

# Trust-Sensitive Evolution of DL-Lite Knowledge Bases\*

Dmitriy Zheleznyakov, Evgeny Kharlamov, Ian Horrocks

Department of Computer Science, University of Oxford, UK

{f\_name.l\_name}@cs.ox.ac.uk

## Abstract

Evolution of Knowledge Bases (KBs) consists of incorporating new information in an existing KB. Previous studies assume that the new information should be fully trusted and thus completely incorporated in the old knowledge. We suggest a setting where the new knowledge can be partially trusted and develop model-based approaches (MBAs) to KB evolution that rely on this assumption. Under MBAs the result of evolution is a set of interpretations and thus two core problems for MBAs are closure, i.e., whether evolution result can be axiomatised with a KB, and approximation, i.e., whether it can be (maximally) approximated with a KB. We show that DL-Lite is not closed under a wide range of trust-sensitive MBAs. We introduce a notion of s-approximation that improves the previously proposed approximations and show how to compute it for various trust-sensitive MBAs.

## Introduction

Recent years have witnessed a strong and increasing interest in Description Logic (DL) knowledge bases (KBs) (Baader et al. 2003) as a mechanism for representing structured knowledge; in particular, DLs became the foundation for OWL 2, the standard ontology language of the Semantic Web. A DL KB  $\mathcal{K}$  consists of a TBox  $\mathcal{T}$  that models at the intensional level the static and structural aspects of an application domain, and an ABox  $\mathcal{A}$  that models at the extensional level the current state of affairs or data about individuals.

In many applications KBs are subject to changes, for instance, when they are constructed from evolving Web pages (Suchanek and Weikum 2013) or databases (Furche et al. 2012), or created collaboratively (Bollacker et al. 2008; Stearns et al. 2001). A typical scenario for such applications is to incorporate in a given KB  $\mathcal{K}$  an acquired KB  $\mathcal{N}$  that expresses new information. In the case where  $\mathcal{N}$  interacts with  $\mathcal{K}$  in an undesirable way, e.g., by causing the KB or relevant parts of it to become unsatisfiable,  $\mathcal{N}$  cannot simply be added to  $\mathcal{K}$ . Different ways to address this problem are possible, corresponding to different approaches for *KB evolution*.

Knowledge evolution in the context of DL KBs has recently attracted a lot of attention from both practical and foundational perspectives, see e.g. (Fridman Noy et al. 2004; Haase and Stojanovic 2005; Flouris et al. 2008; Konev, Walther, and Wolter 2008; Cuenca Grau et al. 2012; Wu and Lécué 2014). *Model-based approaches* (MBAs) are the most commonly studied. Under MBAs, the result of evolution is the *set of first-order interpretations*  $\mathcal{M}$  that are models of  $\mathcal{N}$ , minimally distant from the models of  $\mathcal{K}$ . The latter condition corresponds to a widely accepted *principle of minimal change* (Eiter and Gottlob 1992). Depending on how minimality and distance are defined, one can obtain various evolution semantics and a number of them have been introduced and studied (Qi and Du 2009; Calvanese et al. 2010; Wang, Wang, and Topor 2010; Kharlamov and Zheleznyakov 2011; Liu et al. 2011; Kharlamov, Zheleznyakov, and Calvanese 2013; Zhuang et al. 2014; Qi et al. 2015; Wang et al. 2015; Zhuang et al. 2016).

To the best of our knowledge, all previous studies of KB evolution assume that the new knowledge  $\mathcal{N}$  should be *fully* trusted and thus completely taken on board (see Related Work section). However, this assumption does not hold in a wide range of important applications (Suchanek and Weikum 2013) where  $\mathcal{N}$  comes from a *partially trusted* source, e.g., from the Web or from a source with a limited expertise.

In this work we address this issue for MBAs and study how an external notion of trust could be used in order to determine how new knowledge should be integrated with existing knowledge. Following Hunter and Booth (2015), who studied trust in the context of propositional belief revision, we assume that the knowledge provider has expertise that is restricted to a particular area and thus cannot distinguish between certain states of the application domain—first-order interpretations in our case. We formalise such a notion of trust as an equivalence relation on first-order interpretations and introduce four natural classes of trust. Then we use trust as an external mechanism to relativise arbitrary interpretations (not necessarily models of  $\mathcal{N}$ ) to models of  $\mathcal{N}$  by considering equivalence classes of the latter’s models. This allows us to define the result of evolution as a set of minimally distinct interpretations  $\mathcal{M}$  selected from these equivalence classes instead of just models of  $\mathcal{N}$  as in classical MBAs. Our trust-sensitive evolution is generic in the sense that it is applicable to KBs of *any* DL, and is backwards-compatible with classi-

\*This research has been partially supported by the EU project Optique (FP7-IP-318338), the Royal Society, the EPSRC grants ED<sup>3</sup>, DBonto, and MaSI<sup>3</sup>.  
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cal MBAs in the sense that it coincides with them whenever the model of trust assumes that the knowledge provider is an expert in everything.

Since evolution under MBAs is defined as a set of interpretations  $\mathcal{M}$ , in practice one would want to efficiently axiomatise this  $\mathcal{M}$  as a KB or, whenever this is impossible, to efficiently ‘closely’ approximate it with a KB. Thus, the two core KB evolution problems for a given DL  $\mathcal{L}$  and MBA semantics are *closure* of  $\mathcal{L}$  under the semantics, i.e., whether and how for every  $\mathcal{K}$  and  $\mathcal{N}$  in  $\mathcal{L}$ , the corresponding  $\mathcal{M}$  under the semantics can be axiomatised in  $\mathcal{L}$ , and *approximation* of the semantics in  $\mathcal{L}$  whenever  $\mathcal{L}$  is not closed.

We study the closure and approximation problems for DL-Lite—a tractable DL behind the QL profile of OWL 2—under trust-sensitive MBAs for various models of trust. Firstly, we show that DL-Lite is not closed under *any* trust-sensitive semantics. It was known that DL-Lite is not closed under many classical MBAs (Calvanese et al. 2010; Kharlamov, Zheleznyakov, and Calvanese 2013; Qi et al. 2015) and our results in particular imply the non-closure of DL-Lite under those classical MBAs for which this problem remained open.

We next turn our attention to the approximation problem for DL-Lite and an important practical setting of ABox evolution where the TBox is static and only the ABox evolves. A widely studied approach for this setting is sound approximation, where  $\mathcal{M}$  is approximated with a KB whose set of models contains  $\mathcal{M}$ . For classical MBAs De Giacomo et al. (2009), Kharlamov, Zheleznyakov, and Calvanese (2013), and Qi et al. (2015) proposed algorithms to compute maximal sound approximations for various semantics. Here we propose the notion of *s-approximation*—a KB that may use special predicates and constants—and show that in general it improves sound approximations by better capturing  $\mathcal{M}$  for both classical and trust-sensitive MBAs. Moreover, we show that s-approximations are also better in preserving Boolean queries satisfied by  $\mathcal{M}$  and we determine an important class of such queries. Finally, we develop polynomial time algorithms to compute maximal sound s-approximations for several trust-sensitive and classical evolution semantics.

## Preliminaries

**Description Logics.** We assume standard definitions of first-order logic signature, sentences, interpretations, satisfiability, and entailment. We further assume a fixed signature with disjoint countable sets of unary and binary predicates and constants, and that all interpretations are over this signature and the same countable domain  $\Delta$ . Let PW (*Possible Worlds*) denote the class of all such interpretations. Whenever convenient we treat interpretations as sets of atoms.

