $\mathcal{ST}(a) <_* \mathcal{ST}(b)$. Then, by Principle 2, $\sigma(a) \ll \sigma(b)$. For $\ll = <$, $\sigma(a) \ll \sigma(b)$ amounts to $\sigma(a) < \sigma(b)$, while for $\ll = <_\perp$, it amounts to $\sigma(a) < \sigma(b)$ unless $\sigma(a) = \sigma(b) = \perp$;
- if $* = \sigma\not\perp$, by Definitions 3 and 6, $\mathcal{ST}(a) \simeq_* \mathcal{ST}(b)$ and then, by Principle 2, $\sigma(a) = \sigma(b)$.

Note that for $\ll = <_\perp$ ($\ll = <_\top$), if a shaping triple already gives rise to a $\perp$ ($\top$, resp.) strength, a less (more, resp.) boosting triple cannot produce a strictly lesser (greater, resp.) strength. For $\ll = <$, strict monotonicity and the attainment of extreme values are somehow incompatible.

Proposition 17. If σ is monotonic for some $*$,$\ll$ then it satisfies GP6 for the same $*$,$\ll$; if σ is strictly monotonic for some $*$,$\ll$ then it satisfies GP6 to GP11 for the same $*$,$\ll$.

Note that the reverse of Proposition 17 does not hold. For example, if a, b are arguments in a QBAF such that $\mathcal{R}^-_*(b) < \mathcal{R}^-_*(a)$, $\mathcal{R}^+_*(b) > \mathcal{R}^-_*(a)$ and $\tau(b) > \tau(a)$ but $\sigma(a) > \sigma(b)$ then GP6–GP11 may hold while σ is not (strictly) monotonic.

5 Principles vs. Existing Approaches

In this section we show that suitable instances (i.e. types of QBAF, $\mathbb{I}$, * and $\ll$) of our principles are satisfied by several gradual evaluation methods considered in the literature for various kinds of argumentation framework. Thus, by virtue of Propositions 16, 17, for each approach we synthetically prove that it satisfies the implied GPs and any literature and other property they are equivalent to or that the GPs imply. This shows that the level of generalisation afforded by the principles is well-chosen, by showing that the principles, for specific choices of parameters, are satisfied by existing concrete gradual semantics for various instances of QBAFs.

Note that if a principle is satisfied for $\ll = <$ then it is also satisfied for every possible $\ll$ since $<$ is the strongest possible instance of $\ll$.

5.1 Besnard & Hunter Categorizer

(Besnard and Hunter 2001) introduce the notion of *h-categoriser*, which basically is a function providing a quantitative evaluation of argument strength for tree-shaped abstract argumentation frameworks (aQBAFfs). In our notation, the h-categoriser is defined as follows, for $\alpha \in \mathcal{X}$:

$$\sigma(\alpha) = \frac{1}{1 + \sum_{\beta \in \mathcal{R}^-(\alpha)} \sigma(\beta)}. \tag{1}$$

It can be noted that $0 < \sigma(\alpha) \leq 1$, thus in this approach $\mathbb{I} = (0, 1]$ with $\top = 1$ and no bottom value $\perp$ in $\mathbb{I}$. Consequently, in this approach there is no difference between $\sigma\perp$ and $\sigma\not\perp$.

Proposition 18. The h-categoriser is strictly balanced and strictly monotonic for any choice of $*$ and $\ll$ (i.e. for $\ll = <$).

This implies that the h-categoriser satisfies, in addition to GP1–GP11 (by Propositions 16 and 17), also the implied Ps (e.g. P1) and the "missing" properties (e.g. $P1_{\sigma\not\perp}$).[9]

5.2 Leite & Martins Social Model

(Leite and Martins 2011) introduce a generic gradual evaluation method for social abstract argumentation frameworks (aQBAFs). The method is defined in terms of *well-behaved social abstract argumentation semantic framework*, which is a 5-tuple $\mathcal{S} = \langle L, \tau, \wedge, \vee, \neg \rangle$ where: L is a totally ordered set including $\top$ and $\perp$; τ is a vote aggregation function which assigns a base score in L to each argument α based on the positive and negative votes received by α in the considered social context; the operator $\wedge : L \times L \to L$ is continuous, commutative, associative, monotonic w.r.t. both arguments, and $\top$ is its identity element; the operator $\vee : L \times L \to L$ is continuous, commutative, associative, monotonic w.r.t. both arguments, and $\perp$ is its identity element; the operator $\neg : L \to L$ is antimonotonic, continuous, with $\neg\top = \perp$, $\neg\perp = \top$, and $\neg\neg x = x$. Then $\sigma : \mathcal{X} \to L$ is a $\mathcal{S} - model$ iff for $\alpha \in \mathcal{X}$:

$$\sigma(\alpha) = \tau(\alpha) \wedge \neg \vee \{\sigma(\beta) \mid \beta \in \mathcal{R}^-(\alpha)\} \tag{2}$$

