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Qualitative Spatio-Temporal Stream Reasoning with Unobservable Intertemporal Spatial Relations Using Landmarks

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Abstract

Qualitative spatio-temporal reasoning is an active research area in Artificial Intelligence. In many situations there is a need to reason about intertemporal qualitative spatial relations, i.e. qualitative relations between spatial regions at different time-points. However, these relations can never be explicitly observed since they are between regions at different time-points. In applications where the qualitative spatial relations are partly acquired by for example a robotic system it is therefore necessary to infer these relations. This problem has, to the best of our knowledge, not been explicitly studied before. The contribution presented in this paper is two-fold. First, we present a spatio-temporal logic MSTL, which allows for spatio-temporal stream reasoning. Second, we define the concept of a landmark as a region that does not change between time-points and use these landmarks to infer qualitative spatio-temporal relations between non-landmark regions at different time-points. The qualitative spatial reasoning is done in RCC-8, but the approach is general and can be applied to any similar qualitative spatial formalism.

Introduction

In many situations there is a need to reason about the spatial relations between entities at different time-points. For example, 'to take someone's place' implies a specific change in spatial relations for two objects across two time-points. Qualitative representations are especially useful in situations where no quantitative representation of spatial regions exist, for instance because they were provided by a person, or because the regions represent abstract entities for which no exact spatial information is available. A qualitative representation provides a more abstract representation which reduces the complexity of the reasoning by focusing on the salient aspects. It also handles some forms of uncertainty by considering equivalence classes rather than values, and it provides a natural human-computer interface as people often think and communicate in terms of qualitative representations.

With the amount of data that is continuously produced, AI applications such as robotic systems are often tasked with handling incrementally available information. These data flows are commonly modeled as streams, and the reasoning over such streams of information is called *stream reasoning*.

Formalisms such as Metric Temporal Logic (MTL) (Koymans 1990) in combination with techniques such as progression can be used to apply logic-based temporal stream reasoning, and formalisms such as $\mathcal{ST}_0$ (Wolter and Zakharyaschev 2000) extend temporal formalisms with qualitative spatial expressions. One problem with reasoning about qualitative spatial relations between different time-points is that these relations in many cases (such as in robotic systems) cannot be observed, and must therefore be inferred.

The motivation for our work is to propose and evaluate empirically a novel stream reasoning applicable solution to the problem of reasoning over unobserved intertemporal spatial relations. Our suggested solution makes use of *landmarks*. We are particulary interested in the effectiveness of using landmarks for these reasoning purposes, and how well our solution scales. In this paper we define the concept of a landmark as a region that does not change *between* time-points and use these landmarks to infer qualitative spatio-temporal relations between spatial entities at different time-points. Landmarks provide a kind of 'anchor' or frame of reference in relation to which other spatial objects are observed to change over time. We make use of the well-known Region Connection Calculus RCC-8 (Randell, Cui, and Cohn 1992) to represent qualitative spatial relations between regions, but our approach is general in the sense that it can be applied to other qualitative spatial formalisms that are based on transitive spatial relations. Our work complements other temporalisations of RCC-8, including $\mathcal{ST}_1$ (Wolter and Zakharyaschev 2000).

Related Work

The temporal logic MTL captures reasoning over time using temporal operators such as 'until' and 'since', from which temporal operators $\mathcal{G}$ and $\mathcal{F}$ for 'it is always going to be the case' and 'at least once in the future' can be constructed. It does not attempt to handle spatial reasoning. MTL has been used in previous working solutions to temporal stream reasoning (Doherty, Kvarnström, and Heintz 2009).

Qualitative spatio-temporal reasoning is concerned with reasoning over time and space, in particular reasoning about spatial change (Cohn and Renz 2008). Several qualitative spatio-temporal reasoning formalisms have been created by combining a spatial formalism with a temporal one. Examples are STCC (Gerevini and Nebel 2002) and ARCC-

8 (Bennett et al. 2002) which both combine the Region Connection Calculus RCC-8 with Allen's Interval Algebra (Allen 1983). RCC-8 provides and formalisation for topological reasoning over abstract regions based on their spatial relations. The reasoning is qualitative and uses a subset of RCC, which builds up a range of spatial relations starting from the 'connected' relation. Using composition-table based reasoning in RCC-8 (Cui, Cohn, and Randell 1993), new spatial relations can be inferred from incomplete spatial knowledge.