In DLs (Baader et al. 2003), the domain of interest is modelled by means of *concepts*, that are formulae with one free variable, denoting sets of objects, *roles*, that are formulae with two free variables, denoting binary relations between objects, and *constants*, denoting objects. In order to support such modelling, DLs provide a specialised variable-free syntax and operators for constructing concepts and roles from unary predicates (called *atomic concepts*) and binary predicates (called *atomic roles*). A DL KB  $\mathcal{K} = (\mathcal{T}, \mathcal{A})$  consists

of a TBox  $\mathcal{T}$  that is a finite set of sentences (called *TBox assertions*) over concepts and roles, and an ABox  $\mathcal{A}$  that is a finite set of sentences (called *ABox* or *membership assertions*) of the form  $C(a)$  and  $R(a, b)$ , where  $C$  is a concept,  $R$  is a role and  $a, b$  are constants. A DL  $\mathcal{L}$  is a recursive set of KBs closed under renaming of constants and the subset relation.

All the logics of the DL-Lite family have the following constructs for complex concepts and roles (Calvanese et al. 2007): (i)  $B ::= A \mid \exists R$ , (ii)  $C ::= B \mid \neg B$ , and (iii)  $R ::= P \mid P^-$ , where  $A$  and  $P$  are an atomic concept and role,  $B$  and  $C$  are *basic* and *general* concepts, and  $R$  is a *basic* role. A DL-Lite<sub>core</sub> TBox consists of *concept inclusions* assertions  $B \sqsubseteq C$ . DL-Lite extends DL-Lite<sub>core</sub> by allowing in a TBox *role inclusion* assertions  $R_1 \sqsubseteq R_2$  and *functionality* assertions ( $\text{funct } R$ ) in a way that if  $R_1 \sqsubseteq R_2$  appears in a TBox, then neither ( $\text{funct } R_2$ ) nor ( $\text{funct } R_2^-$ ) appears in the TBox.<sup>1</sup> This syntactic restriction ensures the tractability of the logic. ABoxes in DL-Lite<sub>core</sub> and DL-Lite consist of membership assertions of the form  $C(a)$  and  $P(a, b)$ .

The semantics for concepts and roles is defined in the standard way under an assumption that  $a^{\mathcal{I}} = a$  for each constant  $a$ . That is,  $A^{\mathcal{I}} \subseteq \Delta$ ,  $P^{\mathcal{I}} \subseteq \Delta \times \Delta$ ,  $(P^-)^{\mathcal{I}} = \{(b, a) \mid (a, b) \in P^{\mathcal{I}}\}$ ,  $(\neg B)^{\mathcal{I}} = \Delta \setminus B^{\mathcal{I}}$ , and  $(\exists R)^{\mathcal{I}} = \{a \mid \text{there exists } b \text{ s.t. } (a, b) \in R^{\mathcal{I}}\}$ . The semantics of assertions is also defined in the standard way:  $\mathcal{I} \models D_1 \sqsubseteq D_2$  if  $D_1^{\mathcal{I}} \subseteq D_2^{\mathcal{I}}$ ,  $\mathcal{I} \models (\text{funct } R)$  if the relation  $R^{\mathcal{I}}$  is a function,  $\mathcal{I} \models C(a)$  if  $a^{\mathcal{I}} \in C^{\mathcal{I}}$ , and  $\mathcal{I} \models P(a, b)$  if  $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in P^{\mathcal{I}}$ .

For  $\xi$  a concept, role, (set of) assertions(s), or (set of) interpretation(s),  $\text{pred}(\xi)$  denotes the set of atomic concepts and roles in  $\xi$ .  $\text{Mod}(\mathcal{K})$  denotes the set of all interpretations  $\mathcal{I}$  that are models of  $\mathcal{K}$ , i.e.,  $\mathcal{I} \models \varphi$  for each assertion  $\varphi$  in  $\mathcal{K}$ .

**Classical Model-Based Evolution.** A classical evolution setting consists of an old KB  $\mathcal{K}$  and a new KB  $\mathcal{N}$ . Under classical MBAs, the evolution result is the subset of  $\text{Mod}(\mathcal{N})$  that is minimally distant from  $\text{Mod}(\mathcal{K})$ . We now formally introduce classical MBAs (Calvanese et al. 2010).

Let  $\mathcal{I}$  and  $\mathcal{J}$  be interpretations. Recall that  $\mathcal{I} \ominus \mathcal{J}$  denotes the symmetric difference  $(\mathcal{I} \setminus \mathcal{J}) \cup (\mathcal{J} \setminus \mathcal{I})$ . A *distance function*  $\text{dist}$  between  $\mathcal{I}$  and  $\mathcal{J}$  can be defined in one of the following ways. Distances based on atoms are defined as (i) a set  $\mathcal{I} \ominus \mathcal{J}$ , denoted  $\text{dist}_{\{\}}^{\text{a}}(\mathcal{I}, \mathcal{J})$ , or (ii) a number  $|\mathcal{I} \ominus \mathcal{J}|$ , denoted  $\text{dist}_{\#}^{\text{a}}(\mathcal{I}, \mathcal{J})$ . Distances based on predicates are defined as (i) a set  $\{\alpha \mid \alpha \text{ is a predicate and } \alpha^{\mathcal{I}} \neq \alpha^{\mathcal{J}}\}$ , denoted  $\text{dist}_{\{\}}^{\text{p}}(\mathcal{I}, \mathcal{J})$ , or (ii) a number  $|\text{dist}_{\{\}}^{\text{p}}(\mathcal{I}, \mathcal{J})|$  denoted  $\text{dist}_{\#}^{\text{p}}(\mathcal{I}, \mathcal{J})$ . Distances returned by  $\text{dist}_{\{\}}^x$ , where  $x \in \{\text{a}, \text{p}\}$  are sets and thus can be compared via set inclusion:  $\text{dist}_{\{\}}^x(\mathcal{I}_1, \mathcal{J}_1) \leq \text{dist}_{\{\}}^x(\mathcal{I}_2, \mathcal{J}_2)$  if  $\text{dist}_{\{\}}^x(\mathcal{I}_1, \mathcal{J}_1) \subseteq \text{dist}_{\{\}}^x(\mathcal{I}_2, \mathcal{J}_2)$ . Distances returned by  $\text{dist}_{\#}^x$  are natural numbers and thus can be compared numerically.<sup>2</sup>

Let  $\mathcal{S}$  and  $\mathcal{S}'$  be sets of interpretations and  $\text{dist}$  a distance function. The subset  $\text{min}_{\text{dist}, \mathcal{S}}(\mathcal{S}')$  of  $\mathcal{S}'$  that consists of interpretations minimally distant from  $\mathcal{S}$  is defined as follows:

<sup>1</sup>In (Calvanese et al. 2010) this DL is referred to as DL-Lite<sub>eFR</sub>.

<sup>2</sup>Note that for infinite  $\text{dist}_{\{\}}^x(\mathcal{I}, \mathcal{J})$ , we assume that: for any  $\mathcal{I}'$  and  $\mathcal{J}'$  if  $\text{dist}_{\{\}}^x(\mathcal{I}', \mathcal{J}')$  is (i) finite, then  $\text{dist}_{\{\}}^x(\mathcal{I}', \mathcal{J}') < \text{dist}_{\{\}}^x(\mathcal{I}, \mathcal{J})$ , or (ii) infinite, then  $\text{dist}_{\{\}}^x(\mathcal{I}', \mathcal{J}') = \text{dist}_{\{\}}^x(\mathcal{I}, \mathcal{J})$ .

$\{\mathcal{J} \in \mathcal{S}' \mid \text{there is } \mathcal{I} \in \mathcal{S} \text{ s.t. for each } \mathcal{I}' \in \mathcal{S}, \text{ and}$   
for each  $\mathcal{J}' \in \mathcal{S}'$  it holds  $\text{dist}(\mathcal{I}', \mathcal{J}') \not\prec \text{dist}(\mathcal{I}, \mathcal{J})\}$ .

We now define *selectors* that choose those interpretations of  $\mathcal{S}'$  that are minimally distant from  $\mathcal{S}$ .