It can be noted that $\perp$ is the identity element of $\vee$ and thus this approach fits the option $* = \sigma\not\perp$. In (Leite and Martins 2011) there is no remark about whether the monotonicity properties of $\wedge$, $\vee$, and $\neg$ are to be interpreted as strict. By interpreting them as strict then a well-behaved $\mathcal{S} - model$ is balanced[10]. Monotonicity is part of the definition of $\mathcal{S} - model$, and is strict if the monotonicity properties of $\wedge$, $\vee$, and $\neg$ are strict. Formally:

Proposition 19. A well behaved $\mathcal{S} - model$ is monotonic with $* = \sigma\not\perp$. If the monotonicity properties of $\wedge$, $\vee$, and $\neg$ are strict, then a well behaved $\mathcal{S} - model$ is strictly balanced and strictly monotonic with $* = \sigma\not\perp$ and for any choice of $\ll$ (i.e. for $\ll = <$).

[9]We will not repeat this same remark for other formalisms.

[10]This interpretation is natural as non-strict monotonicity might lead to flat evaluation methods annihilating all differences. The instance in (Leite and Martins 2011) satisfies strict monotonicity.

5.3 Amgoud & Ben-Naim Support Semantics

(Amgoud and Ben-Naim 2016b) introduce three gradual semantics for support argumentation frameworks (sQBAFs).

According to the **top-based semantics** the strength of arguments ranges in $\mathbb{I}=[0,1]$ and is related to their base score and to the strength of their strongest supporter (see Theorem 3 of (Amgoud and Ben-Naim 2016b)) according to the following equation, for $\alpha \in \mathcal{X}$:

$$\sigma(\alpha) = \tau(\alpha) + (1 - \tau(\alpha)) \max_{\beta \in \mathcal{R}^+(\alpha)} \sigma(\beta) \tag{3}$$

where $\max_{\beta \in \mathcal{R}^+(\alpha)} = 0$ if $\mathcal{R}^+(\alpha) = \emptyset$. Accordingly, supporters with $\perp$ strength have no effect, which means that this approach fits the option $* = \sigma\not\perp$.

Proposition 20. The top-based semantics is strictly balanced and monotonic with $* = \sigma\not\perp$ and $\ll = <_\top$.

The top-based semantics is not strictly monotonic since clearly there can be cases where $\mathcal{ST}(\alpha) <_{\sigma\not\perp} \mathcal{ST}(\beta)$ but $\tau(\alpha) = \tau(\beta)$ and $\max_{\gamma \in \mathcal{R}^+(\alpha)} \sigma(\gamma) = \max_{\delta \in \mathcal{R}^+(\beta)} \sigma(\delta)$.

The **reward-based semantics** is based on the notion of *founded* argument: $\alpha \in \mathcal{X}$ is founded iff there is a finite sequence $\alpha_0 \dots \alpha_n$, for $n \geq 0$, such that $\tau(\alpha_0) > 0$, $\alpha_n = \alpha$, and for every $i = 0, \dots n-1$: $\alpha_i \in \mathcal{R}^+(\alpha_{i-1})$. Then, the strength of arguments ranges in $\mathbb{I}=[0,1]$ and is related to their base score and to the strength of their supporters (see Theorem 6 of (Amgoud and Ben-Naim 2016b)) according to the following equation, for $\alpha \in \mathcal{X}$:

$$\sigma(\alpha) = \tau(\alpha) + (1 - \tau(\alpha)) \left(\sum_{j=1}^{n(\alpha)-1} \frac{1}{2^j} + \frac{m(\alpha)}{2^{n(\alpha)}} \right) \tag{4}$$

where $n(\alpha) = |\mathcal{R}^+_F(\alpha)|$ and $m(\alpha) = \frac{\sum_{\beta \in \mathcal{R}^+_F(\alpha)} \sigma(\beta)}{n(\alpha)}$ and, by convention, $\sum_{j=1}^{n(\alpha)-1} \frac{1}{2^j} + \frac{m(\alpha)}{2^{n(\alpha)}} = 0$ if $\mathcal{R}^+_F(\alpha) = \emptyset$.