$\mathcal{ST}_0$ represents a language for reasoning over spatio-temporal representations and offers a temporalisation of RCC-8 using temporal operators similar to MTL. It makes use of the temporal operators 'it will always be the case' $\Box$, 'at some point in the future' $\Diamond$, and 'at the next time-point' $\bigcirc$. Its extension $\mathcal{ST}_1$ introduces spatio-temporal representations for spatial relations between two time-points through the 'next' operator, but does not attempt to provide reasoning techniques that handle instantaneous observations as is the aim of this paper.

Previous work (Heintz and de Leng 2014) focused on integrating qualitative spatial reasoning using MTL in combination with RCC-8. The temporal operators were extended to allow for optionally time-bounded versions $\Box_{[t_0,t_1]}$ and $\Diamond_{[t_0,t_1]}$ respectively. The approach was to subdivide regions into static and dynamic regions, where the relations between static regions could be precomputed for performance gains. However, these relations were limited to single time-points.

Landmarks have previously been used for qualitative spatial reasoning (Liu et al. 2011; Li, Liu, and Wang 2013) to refer to known entities or reference objects within single time-points. They are used as reference objects for formulating constraints, and are considered to be known entities or constants from which constraints are formed to unknown entities or variables respectively. In this paper the term 'landmark' is used to similar effect, i.e. referring to a known entity, with the difference being that what is known is the entity's intertemporal relations to itself.

For our empirical evaluations in this paper we make use of and extend the scenario generation techniques presented by (Renz and Nebel 2001). We have extended the Generic Qualitative Reasoner (Gantner, Westphal, and Wölfl 2008) to support the 'next' operator between any two time-points. GQR can be used to compute the algebraic closure given a constraint satisfaction problem (CSP) composed of a set of qualitative (spatial) relations.

MSTL for Spatio-Temporal Objects

To make statements about the spatial and temporal nature of objects, we introduce a hybrid logic called *Metric Spatio-Temporal Logic* (MSTL), which combines elements from MTL and RCC-8. MTL provides the ability to reason over objects in time, but does not include a spatial formalism. We extend these languages by considering temporal objects that are spatial in nature. MSTL is thus similar to $\mathcal{ST}_1$, which seeks to temporalise RCC-8, but thereby restricts its language to spatial relations. Because MSTL is in part based on MTL, statements in MSTL can contain both spatial relations and predicates.

Syntax of MSTL

Spatial relations are of the form $R(r_1, r_2)$ where R is any of $\left\{\mathsf{EC}, \mathsf{EQ}, \mathsf{DC}, \mathsf{PO}, \mathsf{NTTP}, \mathsf{TPP}, \mathsf{NTTP}^{-1}, \mathsf{TPP}^{-1}\right\}$ and r_1, r_2 are spatial objects, also referred to as regions. We call this set $\mathcal{R}_8$ for brevity to indicate that its elements correspond to the RCC-8 relations 'externally connected', 'equals', 'disconnected', 'non-tangential proper part', 'tangential proper part', 'inverse non-tangential proper part' and 'inverse tangantial proper part' respectively. Given n-ary predicate P, binary spatial relation $\mathcal{R}_8$, and variable or constant terms $\tau_1, \ldots, \tau_n$, the following statements are well-formed formulas (wffs) in MSTL:

$$\mathcal{R}_8(\tau_1, \tau_2) \mid P(\tau_1, \ldots, \tau_n) \mid \tau_1 = \tau_2 \mid \tau_1 \neq \tau_2$$

By recursion, for wffs ϕ and ψ and variable x the following statements are also wffs in MSTL:

$$\neg\phi \mid \phi \vee \psi \mid \phi \wedge \psi \mid \phi \rightarrow \psi \mid \forall x[\phi] \mid \exists x[\phi]$$

Finally, temporal notations are also defined by recursion for wff ϕ and integers $n_1, n_2 \in \mathbb{N}$:

$$\bigcirc\phi \mid \Box_{[n_1,n_2]}\phi \mid \Box\phi \mid \Diamond_{[n_1,n_2]}\phi \mid \Diamond\phi$$

The syntax allows us to make complex spatio-temporal statements. Take for example the following statement, where informally $\Box$ means 'it will always be the case', $\Diamond$ means 'at some point in the future', and $\bigcirc$ means 'at the next time-point'. The spatial relation PO is contained in $\mathcal{R}_8$ and stands for 'partially overlapping'.

$$\forall c_1[\forall c_2[c_1 \neq c_2 \wedge Car(c_1) \wedge Car(c_2) \rightarrow (\Box(\mathsf{PO}(\bigcirc c_1, c_2) \wedge Speeding(c_1) \rightarrow \Diamond\mathsf{PO}(c_1, c_2)))]]$$

This wff has the intended meaning 'it is always the case that if a car is speeding and tails another car, they will eventually collide'.