**Definition 1.** A selector, denoted  $\mathbb{S}$ , is a function that maps each pair  $(\mathcal{S}, \mathcal{S}')$  of sets of models into  $2^{\mathcal{S}'}$ . We consider the following selectors, where  $x \in \{\mathbf{a}, \mathbf{p}\}$ , and  $y \in \{\emptyset, \#\}$ :

- a global selector induced by  $\text{dist}_{y,x}^x$ , denoted  $\mathbb{G}_{y,x}^x$ , is defined as  $\min_{\text{dist}_{y,x}^x, \mathcal{S}}(\mathcal{S}')$ ;
- a local selector induced by  $\text{dist}_{y,x}^x$ , denoted  $\mathbb{L}_{y,x}^x$ , is defined as  $\cup_{\mathcal{I} \in \mathcal{S}} \min_{\text{dist}_{y,x}^x, \{\mathcal{I}\}}(\mathcal{S}')$ .

Finally, *classical evolution semantics* for  $\mathcal{K}$  and  $\mathcal{N}$  is defined as  $\mathbb{S}(\text{Mod}(\mathcal{K}), \text{Mod}(\mathcal{N}))$ . Note that, in terms of Kat-suno and Mendelzon (1991), semantics based on local selectors correspond to *knowledge update*, and semantics based on global selectors correspond to *knowledge revision*.

## Trust-Sensitive Model-Based Evolution

In this section we introduce four models of trust and define how they can be incorporated in MBAs.

**Models of Trust.** Our models of trust reflect the assumption that the knowledge provider has a restricted area of expertise, and thus we do not trust facts that are outside this area. In terms of interpretations this means that if two interpretations disagree only on such facts, then the provider cannot distinguish between them.

**Definition 2.** A model of trust is an equivalence relation  $\equiv$  on models. We consider four classes of models of trust:

- total trust, denoted TT, consists of one equivalence relation  $\equiv_{\text{TT}}$  defined as  $\mathcal{I}_1 \equiv_{\text{TT}} \mathcal{I}_2$  iff  $\mathcal{I}_1 = \mathcal{I}_2$ ;
- total distrust, denoted TD, consists of one equivalence relation  $\equiv_{\text{TD}}$  defined as  $\mathcal{I}_1 \equiv_{\text{TD}} \mathcal{I}_2$  for each  $\mathcal{I}_1$  and  $\mathcal{I}_2$ ;
- assertion trust, denoted AT, consists of one equivalence relation  $\equiv_{\Phi}$  for each finite set of assertions  $\Phi$  which is defined as  $\mathcal{I}_1 \equiv_{\Phi} \mathcal{I}_2$  iff either  $\mathcal{I}_1 \models \varphi$  and  $\mathcal{I}_2 \models \varphi$  or  $\mathcal{I}_1 \not\models \varphi$  and  $\mathcal{I}_2 \not\models \varphi$  for each  $\varphi \in \Phi$ .
- predicate trust, denoted PT, consists of one equivalence relation  $\equiv_{\mathcal{P}}$  for each finite set of predicates  $\mathcal{P}$  which is defined as  $\mathcal{I}_1 \equiv_{\mathcal{P}} \mathcal{I}_2$  iff  $p^{\mathcal{I}_1} = p^{\mathcal{I}_2}$  for each  $p \in \mathcal{P}$ .

**Example 3.** Consider a scenario about places where famous researchers Einstein (Ein) and Mendeleev (Men) live (livesIn). Consider the two following models of trust:  $\equiv_{\mathcal{P}_{\text{ex}}} \in \text{PT}$  and  $\equiv_{\Phi_{\text{ex}}} \in \text{AT}$ , where  $\Phi_{\text{ex}} = \{\text{livesIn}(\text{Ein}, \text{us}), \exists \text{livesIn}(\text{Men})\}$  and  $\mathcal{P}_{\text{ex}} = \{\text{livesIn}\}$ . In  $\equiv_{\mathcal{P}_{\text{ex}}}$  we trust that the knowledge provider is an expert in places of residence in general, while in  $\equiv_{\Phi_{\text{ex}}}$  we trust that they can tell whether or not Einstein lives in the USA and Mendeleev lives somewhere. Consider two interpretations  $\mathcal{I}_{\text{ex}}^1 = \{\text{livesIn}(\text{Ein}, \text{us})\}$  and  $\mathcal{I}_{\text{ex}}^2 = \{\text{livesIn}(\text{Ein}, \text{us}), \text{livesIn}(\text{Ein}, \text{ru})\}$ . It is easy to see that  $\mathcal{I}_{\text{ex}}^1 \not\equiv_{\mathcal{P}_{\text{ex}}} \mathcal{I}_{\text{ex}}^2$ , while  $\mathcal{I}_{\text{ex}}^1 \equiv_{\Phi_{\text{ex}}} \mathcal{I}_{\text{ex}}^2$ . ■

We will use models of trust to relativise interpretations to a given one using the following *extender* function.

**Definition 4.** An extender, denoted  $\mathbb{E}$ , is a function that maps each pair  $(\equiv, \mathcal{I})$  where  $\equiv$  is a model of trust and  $\mathcal{I}$  is an interpretation, into  $2^{\text{PW}}$  in the following way:

$$\mathbb{E}(\equiv, \mathcal{I}) = \{\mathcal{J} \in \text{PW} \mid \mathcal{J} \equiv \mathcal{I}\}.$$

For a set of interpretations  $\mathcal{S}$ ,  $\mathbb{E}(\equiv, \mathcal{S}) = \bigcup_{\mathcal{I} \in \mathcal{S}} \mathbb{E}(\equiv, \mathcal{I})$ .

Clearly, for each  $\mathcal{S}$  it holds that  $\mathcal{S} \subseteq \mathbb{E}(\equiv, \mathcal{S})$ , while  $\mathbb{E}(\equiv, \mathcal{S}) \subseteq \mathcal{S}$  does not hold in general.

**Example 5.**  $\mathbb{E}(\equiv_{\mathcal{P}_{\text{ex}}}, \mathcal{I}_{\text{ex}}^1)$  and  $\mathbb{E}(\equiv_{\Phi_{\text{ex}}}, \mathcal{I}_{\text{ex}}^1)$  are, resp.:

$$\begin{aligned} &\{\mathcal{I} \in \text{PW} \mid \{(\text{Ein}, \text{us})\} = \text{livesIn}^{\mathcal{I}}\} \text{ and} \\ &\{\mathcal{I} \in \text{PW} \mid \mathcal{I} \models \text{livesIn}(\text{Ein}, \text{us}), \text{ and } \mathcal{I} \not\models \exists \text{livesIn}(\text{Men})\}. \blacksquare \end{aligned}$$

**Trust-Sensitive Evolution Settings and Semantics.** We distinguish between KB and ABox evolution. In the former case the whole KB changes, while in the latter case the TBox is fixed and only the ABox evolves. The following definition of evolution settings reflects this distinction.

**Definition 6.** Let  $\mathcal{L}$  be a DL and  $\mathcal{C}$  a class of models of trust.

- An  $(\mathcal{L}, \mathcal{C})$ -setting  $\mathcal{E}$  for KB evolution is a quadruple  $(\mathcal{T}, \mathcal{A}, \mathcal{N}, \equiv)$ , where  $(\mathcal{T}, \mathcal{A})$  and  $\mathcal{N}$  are satisfiable  $\mathcal{L}$ -KBs and  $\equiv$  is a model of trust in  $\mathcal{C}$ .
- An  $(\mathcal{L}, \mathcal{C})$ -setting  $\mathcal{E}$  for ABox evolution is a quadruple  $(\mathcal{T}, \mathcal{A}, \mathcal{N}, \equiv)$ , where  $\mathcal{N}$  is an  $\mathcal{L}$ -ABox,  $(\mathcal{T}, \mathcal{A})$  and  $(\mathcal{T}, \mathcal{N})$  are satisfiable  $\mathcal{L}$ -KBs,  $\equiv$  is a model of trust in  $\mathcal{C}$ .

We will refer to  $\mathcal{E}$  as just a  $\mathcal{C}$ -setting (resp., setting) when  $\mathcal{L}$  is (resp.,  $\mathcal{L}$  and  $\mathcal{C}$  are) clear or not important.