Proposition 4 of (Amgoud and Ben-Naim 2016b) proves that if an argument α is founded then $\sigma(\alpha) > 0$. It is also immediate to see that if $\sigma(\alpha) > 0$ then α is founded (from $\sigma(\alpha) > 0$ and Equation (4) we get that either $\tau(\alpha) > 0$ or $\mathcal{R}^+_F(\alpha) \neq \emptyset$). It follows that supporters with zero strength have no effect, and thus this approach fits the option $* = \sigma\not\perp$.

Proposition 21. The reward-based semantics is strictly balanced and strictly monotonic with $* = \sigma\not\perp$ and $\ll = <_\top$.

Finally, according to the **aggregation-based semantics**, the strength of arguments ranges in $\mathbb{I}=[0,1]$ and is related to their base score and to the strength of their supporters (see Theorem 9 of (Amgoud and Ben-Naim 2016b)) according to the following equation, for $\alpha \in \mathcal{X}$:

$$\sigma(\alpha) = \tau(\alpha) + (1 - \tau(\alpha)) \frac{\sum_{\beta \in \mathcal{R}^+(\alpha)} \sigma(\beta)}{1 + \sum_{\beta \in \mathcal{R}^+(\alpha)} \sigma(\beta)} \tag{5}$$

It follows that supporters with zero strength have no effect (they do not affect the sum $\sum_{\beta \in \mathcal{R}^+(\alpha)} \sigma(\beta)$), and thus this approach fits the option $* = \sigma\not\perp$.

Proposition 22. The aggregation-based semantics is strictly balanced and strictly monotonic with $* = \sigma\not\perp$ and $\ll = <_\top$.

5.4 Local Valuation in Bipolar Frameworks

(Amgoud et al. 2008) introduces a generic local gradual evaluation scheme for arguments in bipolar argumentation frameworks (QBAFfs): we analyse here two instances thereof. In both cases arguments' strengths range over the interval $\mathbb{I}=[-1,1]$. No explicit notion of base score is given in (Amgoud et al. 2008): for our purposes here, we assume that $\tau(\alpha) = 0$ for each $\alpha \in \mathcal{X}$. The first instance, which we call **LocMax**, is defined as follows, for $\alpha \in \mathcal{X}$:

$$\sigma(\alpha) = \frac{\max_{\beta \in \mathcal{R}^+(\alpha)} \sigma(\beta) - \max_{\gamma \in \mathcal{R}^-(\alpha)} \sigma(\gamma)}{2} \tag{6}$$

Under the convention that when applied to the empty set the $\max$ operator returns $\perp = -1$, we get that supporters and attackers with $\perp$ strength have no effect, and thus this approach fits the option $* = \sigma\not\perp$. To satisfy balance it also requires the choice $\ll = <_0$, where $m <_0 n$ iff $m < n$ or $m = n = 0$.

Proposition 23. LocMax is balanced and monotonic with $* = \sigma\not\perp$ and $\ll = <_0$.

LocMax is not strictly balanced since, for example, $\sigma(\alpha) < \tau(\alpha)$ can occur when the maximum strength of the attackers is greater than that of the supporters but the attackers and supporters are incomparable for any choice of $*$.

Instance two, called **LocSum**, is given, for $\alpha \in \mathcal{X}$, by:

$$\sigma(\alpha) = \frac{1}{1 + h(\mathcal{R}^-(\alpha))} - \frac{1}{1 + h(\mathcal{R}^+(\alpha))} \tag{7}$$

where, for $S \subseteq \mathcal{X}$, $h(S) \triangleq \sum_{\beta \in S} \frac{\sigma(\beta)+1}{2}$. Under the convention that when applied to the empty set the sum returns 0, we get that supporters and attackers with a strength of $-1 = \perp$ have no effect, and thus this approach fits the option $* = \sigma\not\perp$.

Proposition 24. LocSum is balanced and strictly monotonic with $* = \sigma\not\perp$ and for any choice of $\ll$ (i.e. with $\ll = <$).

LocSum is not strictly balanced since, for example, $\sigma(\alpha) < 0$ can occur when $h(\mathcal{R}^-(\alpha)) > h(\mathcal{R}^+(\alpha))$ but the attackers and supporters are incomparable for any choice of $*$.

