Chaotic Time Series Prediction Using a Photonic Reservoir Computer with Output Feedback

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Reservoir Computing is a bio-inspired computing paradigm for processing time dependent signals (Jaeger and Haas 2004; Maass, Natschläger, and Markram 2002). It can be easily implemented in hardware. The performance of these analogue devices matches digital algorithms on a series of benchmark tasks (see e.g. (Soriano et al. 2015) for a review). Their capacities could be extended by feeding the output signal back into the reservoir, which would allow them to be applied to various signal generation tasks (Antonik et al. 2016b). In practice, this requires a high-speed readout layer for real-time output computation. Here we achieve this by means of a field-programmable gate array (FPGA), and demonstrate the first photonic reservoir computer with output feedback. We test our setup on the Mackey-Glass chaotic time series generation task and obtain interesting prediction horizons, comparable to numerical simulations, with ample room for further improvement. Our work thus demonstrates the potential offered by the output feedback and opens a new area of novel applications for photonic reservoir computing.

Theory and methods

Reservoir computing. A general reservoir computer is described in (Lukoševičius and Jaeger 2009). In our implementation we use a sine transfer function and a ring topology to simplify the interconnection matrix, so that only the first neighbour nodes are connected (Paquot et al. 2012). The system is trained offline, using ridge regression algorithm.

Mackey-Glass chaotic series generation task. The Mackey-Glass delay differential equation is given by (Mackey and Glass 1977)

$$\frac{dx}{dt} = \beta \frac{x(t-\tau)}{1+x^n(t-\tau)} - \gamma x \tag{1}$$

with τ, γ, β, $n > 0$. To obtain chaotic dynamics, we set the parameters as in (Jaeger and Haas 2004): $\beta = 0.2$, $\gamma = 0.1$, $\tau = 17$ and $n = 10$. The equation was solved using the RK4 method with a stepsize of 1.0.

During the training phase, the reservoir computer receives the Mackey-Glass time series as input and is trained to predict the next value of the series from the current one. Then, the reservoir input is switched from the teacher sequence to the reservoir output signal, and the system is left running autonomously. To evaluate the system performance, we compute the number of correctly predicted steps.

Experimental setup

Our experimental setup, schematised in figure 1, consists of two main components: the opto-electronic reservoir and the FPGA board. The former is based on previously published works (Paquot et al. 2012). The reservoir size N depends on the delay created by the fibre spool (Spool). We performed experiments with two spools of approximately 1.6 km and 10 km and, correspondingly, reservoirs of 100 and 600 neurons.

The FPGA board is used to interface the opto-electronic setup with a personal computer, running Matlab, and implements the input and readout layers of the reservoir computer (Antonik et al. 2016a). As the neurons are processed sequentially, due to propagation delay within the setup, the output signal can only be computed in time to update the 24-th neuron. For this reason, we set the first 23 elements of the input mask to zero. That way, all neurons contribute to solving the task, but the first 23 do not "see" the input signal.

Results

Numerical simulations. While this work focuses on experimental results, we also developed three numerical models of the setup in order to have several points of comparison: (a) the idealised model incorporates the core characteristics of our reservoir computer, disregarding experimental considerations, and is used to define maximal achievable performance, (b) the noiseless experimental model emulates the most influential features of the experimental setup, but neglects the noise, that is taken into account by (c) the noisy experimental model.

Experimental results. The system was trained over 1000 input samples and was running autonomously for 600 timesteps. We discovered that the noise inside the opto-electronic reservoir makes the outcome of an experiment inconsistent. That is, several repetitions of the experiment with same parameters may result in significantly different prediction lengths. While the system produced several very good