**Example 7.** Consider  $\mathcal{E}_{\text{ex}} = (\mathcal{T}_{\text{ex}}, \mathcal{A}_{\text{ex}}, \mathcal{N}_{\text{ex}}, \equiv_{\Phi_{\text{ex}}})$ , a (DL-Lite, AT)-setting for ABox evolution, where the TBox is  $\mathcal{T}_{\text{ex}} = \emptyset$ , ABox is  $\mathcal{A}_{\text{ex}} = \{\text{livesIn}(\text{Men}, \text{ru})\}$ , and the new ABox is  $\mathcal{N}_{\text{ex}} = \{\text{livesIn}(\text{Ein}, \text{us}), \text{livesIn}(\text{Men}, \text{us})\}$ . ■

We are now ready to show how models of trust can provide an external mechanism to guide evolution semantics. Intuitively, models of trust work like filters that are applied to the (models of the) new knowledge  $\mathcal{N}$  before performing the evolution. Recall that the classical MBAs ‘pick’ interpretations  $\mathcal{J}$  from  $\text{Mod}(\mathcal{N})$  that comes from the knowledge provider. In our case, however, we know that the knowledge provider cannot distinguish between any two  $\equiv$ -equivalent interpretations, i.e., any  $\mathcal{J}'$  that is  $\equiv$ -equivalent to  $\mathcal{J}$  is as ‘good’ as  $\mathcal{J}$ , and therefore, trust-sensitive evolution ‘picks’ interpretations from  $\mathbb{E}(\equiv, \text{Mod}(\mathcal{N}))$  that extends  $\text{Mod}(\mathcal{N})$  with all such  $\mathcal{J}'$ s. This approach corresponds to how Hunter and Booth (2015) introduced trust in the evolution of propositional theories.

**Definition 8.** Let  $\mathbb{S}$  be a selector. Then a trust-sensitive evolution semantics  $\text{sem}_{\mathbb{S}}$  maps each setting  $\mathcal{E} = (\mathcal{T}, \mathcal{A}, \mathcal{N}, \equiv)$  to a set of interpretations  $\mathbb{S}(\text{Mod}(\mathcal{T}, \mathcal{A}), \mathbb{E}(\equiv, \mathcal{M}'))$ , where  $\mathcal{M}'$  is equal to  $\text{Mod}(\mathcal{N})$  if  $\mathcal{E}$  is for KB evolution and to  $\text{Mod}(\mathcal{T}, \mathcal{N})$  if  $\mathcal{E}$  is for ABox evolution.

**Example 9.** Consider two sets of interpretations:  $\mathcal{M}_{\text{ex}} = \text{Mod}(\mathcal{T}_{\text{ex}}, \mathcal{A}_{\text{ex}})$ ,  $\mathcal{M}'_{\text{ex}} = \text{Mod}(\mathcal{T}_{\text{ex}}, \mathcal{N}_{\text{ex}})$ . Then, the evolution result  $\text{sem}_{\mathbb{G}_{\emptyset}^{\mathbf{a}}}(\mathcal{E}_{\text{ex}}) = \mathbb{G}_{\emptyset}^{\mathbf{a}}(\mathcal{M}_{\text{ex}}, \mathbb{E}(\equiv_{\Phi_{\text{ex}}}, \mathcal{M}'_{\text{ex}}))$  is equal to

$$\{\mathcal{J} \in \text{PW} \mid \{\text{livesIn}(\text{Ein}, \text{us}), \text{livesIn}(\text{Men}, \text{ru})\} \subseteq \mathcal{J}\}.$$

For classical MBAs the evolution result for  $\mathcal{E}_{\text{ex}}$  under  $\mathbb{G}_{\{\}}^{\text{a}}$  is:

$$\{\mathcal{J} \in \text{PW} \mid \{\text{livesIn}(\text{Ein}, \text{us}), \text{livesIn}(\text{Men}, \text{ru}), \\ \text{livesIn}(\text{Men}, \text{us})\} \subseteq \mathcal{J}\}. \quad \blacksquare$$

In practice one would expect the result of evolution to be a KB. Thus, a natural problem to study for MBAs is how evolution results can be axiomatised. Observe that the result of trust-sensitive evolution from Example 9 can be axiomatised respectively as

$$(\emptyset, \{\text{livesIn}(\text{Ein}, \text{us}), \text{livesIn}(\text{Men}, \text{ru})\}),$$

while the evolution result from Example 9 under the classical MBA  $\mathbb{G}_{\{\}}^{\text{a}}$  can be axiomatised as

$$(\emptyset, \{\text{livesIn}(\text{Ein}, \text{de}), \text{livesIn}(\text{Men}, \text{ru}), \text{livesIn}(\text{Men}, \text{us})\}).$$

In the classical case, the resulting KB is the union of the old  $\mathcal{A}_{\text{ex}}$  and the new knowledge  $\mathcal{N}_{\text{ex}}$ . In the trust-sensitive case, the semantics *rejects* the new knowledge about Mendeleev since there is no trust in the fact that he is a US born.

### Closure of DL-Lite Under Evolution

We now turn our attention to DL-Lite and show that evolution results in general cannot be axiomatised as DL-Lite KBs. We start with a definition of the closure problem.

**Definition 10.** Let  $\mathcal{L}$  be a DL,  $\mathcal{C}$  a class of models of trust, and  $\text{sem}$  a trust-sensitive evolution semantics. Then,  $\mathcal{L}$  is closed under  $\text{sem}$  for  $\mathcal{C}$  if for every  $(\mathcal{L}, \mathcal{C})$ -setting  $\mathcal{E}$  there is an  $\mathcal{L}$ -KB  $\mathcal{K}$  such that  $\text{Mod}(\mathcal{K}) = \text{sem}(\mathcal{E})$ .

**Total Trust and Total Distrust.** The main reason why we introduce TT and TD is to verify on these extreme cases whether trust-sensitive evolution semantics behave in an intuitive way. In particular, we expect backward compatibility of trust-sensitive MBAs with the classical ones, that is,  $\text{sem}_{\mathbb{S}}$  should coincide with the corresponding classical semantics  $\mathbb{S}$  in the case of TT. The following proposition confirms that this is indeed the case.

**Proposition 11.** Let  $\mathcal{E} = (\mathcal{T}, \mathcal{A}, \mathcal{N}, \equiv_{\text{TT}})$  be a  $(\mathcal{L}, \text{TT})$ -setting for some DL  $\mathcal{L}$ . Then for any selector  $\mathbb{S}$ , it holds that  $\text{sem}_{\mathbb{S}}(\mathcal{E}) = \mathbb{S}(\text{Mod}(\mathcal{T}, \mathcal{A}), \mathcal{M}')$ , where  $\mathcal{M}'$  is equal to  $\text{Mod}(\mathcal{N})$  if  $\mathcal{E}$  is for KB and  $\text{Mod}(\mathcal{T}, \mathcal{N})$  if  $\mathcal{E}$  is for ABox evolution.

The proposition implies that all non-closure results for DL-Lite under classical MBAs are inherited by trust-sensitive MBAs for TT. In particular, it is known that for ABox evolution DL-Lite is not closed under six out of eight MBAs: Calvanese et al. (2010) showed the non-closure under  $\mathbb{L}_{\{\}}^{\text{a}}$  and  $\mathbb{L}_{\{\}}^{\text{p}}$ , Kharlamov, Zheleznyakov, and Calvanese (2013) under  $\mathbb{L}_{\{\}}^{\text{a}}$  and  $\mathbb{L}_{\{\}}^{\text{p}}$ , and finally Qi et al. (2015) under  $\mathbb{G}_{\{\}}^{\text{a}}$  and  $\mathbb{G}_{\{\}}^{\text{p}}$ . For KB evolution Calvanese et al. (2010) showed the non-closure under all eight MBAs. Thus, the remaining open problem for trust-sensitive MBAs for TT is the closure under  $\mathbb{G}_{\{\}}^{\text{a}}$  and  $\mathbb{G}_{\{\}}^{\text{p}}$  for ABox evolution. The following theorem closes this gap.