5.5 Semantics for Weighted Argumentation

(Amgoud et al. 2017) gives three semantics for Weighted Argumentation Frameworks (aQBAFs), analysed below.

The **weighted max-based semantics** is such that (Theorems 4 and 5 of (Amgoud et al. 2017)) for $\alpha \in \mathcal{X}$:

$$\sigma(\alpha) = \frac{\tau(\alpha)}{1 + max_{\beta \in \mathcal{R}^-(\alpha)} \sigma(\beta)}. \tag{8}$$

Note that attackers with 0 strength have no actual effect in Eq. 8, and thus this approach fits the option $* = \sigma\not\perp$.

Proposition 25. The weighted max-based semantics is strictly balanced and monotonic with $* = \sigma\not\perp$ and for any choice of $\ll$ (i.e. with $\ll = <$).

The weighted max-based semantics is not strictly monotonic as clearly there can be cases where $\mathcal{ST}(\alpha) <_{\sigma\not\perp} \mathcal{ST}(\beta)$ but $\tau(\alpha) = \tau(\beta)$ and $\max_{\gamma \in \mathcal{R}^+(\alpha)} \sigma(\gamma) = \max_{\delta \in \mathcal{R}^+(\beta)} \sigma(\delta)$.

The **weighted card-based semantics** is such that (Theorems 6 and 7 of (Amgoud et al. 2017)) for $\alpha \in \mathcal{X}$:

$$\sigma(\alpha) = \frac{\tau(\alpha)}{1 + |\mathcal{R}^-(\alpha)| + \frac{\sum_{\beta \in \mathcal{R}^-(\alpha)} \sigma(\beta)}{|\mathcal{R}^-(\alpha)|}}, \tag{9}$$

$$\text{where } \frac{\sum_{\beta \in \mathcal{R}^-(\alpha)} \sigma(\beta)}{|\mathcal{R}^-(\alpha)|} = 0 \text{ if } |\mathcal{R}^-(\alpha)| = 0.$$

Note that attackers with 0 strength play a role in (9) (e.g. through the term $|\mathcal{R}^-(\alpha)|$), and thus for this approach $* = \sigma\bot$.

Proposition 26. The weighted card-based sem. is strictly balanced and strictly monotonic for $* = \sigma\bot$ and any $\ll$.

The **weighted h-categorizer semantics** generalises the h-categorizer given in Section 5.1 so that, for $\alpha \in \mathcal{X}$:

$$\sigma(\alpha) = \frac{\tau(\alpha)}{1 + \sum_{\beta \in \mathcal{R}^-(\alpha)} \sigma(\beta)}. \tag{10}$$

Note that attackers with 0 strength have no actual effect in Eq. 10, and thus this approach fits the option $* = \sigma\not\perp$.

Proposition 27. The weighted h-categoriser is strictly balanced and strictly monotonic with $* = \sigma\not\perp$ and for any $\ll$.

6 Conclusion

This paper provides a twofold answer to the question stated in the title. On the one hand, the analysis in Section 3 shows that considering systematically all possible instances of some common, intuitive patterns leads to the identification of a large number of distinct properties, only partially covered in the literature to date. We have explicitly formalised four such novel properties and left others implicit. On the other hand, the results in Section 4 show that considering such a wide spectrum of properties explicitly may not be necessary since the principles of balance and monotonicity are sufficient to imply the satisfaction of many other properties. The use of these principles can hence greatly simplify the design and analysis of actual argumentation formalisms. The results in Section 5 confirm their practical value, by showing that a variety of gradual methods, conceived in different contexts, all satisfy suitable instances of both balance and monotonicity. It is our principles' simplicity and power, confirmed by these results, which make this work an advancement on previous literature. Other groupings and principles may also be possible: identifying them would be a natural continuation of our work. Given the foundational nature of this work, a variety of other future research directions can be considered. Among the many ones, we mention a study about which combinations of the parameters $*$ and $\ll$ are more appropriate with respect to the needs of different argumentative contexts (e.g. epistemic vs. practical reasoning, monological or dialogical argumentation), a comparison with the properties of traditional non-gradual semantics, and the study of possible relationships with other properties considered in the literature and not included in the analysis of this paper, such as cardinality precedence, quality precedence or compensation (Amgoud et al. 2017).

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