Human-Like Spatial Reasoning Formalisms

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Introduction

My work on the PhD thesis concerns human-like reasoning about relations between spatial objects and the way they change in time. In particular, my research is focused on logic-based reasoning systems that model human spatial reasoning methods and may enable better understanding of humans reasoning mechanisms in future. Importantly, such formalisms are also interested from the practical point of view – they have a number of potential applications, e.g., in robotics, architecture design, databases, among others.

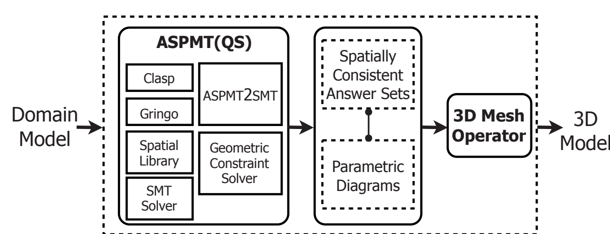
The work I have accomplished so far consists of two parts. The first part amounts to constructing ASPMT(QS) – a general framework for spatial reasoning within the paradigm of Answer Set Programming Modulo Theories. ASPMT(QS) constitutes the only existing spatial reasoning system capable of supporting the key non-monotonic spatial reasoning features (e.g., spatial inertia, ramification) in the context of ASP, and integrating geometrical and qualitative spatial information. As a result, our system enables conterfactual reasoning, explanation and diagnosis, as well as belief revision about spatial objects and their change in time. The later part consists of constructing $\mathsf{HS}^{\Box,@,i}_{horn}$ – a hybrid version of a Horn fragment of the well-known Halpern-Shoham (HS in short) logic which is a logic for reasoning about relations between intervals (Halpern and Shoham 1991). $\mathsf{HS}^{\Box,@,i}_{horn}$ logic provides *referentiality* (see the next part of this abstract), which is a crucial construct in knowledge representation but not available in sub-propositional HS fragments investigated in the literature (Bresolin et al. 2016). My main complexity result is that $\mathsf{HS}^{\Box,@,i}_{horn}$ is decidable (NP-complete) over reflexive, and irreflexive and dense time frames.

Answer Set Programming Modulo Theories with Qualitative Space

This joint work with Mehul Bhatt and Carl Schultz was accomplished during my research scholarship in Germany funded by DAAD and published in the best paper award with narrower logic programming focus paper during LPNMR'15 conference (Wałęga, Bhatt, and Schultz 2015), and in an extended version as a TPLP journal paper (Wałęga, Bhatt, and Schultz 2016). The approach we use in ASPMT(QS) is based on Answer Set Programming Modulo Theories (Bartholomew and Lee 2014) extended to spatial domains. Spatial reasoning is performed in an analytic manner, i.e., relations are encoded as polynomial constraints. The main reasoning task, i.e., determining whether a spatial configuration is consistent, is equivalent to determining whether a particular system of polynomial constraints is satisfiable. The reasoning method uses Satisfiability Modulo Theories (SMT) with real nonlinear arithmetic, and can be performed in a sound and complete manner. Moreover, this approach enables us to express a number of relations from the well-known qualitative approaches, as we have proved in (Wałęga, Bhatt, and Schultz 2016):

Proposition 1 *Each relation of Interval Algebra, Rectangle Algebra, Left-Right Algebra, Region Connection Calculus–5 in the domain of convex polygons with a finite number of vertices and Cardinal Direction Calculus may be defined in ASPMT(QS).*

We have built ASPMT(QS) implementation upon the system ASPMT2SMT (Bartholomew and Lee 2014) – a compiler that translates a tight fragment of ASPMT into SMT instances. Then, SMT solver is used to compute spatially consistent answer sets and finally every (spatially consistent) answer set is used to generate a qualitatively distinct, 3D model as presented in the following pipeline graph.



We have presented a number of possible applications of ASPMT(QS), e.g., in architecture design, spatio-temporal reasoning, abductive reasoning, and people tracking. A minimal prototypical implementation of ASPMT(QS) is available online publicly from Docker Hub: https://hub.docker.com/r/spatialreasoning/aspmtqs/. It contains the core system, minimal working examples, short description and installation instructions.

Hybridization of Halpern-Shoham Logic

Another part of accomplished work was done independently by myself and recently submitted to the 7th Indian Conference on Logic and its Applications (ICLA). The work consists of extending expressive power of Halpern-Shoham logic fragments. HS is an elegant interval-based temporal logic that introduces one modal operator for each of the well-known Allen relations (Allen 1983). The Allen relations form a jointly exhaustive and pairwise disjoint set of binary relations between nonidentical intervals, namely: *begins*, *during*, *ends*, *overlaps*, *adjacent to*, *later than*, and their inverses. HS is highly expressive, in particular it is strictly more expressive than any point-based temporal logic over linear orders of underlying time structure. On the other hand, HS is undecidable for the most interesting linear orders including $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$ (Halpern and Shoham 1991). One, recently studied in literature approach for reducing the complexity of HS is to investigate sub-propositional languages such as *Horn* and *core* fragments (Bresolin et al. 2016). However, logics obtained in this way are not *referential* any more, where by referentiality, we mean a possibility to label intervals and then to refer to a chosen interval with a concrete label. This kind of reference is possible in full HS and is a crucial construct in temporal knowledge representation (Areces, Blackburn, and Marx 2000). The most straightforward way to restore the referentiality in HS fragments is to hybridize them, i.e., to add the second sort of expressions to the language (the so-called nominals), i.e., primitive formulas each of which is true at exactly one interval, and satisfaction operators indexed by nominals that enable to access a particular interval denoted by this nominal. Surprisingly, although hybridization of interval temporal logics was already recognised as a promising line of research (Blackburn 2000), it has received only limited attention from the research community.