**Theorem 12.** For ABox evolution, DL-Lite is not closed under  $\text{sem}_{\mathbb{S}}$  for TT, where  $\mathbb{S} \in \{\mathbb{G}_{\{\}}^{\text{a}}, \mathbb{G}_{\{\}}^{\text{p}}\}$ .

*Proof (Sketch).* Regarding  $\mathbb{G}_{\{\}}^{\text{p}}$ , one can check that for the TT-setting with  $\mathcal{A} = \{\neg \exists R^-(a)\}$ ,  $\mathcal{N} = \{\exists R^-(a)\}$ , and  $\mathcal{T} = \{A \sqsubseteq \exists R, \exists R \sqsubseteq A\}$ , the set of interpretations obtained by evolution satisfies  $\forall x. R(x, a) \rightarrow \exists y. (y \neq a \wedge R(x, y))$ . One can show that this set is not axiomatisable in DL-Lite. The non-closure for the case of  $\mathbb{G}_{\{\}}^{\text{a}}$  can be shown similarly.  $\square$

In the case of TD, regardless of the DL  $\mathcal{L}$ , selector  $\mathbb{S}$ , and  $(\mathcal{L}, \text{TD})$ -setting  $\mathcal{E} = (\mathcal{T}, \mathcal{A}, \mathcal{N}, \equiv_{\text{TD}})$ , it is easy to see that  $\text{sem}_{\mathbb{S}}(\mathcal{E}) = \text{Mod}(\mathcal{T}, \mathcal{A})$  for both KB and ABox evolution. Thus,  $\text{sem}_{\mathbb{S}}$  satisfies our intuition: it rejects the new information  $\mathcal{N}$  as it is distrusted.

**Assertion and Predicate Trust.** We denote with  $\mathcal{S}$  the set of all introduced semantics:  $\mathcal{S} = \bigcup_{x,y,z} \{\text{sem}_{\mathbb{Z}_y^x}\}$ , where  $x \in \{\text{a}, \text{p}\}$ ,  $y \in \{\{\}, \#\}$ , and  $\mathbb{Z} \in \{\mathbb{L}, \mathbb{G}\}$ .

Observe that for each  $(\mathcal{L}, \text{TT})$ -setting  $\mathcal{E}_{\text{TT}} = (\mathcal{T}, \mathcal{A}, \mathcal{N}, \equiv_{\text{TT}})$  one can construct an  $(\mathcal{L}, \text{PT})$ -setting  $\mathcal{E}_{\text{PT}} = (\mathcal{T}, \mathcal{A}, \mathcal{N}, \equiv_{\text{P}})$ , where  $\mathcal{P} = \text{pred}(\mathcal{T} \cup \mathcal{A} \cup \mathcal{N})$ , such that for each  $\text{sem} \in \mathcal{S}$  we have  $\text{sem}(\mathcal{E}_{\text{TT}}) = \text{sem}(\mathcal{E}_{\text{PT}})$ . Therefore, all the non-closure results for TT are inherited by PT.

Finally, we turn our attention to AT and show the non-closure of DL-Lite under various trust-sensitive semantics.

**Theorem 13.** For AT it holds that:

- For KB evolution and each  $\text{sem} \in \mathcal{S}$ , DL-Lite is not closed under  $\text{sem}$ ; this holds already in the case when the new information consists of one TBox assertion.
- For ABox evolution and each  $\text{sem} \in \mathcal{S}$  DL-Lite is not closed under  $\text{sem}$ .

In order to prove these results one can show that for each semantics  $\text{sem}$  considered in the theorem, there is a setting  $\mathcal{E}$ , such that  $\text{sem}(\mathcal{E})$  is a set of models that satisfies a so-called *genuine disjunction*. That is,  $\text{sem}(\mathcal{E})$  satisfies  $\varphi \vee \psi$ , for some ABox assertions  $\varphi$  and  $\psi$ , but does not satisfy either  $\varphi$  or  $\psi$ . By Lemma 1 from (Calvanese et al. 2010) such a set of interpretations is not axiomatisable in DL-Lite.

### Approximation of Evolution in DL-Lite

Since DL-Lite is not closed under the trust-sensitive MBAs, we turn our attention to approximation of evolution results. In this section we focus on ABox evolution and thus all settings are for ABox evolution.

A *sound approximation* of  $\text{sem}(\mathcal{E})$  is a KB  $\mathcal{K}$  such that  $\text{sem}(\mathcal{E}) \subseteq \text{Mod}(\mathcal{K})$ , and it is *maximal* if no other sound approximation  $\mathcal{K}'$  exists s.t.  $\text{Mod}(\mathcal{K}') \subset \text{Mod}(\mathcal{K})$ . Sound approximation of evolution in the context of DLs has been studied for classical MBAs by De Giacomo et al. (2009), Kharlamov, Zheleznyakov, and Calvanese (2013), and Qi et al. (2015). We extend the notion of sound approximation by considering s-approximations which we introduce next. In order to define them, we use the following notation. Let  $\Sigma$  be a signature, then  $\mathcal{I}_{|\Sigma}$  is a sub-interpretation of  $\mathcal{I}$  consisting of all atoms of  $\mathcal{I}$  whose predicates are in  $\Sigma$ , and for a set

of models  $\mathcal{S}$ , we define  $\mathcal{S}|_{\Sigma}$  as  $\{\mathcal{I}|_{\Sigma} \mid \mathcal{I} \in \mathcal{S}\}$ . Finally,  $\mathcal{S} \subseteq_{\Sigma} \mathcal{S}'$ , if  $\mathcal{S}|_{\Sigma} \subseteq \mathcal{S}'|_{\Sigma}$ .

**Definition 14.** *Let  $\mathcal{S}$  be a set of interpretations, and  $\mathcal{K}$  a knowledge base. Then,  $\mathcal{K}$  is a sound s-approximation of  $\mathcal{S}$  if  $\mathcal{S} \subseteq_{\text{pred}(\mathcal{S})} \text{Mod}(\mathcal{K})$ . Moreover,  $\mathcal{K}$  is a maximal sound s-approximation of  $\mathcal{S}$  if no other sound s-approximation  $\mathcal{K}'$  of  $\mathcal{S}$  exists such that  $\text{Mod}(\mathcal{K}') \subseteq_{\text{pred}(\mathcal{S})} \text{Mod}(\mathcal{K})$ . Finally,  $\mathcal{K}$  is an s-axiomatisation of  $\mathcal{S}$  if  $\mathcal{S} = \text{Mod}(\mathcal{K})|_{\text{pred}(\mathcal{S})}$ .*

Note that s-approximations coincide with sound approximations when  $\text{pred}(\mathcal{S}) = \text{pred}(\mathcal{K})$ .

## Total Trust

In this section we will study  $\text{sem}_{\mathbb{S}}$  evolution in case of TT and  $\mathbb{S} \in \{\mathbb{G}_{\{1\}}^a, \mathbb{G}_{\{\#\}}^a\}$  and use the following notations:  $\text{c1}_{\mathcal{T}}(\mathcal{A})$  is the set of membership assertions  $\varphi$  s.t.  $\mathcal{A} \cup \mathcal{T} \models \varphi$  and  $\text{c1}(\mathcal{T})$  is the set of TBox assertions  $\varphi$  s.t.  $\mathcal{T} \models \varphi$ . It is known (Calvanese et al. 2007) that in DL-Lite  $\text{c1}_{\mathcal{T}}(\mathcal{A})$  and  $\text{c1}(\mathcal{T})$  are finite and can be computed in polynomial time.