Semantics of MSTL

Because we are interested in statements over space and time, we make use of *spatio-temporal models* for MSTL.

Definition 1 (Spatio-Temporal Model)**.** *A* spatio-temporal model *is a tuple of the form $\mathcal{M} = \langle T, <, U, \mathcal{D}, I, \alpha\rangle$, where T represents a set of time-points, $<$ represents an ordering over T, U represents the non-empty universe of the space as a set of points, and $\mathcal{D} = \langle \mathcal{P}, \mathcal{R}\rangle$ represents the domain consisting of predicates $\mathcal{P}$ and spatial objects $\mathcal{R}$. An interpretation $I^t \in I$ maps predicates and constant terms to $\mathcal{P}$ and $\mathcal{R}$ respectively for every time-point T. For constant terms this mapping will be the same for all t, but for predicates this is not necessarily the case. A spatial assignment function α associates at every time-point in T every spatial object label in $\mathcal{R}$ to a subset of U.*

From this definition it is clear that we are only considering objects that have some spatial properties associated with them, expressed in the form of spatial relations. Spatial objects therefore are also commonly called *regions* when we only focus on temporal and spatial properties. Alternatively, one could consider a class hierarchy over objects such that regions are a subclass of objects, but this is left for future work and does not impact the focus of this paper.

Definition 2 (Truth)**.** *The MSTL statement that a spatio-temporal formula ϕ holds in $\mathcal{M} = \langle T, <, U, \mathcal{D}, I, \alpha \rangle$ at time-point $t \in T$ is defined recursively.*

$$\mathcal{M}, t \models P(\tau_1, \ldots, \tau_n) \text{ iff } \left\langle I^t(\tau_1), \ldots, I^t(\tau_n) \right\rangle \in I^t(P)$$
$$\mathcal{M}, t \models \forall x[\phi] \text{ iff } \forall r \in \mathcal{R} : \mathcal{M}, t \models \phi[x/r]$$
$$\mathcal{M}, t \models \exists x[\phi] \text{ iff } \exists r \in \mathcal{R} : \mathcal{M}, t \models \phi[x/r]$$
$$\mathcal{M}, t \models \neg\phi \text{ iff } \mathcal{M}, t \not\models \phi$$
$$\mathcal{M}, t \models \phi \vee \psi \text{ iff } \mathcal{M}, t \models \phi \text{ or } \mathcal{M}, t \models \psi$$
$$\mathcal{M}, t \models \phi \wedge \psi \text{ iff } \mathcal{M}, t \models \phi \text{ and } \mathcal{M}, t \models \psi$$
$$\mathcal{M}, t \models \phi \rightarrow \psi \text{ iff } \mathcal{M}, t \not\models \phi \text{ or } \mathcal{M}, t \models \psi$$
$$\mathcal{M}, t \models \Box_{[t_1,t_2]}\phi \text{ iff } \forall t_1 \leq t' \leq t_2 : \mathcal{M}, t' \models \phi$$
$$\mathcal{M}, t \models \Box\phi \text{ iff } \mathcal{M}, t \models \Box_{[0,\infty]}\phi$$
$$\mathcal{M}, t \models \Diamond_{[t_1,t_2]}\phi \text{ iff } \exists t_1 \leq t' \leq t_2 : \mathcal{M}, t' \models \phi$$
$$\mathcal{M}, t \models \Diamond\phi \text{ iff } \mathcal{M}, t \models \Diamond_{[0,\infty]}\phi$$
$$\mathcal{M}, t \models \bigcirc\phi \text{ iff } \mathcal{M}, suc(t) \models \phi$$
$$\mathcal{M}, t \models \mathsf{C}(r_1, r_2) \text{ iff } \alpha(r_1, t) \cap \alpha(r_2, t) \neq \emptyset$$

From the RCC 'connected' spatial relation C, the usual semantics of all RCC-8 relations can be recursively defined, but here they are left out for the sake of brevity.

