Complexity of Optimally Defending and Attacking a Network

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Introduction

Networks are widely studied across numerous scientific fields. Mostly, the objective of these studies is to establish and maintain a stable and fully functional network. However, sometimes an unstable or dysfunctional network is equally desirable. Examples include epidemic, terrorist and drug trafficking networks. Strategic aspects of network analysis is an important research area in many fields including AI (Michalak, Rahwan, and Wooldridge. 2017). Within this area, identifying the most important nodes or edges is a fundamental problem (Zheng, Dunagan, and Kapoor 2011). Applications include disrupting the spread of an epidemic (Kovács and Barabási 2015), weakening a terrorist network (Michalak et al. 2015), pinpointing the most vulnerable nodes (Holme et al. 2002) and blocking a contagion for network security (Zheng, Dunagan, and Kapoor 2011). The problem is relevant to various fields and sectors such as epidemiology, sociology, physics, security and logistics.

We focus on the problem of optimizing the performance of a network by identifying the most critical nodes or edge whose removal has significant impact on the performance of the network. We choose to quantify the network performance by Inverse Geodesic Length (IGL). Formally, $IGL(G) = \sum_{\{u,v\} \subseteq V, u \neq v} \frac{1}{d(u,v)}$ where $d(u,v)$ denotes the distance between u and v. Our choice is driven by two factors. One, IGL has been frequently studied in the relevant literature as a global measure of robustness of a network. Two, IGL remains effective irrespective of the input graph structure. Interestingly, despite its widespread use as a network performance measure, the optimization problem with respect to IGL has not been examined previously.

Parameterized complexity analysis: A parameterized decision problem Π is in *FPT (Fixed Parameter Tractable)*, if there is an algorithm solving any instance x with parameter k in time $f(k) \cdot |x|^c$, where $f(k)$ is a computable function of k and c is a constant. Denoted by $W[1]$ is a class of parameterized decision problems that are considered unlikely to be in FPT (Downey and Fellows 2013). A para-NP-hard problem is NP-hard even for constant values of the parameter. See (Cygan et al. 2015) for a detailed discussion on parameterized complexity.

Related Work

The problem of optimizing the network performance by identifying critical nodes or edges has been considered both in social network analysis and artificial intelligence; see, e.g., (Lindelauf, Hamers, and Husslage 2013; Michalak et al. 2015). A common trend among these studies is to rank the vertices by their importance using a heuristic (Michalak et al. 2015; Szczepanski, Michalak, and Rahwan 2015). To measure the quality of their ranking they either use expert domain knowledge on the importance of vertices in the network or existing network performance measures like *component order connectivity* (size of the largest connected component) (Gross et al. 2013) and *inverse geodesic length (IGL)*. IGL has been used to examine network vulnerability (Holme et al. 2002) and the effect of critical nodes (Barabási and Albert 1999) in the network security domain. It has also been used to identify influential nodes in a social network (Morone and Makse 2015). Game-theoretic values and centrality measures, such as the Shapley value and betweenness centrality of the nodes, have been used as heuristics to delete nodes with large impact on network performance (Holme et al. 2002; Michalak et al. 2015; Szczepanski, Michalak, and Rahwan 2015). Particularly, Szczepanski, Michalak, and Rahwan delete the nodes with the highest Shapley value as a heuristic to decrease the IGL.

Results

Although the inverse geodesic length is prominent in network analysis, we formally initiate research on the complexity of the optimization problem with respect to IGL. In (Aziz, Gaspers, and Najeebullah 2017) we consider the single-agent, vertex deletion optimization problem corresponding to IGL called MINIMIZE IGL (MINIGL). In MINIGL, given a network G, a budget k, and a target inverse geodesic length T, the question is: does there exist a set $X \subseteq V(G)$, such that $|X| \leq k$ and $IGL(G - X) \leq T$? Our main focus is a parameterized complexity analysis of the problem. We observe that MINIGL is equivalent to VERTEX COVER when $T = 0$ and is therefore NP-Complete. This also implies that it is para-NP-hard for parameter T. For parameter k, we give reductions from CLIQUE on regular graphs to show that MINIGL is $W[1]$-hard and NP-complete even when restricted to bipartite and split graphs. For parameter $k + T$, we give a kernel of size $O(k^2 + T)$, which establishes that the problem is FPT for $k + T$. Next, we consider structural parameters. Our choice of parameters is supported by the relevant literature as well as by the empirical analysis of the real-world datasets. The vertex cover number is one of the well-known and widely studied structural parameters (Fomin et al. 2014) , whereas recently introduced alternatives neighborhood diversity (Lampis 2012) and twin cover number (Ganian 2015) are less restrictive generalizations of vertex cover. Our main result is that MINIGL is FPT parameterized by twin cover number. Since a vertex cover is a twin cover, this also implies tractability for vertex cover number. However, we give a faster algorithm for this parameter. We also provide an FPT algorithm parameterized by neighborhood diversity and the deletion budget combined.

Future Directions

A natural future direction is to consider MINIGL-ED, the edge deletion counterpart of MINIGL, we aim to approach it on similar lines as that of MINIGL. Intuitively, a multi-agent setting for MINIGL (respectively, MINIGL-ED) problem seems more relevant in practice. In what follows, we present multi-agent settings for the optimization problems corresponding to IGL.

Stackelberg Game with IGL Payoffs

A Stackelberg game is a strategic game in which at least one player is defined as the leader who can make a decision and commit to a strategy before other players who are defined as followers. We consider a two-player Stackelberg game on a graph G, let defender d and attacker x be the two players. We designate d to be the leader. The set of actions for d is to protect $S_d \subseteq V(G)$ with $k_d = |S_d|$ and the set of actions for x is to remove $S_x \subseteq V(G)$ with $k_x = |S_x|$. A vertex can only be removed if it is not protected. We formalize the DEFENDER-STACKELBERG GAME (D-SIGL) problem as follows; Given a graph G, integers k_d, k_x and a target inverse geodesic length T, does there exist a set of vertices S_d with $|S_d| = k_d$ such that $IGL(G - S_x) \geq T$ for all $S_x \subseteq (V(G) \setminus S_d)$ where $|S_x| = k_x$. We plan to conduct a comprehensive computational and parameterized complexity analysis of D-SIGL. We intend to consider parameters k_d, k_x, T, tree-width, vertex cover and their combinations.

Cooperative Game with IGL Payoffs

A *cooperative game* is defined by a pair (N, v) where $N = \{1, 2, \ldots, n\}$ is the set of agents and $v : 2^N \to \mathbb{R}$ is a *valuation function* that associates with each coalition $S \subseteq N$ a value $v(S)$ with $v(\emptyset) = 0$. $v(S)$ can be considered as the value generated when players in coalition S cooperate. The Shapley value $\phi(N, v)$ specifies a value $\phi_i(N, v)$ for each player where, $\phi_i(N, v) = \frac{1}{|N|!} \sum_{S \subseteq N \setminus \{i\}} (|S|!)(|N| - |S| - 1)! \cdot (v(S \cup \{i\}) - v(S))$. We consider a cooperative game IGL-COOP defined on a graph G with $N = V(G)$ and $v(S) = IGL(G) - IGL(G[V(G) \setminus S])$. Note that $v(\emptyset) = 0$ and $v(V(G)) = IGL(G)$. Based on IGL-COOP, each vertex has a corresponding Shapley value. We refer to the Shapley value as SH-IGL and aim to find answers to the following questions; What is the complexity of computing SH-IGL? What is the complexity of comparing the SH-IGL of two vertices? Can we find an axiomatic characterization of the SH-IGL?

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