Let  $\mathcal{E} = (\mathcal{T}, \mathcal{A}, \mathcal{N}, \equiv_{\text{TT}})$  be a TT-setting. Qi et al. (2015) showed that the algorithm `AtAlg`, introduced by Kharlamov and Zheleznyakov (2011), computes a maximal sound approximation  $\mathcal{K}'$  of  $\mathbb{S}(\text{Mod}(\mathcal{T}, \mathcal{A}), \text{Mod}(\mathcal{T}, \mathcal{N}))$ , where  $\mathbb{S} \in \{\mathbb{G}_{\{1\}}^a, \mathbb{G}_{\{\#\}}^a\}$ . Given  $\mathcal{E}$ , `AtAlg` returns an ABox  $\mathcal{N} \cup \mathcal{A}'$ , where  $\mathcal{A}'$  is the maximal subset of  $\text{c1}_{\mathcal{T}}(\mathcal{A})$  such that  $(\mathcal{T}, \mathcal{N} \cup \mathcal{A}')$  is satisfiable. By Proposition 11,  $\mathcal{K}'$  is also a maximal sound approximation of  $\text{sem}_{\mathbb{S}}(\mathcal{E})$ . However,  $\mathcal{K}'$  is not necessarily their maximal sound s-approximation, as illustrated next.

**Example 15.** *Consider a TT-setting  $\mathcal{E}_{\text{ex}}^2 = (\mathcal{T}_{\text{ex}}^2, \mathcal{A}_{\text{ex}}^2, \mathcal{N}_{\text{ex}}^2, \equiv_{\text{TT}})$  over the signature  $\Sigma_{\text{ex}} = \{\text{livesIn}, \text{place}\}$ , where  $\mathcal{T}_{\text{ex}}^2 = \{\exists \text{livesIn}^- \sqsubseteq \text{place}\}$ ,  $\mathcal{A}_{\text{ex}}^2 = \{\exists \text{livesIn}(\text{Men})\}$ , and  $\mathcal{N}_{\text{ex}}^2 = \{\neg \exists \text{livesIn}(\text{Men})\}$ . The maximal sound approximation obtained with `AtAlg` is  $(\mathcal{T}_{\text{ex}}^2, \mathcal{N}_{\text{ex}}^2)$ . However, any model  $\mathcal{J}$  from  $\text{Mod}(\mathcal{T}_{\text{ex}}^2, \mathcal{N}_{\text{ex}}^2)$  with  $\text{place}^{\mathcal{J}} = \emptyset$  is not in  $\mathcal{M} = \text{sem}_{\mathbb{G}_{\{1\}}^a}(\mathcal{E}_{\text{ex}}^2)$ . We can rule out such models by introducing a fresh role  $P$  that would ‘enforce’ the existence of an element from  $\Delta$  in the interpretation of `place`. Indeed, consider a KB  $\mathcal{K}^s = (\mathcal{T}^s, \mathcal{A}^s)$ , where  $\mathcal{T}^s = \mathcal{T}_{\text{ex}}^2 \cup \{\exists P^- \sqsubseteq \text{place}\}$  and  $\mathcal{A}^s = \{\exists P(a^*)\}$  with  $P$  and  $a^*$  a fresh role and constant, respectively. Note also that  $\mathcal{K}^s$  is a sound s-approximation of  $\mathcal{M}$  and  $\text{Mod}(\mathcal{K}^s) \subseteq_{\Sigma_{\text{ex}}} \text{Mod}(\mathcal{K})$  holds as no model  $\mathcal{J}$  of  $\mathcal{K}^s$  is such that  $\text{place}^{\mathcal{J}} = \emptyset$ .*

In contrast to `AtAlg`, for a given TT-setting  $\mathcal{E}$ , Algorithm 1 (`TT-SApprox`) provides a maximal sound s-approximation for  $\text{sem}_{\mathbb{S}}(\mathcal{E})$ , where  $\mathbb{S} \in \{\mathbb{G}_{\{1\}}^a, \mathbb{G}_{\{\#\}}^a\}$ . One can follow the steps of the algorithm in Example 15. `TT-SApprox` first computes the maximal sound approximation using `AtAlg` (Line 1). Then, in the spirit of Example 15, the algorithm finds general concepts  $C$  whose interpretations should not be empty in the resulting set of models (Line 8) and ensures that this will not happen via introducing new TBox and ABox assertions with fresh roles and constants (Lines 6 and 9) as in the example. Finally, if an interpretation of  $C$  should contain at least  $n$  constants, then the algorithm ensures that by making disjoint the non-empty ranges of the introduce properties that are ‘pointing’ to  $C$  (Lines 10-11).

## Algorithm 1: TT-SApprox

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**INPUT** : a (DL-Lite<sub>core</sub>, TT)-setting  $\mathcal{E} = (\mathcal{T}, \mathcal{A}, \mathcal{N}, \equiv_{\text{TT}})$   
**OUTPUT**: a DL-Lite<sub>core</sub> KB  $(\mathcal{T}', \mathcal{A}')$

- 1 **set**  $\mathcal{T}' = \mathcal{T}$  **and**  $\mathcal{A}' = \text{AtAlg}(\mathcal{E})$
- 2 **introduce** a fresh constant  $a^*$  not occurring in  $\mathcal{T}, \mathcal{A}$ , nor  $\mathcal{N}$
- 3 **for each** basic role  $R$  over  $\text{pred}(\mathcal{A}, \mathcal{T})$  **do**
- 4     **introduce** a fresh atomic role  $P_R \notin \text{pred}(\mathcal{E})$
- 5     **if**  $\exists R(a) \in \text{c1}_{\mathcal{T}}(\mathcal{A})$  for some  $a$  **then**
- 6         **set**  $\mathcal{A}' = \mathcal{A}' \cup \{\exists P_R(a^*)\}$
- 7     **for each**  $\exists R^- \sqsubseteq C \in \text{c1}(\mathcal{T})$  for some general concept  $C$  **do**
- 8         **if**  $\exists R(a) \in \text{c1}_{\mathcal{T}}(\mathcal{A})$  for some  $a$  **and either**  
            $\neg \exists R(a) \notin \text{c1}_{\mathcal{T}}(\mathcal{N})$  **or there is no constant**  $b$  s.t.  
            $C(b) \in \text{c1}_{\mathcal{T}}(\mathcal{A})$  **and**  $\neg C(b) \in \text{c1}_{\mathcal{T}}(\mathcal{N})$  **then**
- 9             **set**  $\mathcal{T}' = \mathcal{T}' \cup \{\exists P_R^- \sqsubseteq C\}$
- 10 **for each** pair of introduced roles  $P_R$  and  $P_S$  **do**
- 11     **if**  $\exists R^- \sqsubseteq \neg \exists S^-$  is in  $\text{c1}(\mathcal{T})$  **then set**  
            $\mathcal{T}' = \mathcal{T}' \cup \{\exists P_R^- \sqsubseteq \neg \exists P_S^-\}$
- 12 **return**  $(\mathcal{T}', \mathcal{A}' \cup \mathcal{N})$

---

The following theorem shows that `TT-SApprox` efficiently computes maximal sound s-approximations.