The semantics for the 'next' operator $\bigcirc$ uses a function suc that maps a time-point to its successor time-point. In the case of the commonly used $T = \mathbb{N}$, we for example have $suc(t) = t + 1$ for all time-points $t \in T$. In the general case, we use for $t, t', t'' \in T$:

$$suc(t) = t' \text{ iff } t < t' \wedge \neg\exists t'' : [t < t'' < t'].$$

A powerful extension is to allow for the 'next' operator to be invoked over region variables. In supporting this, we can refer to a particular region at the next time-point, or by recursion any future time-point.

Definition 3 (Next over Regions)**.** *A region term is a spatial object in $\mathcal{R}$, a variable x, or 'next' applied to a region term. The semantics is defined by extending α with $\alpha(\bigcirc r, t) = \alpha(r, suc(t))$ for any time-point $t \in T$.*

Representing Spatial Relations

While the 'next' operator allows for powerful representations, it complicates evaluation of those statements when we consider observations of the world to occur within rather than across time-points. Spatial relations for regions can be partially observed at time-point t and at time-point $suc(t)$ independently, but no observations can be made with regards to the spatial relations between regions at time-point t and regions at time-point $suc(t)$. To better illustrate how these concepts relate, we introduce the spatial relation matrix.

Definition 4 (Spatial Relation Matrix)**.** *A* spatial relation matrix *is an $n \times n$ matrix M^t for time-point $t \in T$ where n denotes the total number of region variables $|\mathcal{R}|$. For every matrix element $M^t_{i,j}$ and region variables $r_i, r_j \in \mathcal{R}$ we have $M^t_{i,j} = (r_i R r_j)$ such that $R \subseteq \mathcal{R}_8$ and $R \neq \emptyset$. The semantics of M^t are then as follows.*

$$M^t_{i,j} = (r_i R r_j) \text{ iff } \mathcal{M}, t \models \bigvee_{R_k \in R} R_k(r_i, r_j)$$

The spatial relation matrix allows us to intuitively represent spatial facts about regions and corresponds to a complete RCC-8 network. Such matrices are also expected as input to qualitative reasoners such as GQR. The main diagonal always consists of the singleton $\{\mathsf{EQ}\}$. Further, the matrix is semi-symmetric; symmetry holds for all relations except for NTTP and TPP, which have inverses NTTP^{-1} and TPP^{-1} respectively. Existing general solvers for qualitative CSPs such as GQR can be used to determine the algebraic closure of spatial relation matrices, i.e. given spatial relation matrix M^t, the algebraic closure $AC(M^t)$ yields a spatial relation matrix N^t such that for every corresponding set of spatial relations $N^t_{i,j} \subseteq M^t_{i,j} \subseteq \mathcal{R}_8$. A small example of a spatial relation matrix for regions r_1, r_2, r_3 at time-point t with partial knowledge is shown below.

$$M^t = \begin{bmatrix} \{\mathsf{EQ}\} & \{\mathsf{NTTP}^{-1}\} & \{\mathsf{PO}, \mathsf{EC}\} \\ \{\mathsf{NTTP}\} & \{\mathsf{EQ}\} & \{\mathsf{DC}\} \\ \{\mathsf{PO}, \mathsf{EC}\} & \{\mathsf{DC}\} & \{\mathsf{EQ}\} \end{bmatrix}$$

Region r_2 is inside of region r_1 but disconnected from region r_3, and region r_1 is partially overlapping or externally connected with region r_3.

A spatial relation matrix can be extended to describe relations between multiple time-points. This is a useful property because it allows us to describe relations between regions at different time-points that are not necessarily consecutive.

Definition 5 (Intertemporal Spatial Relation Matrix)**.** *An* intertemporal spatial relation matrix *M^{t_1,t_2} is a spatial relation matrix describing the spatial relations between regions $r_i, r_j \in \mathcal{R}$ such that we relate r_i at time-point t_1 to r_j at time-point t_2, i.e. relating $\alpha(r_i, t_1)$ to $\alpha(r_j, t_2)$.*

A spatial relation matrix M^t from Definition 4 is then equivalent to an intertemporal spatial relation matrix $M^{t,t}$. Intertemporal spatial relations can thus be represented by an intertemporal spatial relation matrix. For the 'next' operator, this would for example be $M^{t,suc(t)}$. However, we assume that these relations are unobservable and must somehow be inferred from our observations at time-points t and $suc(t)$, represented by M^t and $M^{suc(t)}$.