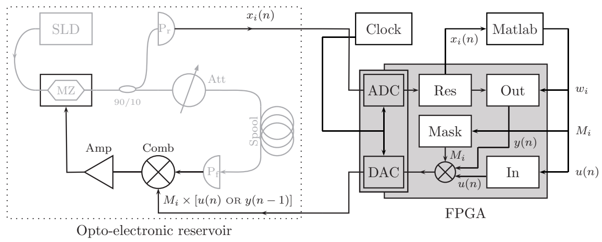

Figure 1: Schematic representation of the experimental setup. Optical and electronic components of the photonic reservoir are shown in grey and black, respectively. It contains an incoherent light source (SLD: Superluminescent Diode), a Mach-Zehnder intensity modulator (MZ), a $90/10$ beam splitter, an optical attenuator (Att), a fibre spool (Spool), two photodiodes (P_r and P_f), a resistive combiner (Comb) and an amplifier (Amp). The FPGA board implements the readout layer and computes the output signal $y(n)$ in real time. It also generates the analogue input signal $u(n)$ and acquires the reservoir states $x_i(n)$. The computer, running Matlab, controls the devices, performs the offline training and uploads all the data – inputs $u(n)$, readout weights w_i and input mask M_i – on the FPGA.

Prediction length	N	
	100	600
experimental	125 ± 14	344 ± 64
numerical (noisy)	120 ± 32	361 ± 87
numerical (noiseless)	121 ± 38	637 ± 252
idealised model	217 ± 156	683 ± 264

Table 1: Summary of experimental and numerical results.

predictions, most of the outcomes were rather poor. We obtained similar behaviour with the noisy experimental model, using the same level of noise as measured experimentally.

Numerical simulations have shown that reducing the noise does not always increase the maximum performance, but only makes the outcome more consistent. For this reason, we measured the performances of our experimental setup by repeating the autonomous run 50 times for each training, and reporting results for the best prediction length.

Table 1 sums up the results obtained experimentally with both reservoir sizes, as well as numerical results obtained with all three models. The prediction lengths were averaged over 10 sequences of the MG series (generated from different starting points), and the uncertainty corresponds to deviations which occurred from one sequence to another. For a small reservoir $N = 100$, experimental results agree with both experimental models, but all three are much lower than the idealised model. We found that this is due to the 23 zeroed input mask elements, as well as the limited resolution of the analog-to-digital converter (see complementary material for details). Prediction lengths obtained with the large reservoir $N = 600$ match the noisy experimental model, but here the noise has a significant impact on the maximal performance achievable.

Perspectives

We report what is, to the best of our knowledge, the first photonic Reservoir Computer capable of generating chaotic time series with significant prediction horizon. Our numerical simulations have shown that reducing the noise inside the opto-electronic reservoir would significantly improve its performance. This can be done by upgrading the components by low noise, low voltage models, thus reducing the effects of electrical noise. Despite these issues, our work demonstrates, for the first time, that photonic reservoir computers are capable of emulating chaotic attractors, which offers new potential applications to this computational paradigm.

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References

Antonik, P.; Duport, F.; Hermans, M.; Smerieri, A.; Haelterman, M.; and Massar, S. 2016a. Online training of an opto-electronic reservoir computer applied to real-time channel equalization. *IEEE Transactions on Neural Networks and Learning Systems* PP(99):1–13.

Antonik, P.; Hermans, M.; Duport, F.; Haelterman, M.; and Massar, S. 2016b. Towards pattern generation and chaotic series prediction with photonic reservoir computers. In *SPIE's 2016 Laser Technology and Industrial Laser Conference*, volume 9732, 97320B.

Jaeger, H., and Haas, H. 2004. Harnessing nonlinearity: Predicting chaotic systems and saving energy in wireless communication. *Science* 304:78–80.

Lukoševičius, M., and Jaeger, H. 2009. Reservoir computing approaches to recurrent neural network training. *Comp. Sci. Rev.* 3:127–149.

Maass, W.; Natschläger, T.; and Markram, H. 2002. Real-time computing without stable states: A new framework for neural computation based on perturbations. *Neural comput.* 14:2531–2560.

Mackey, M. C., and Glass, L. 1977. Oscillation and chaos in physiological control systems. *Science* 197(4300):287–289.

Paquot, Y.; Duport, F.; Smerieri, A.; Dambre, J.; Schrauwen, B.; Haelterman, M.; and Massar, S. 2012. Optoelectronic reservoir computing. *Sci. Rep.* 2:287.

Soriano, M. C.; Brunner, D.; Escalona-Morán, M.; Mirasso, C. R.; and Fischer, I. 2015. Minimal approach to neuro-inspired information processing. *Frontiers in computational neuroscience* 9.