**Theorem 16.** *Let  $\mathcal{E} = (\mathcal{T}, \mathcal{A}, \mathcal{N}, \equiv_{\text{TT}})$  be a (DL-Lite<sub>core</sub>, TT)-setting. Then, `TT-SApprox`( $\mathcal{E}$ ) is a maximal sound s-approximation of  $\text{sem}_{\mathbb{S}}$ , where  $\mathbb{S} \in \{\mathbb{G}_{\{1\}}^a, \mathbb{G}_{\{\#\}}^a\}$ . Moreover, `TT-SApprox` runs in time polynomial in  $|\mathcal{T} \cup \mathcal{A} \cup \mathcal{N}|$ .*

A practical benefit of sound s-approximations is that they preserve important queries that may be lost by sound approximations. We will now introduce a class of such queries. An example query from this class is:

*Is it true that a university has at least five (distinct) researchers  $r_1, \dots, r_5$  such that  $r_1, r_2, r_3$  work on the first project,  $r_2, r_3, r_4$  on the second one, and  $r_1, r_5$  on the third one?*

Let  $\Theta(x)$  be a formula recursively defined as follows:

$$\begin{aligned} \Psi(x) &::= A(x) \mid \exists x' R(x, x') \mid \exists x' R(x', x), \\ \Theta(x) &::= \Psi(x) \mid \neg \Psi(x). \end{aligned}$$

Then,  $\mathcal{Q}$  is the class of queries of the following form:

$$\exists x_1, \dots, x_n \left( \bigwedge_{i \neq j} (x_i \neq x_j) \wedge \bigwedge_{k=1}^m \bigwedge_{X_k \subseteq \{1, \dots, n\}} (\Theta_k(x_i)) \right).$$

**Theorem 17.** *Let  $q \in \mathcal{Q}$  and  $\text{sem} = \text{sem}_{\mathbb{G}_y^a}$  be an evolution semantics, where  $y \in \{1, \#\}$ . Then, there exists a TT-setting  $\mathcal{E}_q$  such that  $\text{sem}(\mathcal{E}_q) \models q$  and `TT-SApprox`( $\mathcal{E}_q$ )  $\models q$ , but `AtAlg`( $\mathcal{E}_q$ )  $\not\models q$ .*

## Predicate Trust

Let  $(\mathcal{T}, \mathcal{A})$  be a DL-Lite KB and  $\equiv_{\mathcal{P}} \in \text{PT}$  for some  $\mathcal{P}$ . A natural approach to this case would be to capture  $\mathbb{E}(\equiv_{\mathcal{P}}, \text{Mod}(\mathcal{T}, \mathcal{A}))$  with some theory, thus reducing the problem to the TT case. On the first glance, it would be sufficient just to remove assertions that are over the ‘wrong’ signature from  $\text{c1}_{\mathcal{T}}(\mathcal{A})$ . This, however, is not enough as shown in the following example.

**Algorithm 2: PT-ExtendSAx**


---

**INPUT** : a DL-Lite<sub>core</sub> KB  $(\mathcal{T}, \mathcal{A})$ ,  
a finite set of predicates  $\mathcal{P}$   
**OUTPUT**: a DL-Lite<sub>core</sub> KB  $(\mathcal{T}', \mathcal{A}')$

- 1 **set**  $\mathcal{T}' = \mathcal{T}$  and  $\mathcal{A}' = \emptyset$
- 2 **introduce** a fresh constant  $a^*$  not occurring in  $\mathcal{T}$  nor  $\mathcal{A}$
- 3 **for each**  $\alpha \in \text{cl}_{\mathcal{T}}(\mathcal{A})$  **do**
- 4 | **if** the predicate of  $\alpha$  is in  $\mathcal{P}$  **then set**  $\mathcal{A}' = \mathcal{A}' \cup \{\alpha\}$
- 5 **for each basic role**  $R$  over  $\text{pred}(\mathcal{T}, \mathcal{A})$  **do**
- 6 | **introduce** a fresh atomic role  $P_R \notin \text{pred}(\mathcal{T}, \mathcal{A})$
- 7 | **if**  $\exists R(a) \in \text{cl}_{\mathcal{T}}(\mathcal{A})$  for some constant  $a$  **then**
- 8 | | **set**  $\mathcal{A}' = \mathcal{A}' \cup \{\exists P_R(a^*)\}$
- 9 | | **for each**  $\exists R^- \sqsubseteq C$  in  $\text{cl}(\mathcal{T})$  with  $C \in \mathcal{P}$  **do**
- 10 | | | **set**  $\mathcal{T}' = \mathcal{T}' \cup \{\exists P_R^- \sqsubseteq C\}$
- 11 **for each pair of fresh atomic roles**  $P_R$  and  $P_S$  **do**
- 12 | **if**  $\exists R^- \sqsubseteq \neg \exists S^-$  is in  $\text{cl}(\mathcal{T})$  **then set**  
 $\mathcal{T}' = \mathcal{T}' \cup \{\exists P_R^- \sqsubseteq \neg \exists P_S^-\}$
- 13 **return**  $(\mathcal{T}', \mathcal{A}')$

---

**Example 18.** Consider  $\equiv_{\mathcal{P}^3_{\text{ex}}} \in \text{PT}$ , where  $\mathcal{P}^3_{\text{ex}} = \{\text{person}\}$  and  $(\mathcal{T}^3_{\text{ex}}, \mathcal{A}^3_{\text{ex}})$ , where  $\mathcal{T}^3_{\text{ex}} = \{\exists \text{livesIn}^- \sqsubseteq \neg \text{person}\}$ ,  $\mathcal{A}^3_{\text{ex}} = \{\text{person}(\text{Men}), \exists \text{livesIn}(\text{Men})\}$ . Consider also  $\mathcal{A}' = \{\text{person}(\text{Men})\}$ , a subset of  $\text{cl}_{\mathcal{T}}(\mathcal{A}^3_{\text{ex}})$  consisting of all the membership assertions over  $\mathcal{P}^3_{\text{ex}}$ . One can see that  $\mathcal{I} = \{\text{person}(x) \mid x \in \Delta\}$  is a model of  $(\mathcal{T}^3_{\text{ex}}, \mathcal{A}')$ , while  $\mathcal{I} \notin \mathbb{E}(\equiv_{\mathcal{P}^3_{\text{ex}}}, \text{Mod}(\mathcal{T}^3_{\text{ex}}, \mathcal{A}^3_{\text{ex}}))$ . However, we can still capture  $\mathbb{E}(\equiv_{\mathcal{P}^3_{\text{ex}}}, \text{Mod}(\mathcal{T}^3_{\text{ex}}, \mathcal{A}^3_{\text{ex}}))$  with  $(\mathcal{T}^s, \mathcal{A}^s)$ , where  $\mathcal{T}^s = \mathcal{T}^3_{\text{ex}} \cup \{\exists P^- \sqsubseteq \neg \text{person}\}$  and  $\mathcal{A}^s = \mathcal{A}' \cup \{\exists P(a^*)\}$  for a fresh role  $P$  and constant  $a^*$ . Indeed, one can check that  $\mathbb{E}(\equiv_{\mathcal{P}^3_{\text{ex}}}, \text{Mod}(\mathcal{T}^3_{\text{ex}}, \mathcal{A}^3_{\text{ex}})) = \text{Mod}(\mathcal{T}^s, \mathcal{A}^s)|_{\text{pred}(\mathcal{T}^3_{\text{ex}}, \mathcal{A}^3_{\text{ex}})}$ .

$\mathbb{E}(\equiv_{\mathcal{P}}, \text{Mod}(\mathcal{K}))$  can still be captured using s-axiomatisation. Consider Algorithm 2 (PT-ExtendSAx) that, for a given  $(\mathcal{T}, \mathcal{A})$  and  $\mathcal{P}$ , returns a maximal sound s-approximation of  $\mathbb{E}(\equiv_{\mathcal{P}}, \text{Mod}(\mathcal{T}, \mathcal{A}))$ . One can follow the steps of the algorithm in Example 18: it finds concepts  $C$  whose interpretation should not be empty in each  $\mathcal{I}$  of  $\mathbb{E}(\equiv_{\mathcal{P}}, \text{Mod}(\mathcal{T}, \mathcal{A}))$  (Line 10) and ensures their non-emptiness (Lines 8 and 9). Finally, as in Algorithm 1, PT-ExtendSAx guarantees that the minimal number of constants in  $C$  is as required (Lines 11-12).

**Theorem 19.** Let  $(\mathcal{T}, \mathcal{A})$  be a DL-Lite<sub>core</sub> KB and let  $\equiv_{\mathcal{P}} \in \text{PT}$  for some finite set of predicates  $\mathcal{P}$ . Then PT-ExtendSAx $(\mathcal{T}, \mathcal{A}, \mathcal{P})$  is an s-axiomatisation of  $\mathbb{E}(\equiv_{\mathcal{P}}, \text{Mod}(\mathcal{T}, \mathcal{A}))$ . Moreover, PT-ExtendSAx runs in time polynomial in  $|\mathcal{T} \cup \mathcal{A} \cup \mathcal{P}|$ .