By combining the four different combinations for intertemporal spatial relation matrices over two time-points t_1 and t_2, we can consisely describe in one matrix the relations between regions at single time-points as well as the relations between those regions at different time-points. This corresponds to an RCC-8 network in which every region is contained twice, i.e. once for every time-point.

Definition 6 (Extended Spatial Relation Matrix)**.** *An* extended spatial relation matrix *$M^{t_1 \cup t_2}$ for $t_1 < t_2$ combines four intertemporal spatial relation matrices as follows:*

$$M^{t_1 \cup t_2} = \begin{bmatrix} M^{t_1,t_1} & M^{t_1,t_2} \\ M^{t_2,t_1} & M^{t_2,t_2} \end{bmatrix}$$

In general, spatial relation matrices can be used to represent uncertainty for spatial relations between regions by using non-singleton sets. This is important because often we can not deduce that a single relation must hold. We can use extended spatial relation matrices to talk about the spatial relations both within individual time-points and between time-points. This makes them a suitable representation tool for intertemporal RCC-8 networks when considering the problem of deducing unobservable intertemporal relations.

Intertemporal Landmarks

Reasoning alone does not allow us to say anything about intertemporal relations, represented by M^{t_1,t_2} and M^{t_2,t_1} in extended spatial relation matrices. These relations cannot be observed, nor can they be inferred from individual time-points. Concretely, observations are limited to M^{t_1,t_1} and M^{t_2,t_2}. This may seem counter-intuitive, but this is because humans often assume a *frame of reference* when observing spatial changes over time. One way around this problem is therefore to make assumptions about some or all intertemporal relations represented by M^{t_1,t_2} and M^{t_2,t_1} in order to establish such a frame of reference. However, bad assumptions can lead to inconsistencies, so special care must be taken. We make use of *landmark* assumptions where some spatial entities are assumed to be non-changing.

Definition 7 (Landmark)**.** *A* landmark *given a set of region variables $\mathcal{R}$ over any two time-points $t, suc(t)$ is a region variable $r \in \mathcal{R}$ that is* rigid *between t and $suc(t)$, i.e.* $\mathsf{EQ}(r, \bigcirc r)$. *The set of landmarks is indicated by $\mathcal{LM} \subseteq \mathcal{R}$ such that $r \in \mathcal{LM}$ implies that landmark r is rigid.*

Example landmark candidates are e.g. buildings, lakes, monuments, trees, roads. These physical entities are unlikely to change during the run-time of a system, and therefore provide a reasonable frame of reference. An immediate effect of landmarks being rigid is that their relations to other landmark regions remain unchanged. Effectively the set of landmarks $\mathcal{LM}$ provides a possible frame of reference with respect to which relations may change over time. Since this affects the truth semantics of statements in MSTL, we introduce a landmark extension to the spatio-temporal model to capture this.

Definition 8 (Landmark-Based Spatio-Temporal Model)**.** *A* landmark-based spatio-temporal model *is a spatio-temporal model $\mathcal{M}_{\mathcal{LM}} = \langle T, <, U, \mathcal{D}, I, \alpha\rangle$ and $\mathcal{LM} \subseteq \mathcal{R}$ represents the landmark set. $\mathcal{M}_{\mathcal{LM}}$ then restricts α such that for all time-points $t \in T$ and all landmark regions $r \in \mathcal{LM}$ it is the case that $\alpha(r,t) = \alpha(r, suc(t))$.*

Landmarks may introduce inconsistencies if we make observations that conflict with the landmark-imposed restriction of α. To illustrate how this might happen, consider an example where at time-point t we make the observation $\mathsf{PO}(r_1, r_2)$, and at time-point $suc(t)$ we make the observation $\mathsf{DC}(r_1, r_2)$. If we only consider the individual time-points, there is no problem. The following extended spatial relation matrix illustrates our ignorance of the intertemporal spatial relations M^{t_1,t_2} and M^{t_2,t_1}.

$$M^{t_1 \cup t_2} = \begin{bmatrix} \{EQ\} & \{PO\} & \mathcal{R}_8 & \mathcal{R}_8 \\ \{PO\} & \{EQ\} & \mathcal{R}_8 & \mathcal{R}_8 \\ \mathcal{R}_8 & \mathcal{R}_8 & \{EQ\} & \{DC\} \\ \mathcal{R}_8 & \mathcal{R}_8 & \{DC\} & \{EQ\} \end{bmatrix}$$

However, if we use landmarks, the choice of $\mathcal{LM}$ results in an assumption about some intertemporal relations. Choosing $\mathcal{LM} = \{r_1, r_2\}$ is inconsistent, because it implies that regions r_1 and r_2 need to be partially overlapping and disconnected at the same time, which is a contradiction. Instead picking $\mathcal{LM} = \{r_1\}$ is consistent, and one could imagine region r_2 'moving away from' region r_1. Naturally, the converse holds as well if we pick region r_2 as our frame of reference.