A particulary interesting sub-propositional fragment of HS is a Horn fragment that allows only box modalities (diamond modalities are forbidden) called $\mathsf{HS}^{\Box}_{horn}$. $\mathsf{HS}^{\Box}_{horn}$ is known to be tractable (P-complete) if the underlying structure of time is reflexive, or irreflexive and dense (Bresolin et al. 2016). On the other hand, this logic is still expressive enough to be applied, e.g., to ontology-based data access over temporal databases. Since $\mathsf{HS}^{\Box}_{horn}$ maintains a good balance between computational complexity and expressive power, it has recently gained attention among researchers working on theoretical, as well as practical aspects of HS.

My work consists of hybridizing $\mathsf{HS}^{\Box}_{horn}$ and studying computational complexity of the obtained logic (which I denote by $\mathsf{HS}^{\Box,@,i}_{horn}$). My main result is that over reflexive, or irreflexive and dense underlying time structures the hybridization of $\mathsf{HS}^{\Box}_{horn}$ results in an NP-complete logic – recall that $\mathsf{HS}^{\Box}_{horn}$ is P-complete over such structures (in contrast to classical modal logic which is PSPACE-complete before and after hybridization). Hence, adding referentiality to $\mathsf{HS}^{\Box}_{horn}$ enables us to maintain decidability but it has a price of reaching NP-completeness, i.e., losing tractability of the logic (if $\mathrm{P} \neq \mathrm{NP}$). The cumulative results on computational complexity of Horn fragments of HS depending on the type of a time frame are presented in the following table – my contributions are denoted by ($\star$).

	Irreflexive	Reflexive
Discrete	$\mathsf{HS}^{\Box}_{horn}$: undecidable	$\mathsf{HS}^{\Box}_{horn}$: P-compl.
Discrete	$\mathsf{HS}^{\Box,@,i}_{horn}$: undecidable	$\mathsf{HS}^{\Box,@,i}_{horn}$: NP-compl. ($\star$)
Dense	$\mathsf{HS}^{\Box}_{horn}$: P-compl.	$\mathsf{HS}^{\Box}_{horn}$: P-compl.
Dense	$\mathsf{HS}^{\Box,@,i}_{horn}$: NP-compl. ($\star$)	$\mathsf{HS}^{\Box,@,i}_{horn}$: NP-compl. ($\star$)

Future Work

My future work plans are to apply the system ASPMT(QS) to a wide range of dynamic domains such as cognitive robotics, computer-aided architecture design, and geographic information systems. In case of my research on interval logics, it seems that hybridization of sub-propositional fragments of HS is a promising line of research and may provide expressive and decidable referential interval logics. Therefore, as a future work I plan to hybridize other fragments (e.g., core fragments) of HS, study their computational complexity, expressive power, and potential areas of application.

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References

Allen, J. F. 1983. Maintaining knowledge about temporal intervals. *Communications of the ACM* 26(11):832–843.

Areces, C.; Blackburn, P.; and Marx, M. 2000. The computational complexity of hybrid temporal logics. *Logic Journal of IGPL* 8(5):653–679.

Bartholomew, M., and Lee, J. 2014. System aspmt2smt: Computing ASPMT Theories by SMT Solvers. In *Logics in Artificial Intelligence*. Springer. 529–542.

Blackburn, P. 2000. Representation, reasoning, and relational structures: a hybrid logic manifesto. *Logic Journal of the IGPL* 8(3).

Bresolin, D.; Kurucz, A.; Muñoz-Velasco, E.; Ryzhikov, V.; Sciavicco, G.; and Zakharyaschev, M. 2016. Horn fragments of the halpern-shoham interval temporal logic. *arXiv preprint arXiv:1604.03515*.

Halpern, J. Y., and Shoham, Y. 1991. A propositional modal logic of time intervals. *Journal of the ACM (JACM)* 38(4):935–962.

Wałęga, P. A.; Bhatt, M.; and Schultz, C. 2015. ASPMT (QS): non-monotonic spatial reasoning with answer set programming modulo theories. In *International Conference on Logic Programming and Nonmonotonic Reasoning*, 488–501.

Wałęga, P. A.; Bhatt, M.; and Schultz, C. 2016. Non-monotonic spatial reasoning with answer set programming modulo theories. *Theory and Practice of Logic Programming* 1–21.