Finally, we will show that PT-ExtendSAx can be used to compute maximal sound s-approximations of  $\text{sem}_{\mathbb{S}}$  for  $\mathbb{S} = \mathbb{L}_{\{j\}}^a$ , that corresponds to a widely accepted Winslett's evolution semantics (De Giacomo et al. 2006; Winslett 1990).

**Theorem 20.** Let  $\mathcal{P}$  be a finite set of predicates and  $\mathcal{E}_{\mathcal{P}} = (\mathcal{T}, \mathcal{A}, \mathcal{N}, \equiv_{\mathcal{P}})$  a (DL-Lite<sub>core</sub>, PT)-setting. Let also  $(\mathcal{T}', \mathcal{N}') = \text{PT-ExtendSAx}((\mathcal{T}, \mathcal{N}), \mathcal{P})$  and  $\mathcal{E}_{\text{TT}} = (\mathcal{T}', \mathcal{A}, \mathcal{N}', \equiv_{\text{TT}})$  be (DL-Lite<sub>core</sub>, TT)-setting. Then, if  $\mathcal{K}$  is a maximal sound s-approximation (resp., s-axiomatisation) of  $\text{sem}_{\mathbb{L}_{\{j\}}^a}(\mathcal{E}_{\text{TT}})$ , then  $\mathcal{K}$  is also a maximal sound s-approximation (resp., s-axiomatisation) of  $\text{sem}_{\mathbb{L}_{\{j\}}^a}(\mathcal{E}_{\mathcal{P}})$ .

**Algorithm 3: AT-ExtendAx**


---

**INPUT** : a DL-Lite KB  $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ ,  
a finite set of membership assertions  $\Phi$   
**OUTPUT**: a DL-Lite KB  $(\mathcal{T}, \mathcal{A}')$

- 1 **set**  $\mathcal{A}' = \emptyset$
- 2 **for each**  $\alpha \in \text{cl}_{\mathcal{T}}(\mathcal{A})$  **do**
- 3 | **if** either  $\alpha$  or  $\neg \alpha$  is in  $\Phi$  **then set**  $\mathcal{A}' = \mathcal{A}' \cup \{\alpha\}$
- 4 **return**  $(\mathcal{T}, \mathcal{A}')$

---

**Assertion Trust**

Let  $(\mathcal{T}, \mathcal{A})$  be a DL-Lite KB and  $\equiv_{\Phi} \in \text{AT}$  for some finite set of membership assertions  $\Phi$ . Firstly, observe that the set of models  $\mathbb{E}(\equiv_{\Phi}, \text{Mod}(\mathcal{T}, \mathcal{A}))$  can be axiomatised and Algorithm 3 (AT-ExtendAx), where  $\neg \alpha = \neg B(a)$  if  $\alpha = B(a)$  and  $\neg \alpha = B(a)$  if  $\alpha = \neg B(a)$  for some basic concept  $B$ , provides this axiomatisation. The algorithm keeps assertions of  $\text{cl}_{\mathcal{T}}(\mathcal{A})$  such that they or their negations are in  $\Phi$ . The following theorem shows the correctness of AT-ExtendAx.

**Theorem 21.** Let  $(\mathcal{T}, \mathcal{A})$  be a DL-Lite KB and  $\equiv_{\Phi} \in \text{AT}$  for some finite set of membership assertions  $\Phi$ . Then,  $\mathbb{E}(\equiv_{\Phi}, \text{Mod}(\mathcal{T}, \mathcal{A})) = \text{Mod}(\text{AT-ExtendAx}(\mathcal{T}, \mathcal{A}, \Phi))$ , and AT-ExtendAx runs in time polynomial in  $|\mathcal{T} \cup \mathcal{A} \cup \Phi|$ .

An immediate consequence of the theorem is that each AT-setting  $\mathcal{E}$  can be transformed into a TT setting  $\mathcal{E}_{\text{TT}}$  with the same evolution result under  $\text{sem}_{\mathbb{S}}$  with any selector  $\mathbb{S}$ .

**Corollary 22.** Let  $\mathbb{S}$  be a selector and  $\Phi$  a finite set of membership assertions. Let  $\mathcal{E}_{\Phi} = (\mathcal{T}, \mathcal{A}, \mathcal{N}, \equiv_{\Phi})$  and  $\mathcal{E}_{\text{TT}} = (\mathcal{T}, \mathcal{A}, \mathcal{N}_{\text{TT}}, \equiv_{\text{TT}})$  be (DL-Lite, AT) and (DL-Lite, TT)-settings, respectively, where  $(\mathcal{T}, \mathcal{N}_{\text{TT}}) = \text{AT-ExtendAx}(\mathcal{T}, \mathcal{N}, \Phi)$ . Then,  $\text{sem}_{\mathbb{S}}(\mathcal{E}_{\Phi}) = \text{sem}_{\mathbb{S}}(\mathcal{E}_{\text{TT}})$ .

**Related Work and Discussions**

**Related Work.** To the best of our knowledge, this is the first work that combines trust and evolution in the context of DLs. The closest research to ours is knowledge management with preferences where either logical formulae or predicates are ordered and thus less preferred elements can be seen as less trusted. However, this rather corresponds to defining levels of importance than trust. Bienvenu, Bourgaux, and Goasdoué (2014) studied inconsistency-tolerant semantics for querying inconsistent KBs. They rely on KB repairs which are subsets of the ABox that are consistent with the TBox, and use various models of preferences to determine the most important repairs. Since they do not select repairs that are of low importance, this can be seen as a trust-based KB repairing. However, their approach is based on formulae and thus closer to so-called *formula-based evolution* (Eiter and Gottlob 1992) rather than to the model-based approach that we study in this paper.

Qi and Du (2009) studied evolution under a modified version of  $\mathbb{G}_{\#}^p$  selector that relies on predicate-based preferences in selecting models of  $\text{Mod}(\mathcal{N})$ . The crucial difference between their and our work is that their evolution result  $\mathcal{M}$  is a subset of  $\text{Mod}(\mathcal{N})$  that consists of the most important models, while in our case we construct  $\mathcal{M}$  by first extending  $\text{Mod}(\mathcal{N})$  according with the model of trust and then choosing

minimally distant elements from this extended set regardless their importance. Note that our approach can be combined with the selector of (Qi and Du 2009), but this requires further investigation.

**Conclusion.** We have formalised the notion of trust and evolution semantics as operators  $\mathbb{E}$  and  $\mathbb{S}$  and have shown how they can be composed to obtain trust-sensitive evolution semantics. This approach can be generalised to any notions of trust and evolution semantics that can be captured via operators. We have applied trust-sensitive MBAs to DL-Lite and have shown that under all of them DL-Lite is not closed. On the one hand, this is expected since DL-Lite has a limited expressive power, and already in the case of classical MBAs can return sets of models  $\mathcal{M}$  whose axiomatisation requires syntactic constructs beyond DL-Lite. On the other hand, this is not self-evident since trust-sensitive MBAs in general return sets of models that are very different from the ones of classical MBAs. Thus, for both classical and trust-sensitive MBAs the main challenge is to find a ‘good’ notion of evolution approximation and to develop (efficient) algorithms to compute ‘optimal’ approximations. We have proposed a novel notion of sound  $s$ -approximation that captures evolution results better than sound approximations previously considered in the evolution context (i.e.,  $s$ -approximations always capture as many and in some cases even more models, recall Example 15), and that preserve queries that may be lost by regular approximations. We have provided polynomial-time algorithms that compute maximal sound  $s$ -approximations for classical and several trust-sensitive MBAs. Our algorithms work either directly on trust-sensitive evolution settings or they can be utilised as subroutines to reduce evolution from trust-sensitive to classical settings. We see these algorithms as a starting point for developing efficient procedures for automated knowledge update and revision. We also believe this work is a timely contribution for the Semantic Web, where applications may depend on third-party information that is only partly trusted.

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