We can show that consistency is guaranteed if only one landmark is chosen, and the above example shows that this does not always hold for the case of $|\mathcal{LM}| \geq 2$. Picking a single landmark corresponds to the case of adding a single connection between two disconnected RCC-8 networks for different time-points. The issue of choosing more than one landmark while retaining consistency is a difficult problem, and is closely related to the Amalgamation Property (Li et al. 2008), as well as the Patchwork Property (Lutz and Miličić 2007; Huang 2012). In the remainder of this paper we will therefore assume that the chosen set of landmarks is always consistent. This corresponds to the assumption that our chosen frame of reference is consistent. Under this assumption, if we run into any inconsistencies, our observations are therefore assumed to have been incorrect.

To further illustrate the impact of the choice of $\mathcal{LM}$, consider again the scenario above and suppose we wish to evaluate the formula $\Box\mathsf{EQ}(r_1, \bigcirc r_1)$ at time-point t. Choosing $\mathcal{LM} = \{r_1\}$ means this formula will evaluate to True, i.e. $\mathcal{M}_{\{r_1\}}, t \models \Box\mathsf{EQ}(r_1, \bigcirc r_1)$. Choosing $\mathcal{LM} = \{r_2\}$ means this formula will evaluate to False, i.e. $\mathcal{M}_{\{r_2\}}, t \not\models \Box\mathsf{EQ}(r_1, \bigcirc r_1)$. Choosing any other consistent $\mathcal{LM}$ we can only conclude $\mathcal{M}_{\mathcal{LM}}, t \models \Box\mathsf{EQ}(r_1, \bigcirc r_1) \vee \neg(\Box\mathsf{EQ}(r_1, \bigcirc r_1))$; we cannot say for certain which one is true. This is specifically caused by the choice of landmark in combination with the observations at the two time-points. The following two statements then hold for the same two observations described earlier:

$$\mathcal{M}_{\{r_1\}}, t \models \Box\mathsf{EQ}(r_1, \bigcirc r_1) \wedge \neg\Box\mathsf{EQ}(r_2, \bigcirc r_2)$$
$$\mathcal{M}_{\{r_2\}}, t \models \Box\mathsf{EQ}(r_2, \bigcirc r_2) \wedge \neg\Box\mathsf{EQ}(r_1, \bigcirc r_1)$$

This clearly shows how landmark choice shapes the frame of reference within which MSTL statements may hold.

Progression of MSTL Statements

In the context of stream reasoning, information is assumed to become incrementally available. Progression is a technique for evaluating temporal logic formulas where we try to determine the truth value of the formula based on the information received thus far. This makes it possible to sometimes determine the truth value for an MSTL formula without having to wait for the entire stream to arrive. The result of progressing a formula through the first state in a sequence is a new formula that holds in the remainder of the state sequence iff the original formula holds in the complete state sequence. If progression returns true (false), the entire formula must be true (false), regardless of future states. The complexity of progression is linear in the size of the formula, but the resulting formula may double in size. This may result in exponentially long formulas in the worst case, but by introducing intervals for temporal operators, the worst-case length can be limited.

Progression of Intratemporal Relations

By combining temporal with spatial reasoning, we effectively need both temporal and spatial evaluation methods. Progression is used to handle temporal aspects across time-points, and has previously been used to evaluate MTL formulas (Doherty, Kvarnström, and Heintz 2009). For every step in the progression, spatial reasoning is performed within that step by using for example GQR. This however does not include spatial reasoning between different time-points, as is the focus of this paper. Therefore, progression needs to be extended to handle intertemporal relations that are the result of the 'next' operator in MSTL. This gives rise to additional rewriting rules based on occurrences of the 'next' operator.

Progressing the 'next' operator when it occurs in front of wffs in MSTL corresponds to rewriting that formula by removing the operator, i.e. during progression $\bigcirc\phi$ is rewritten to ϕ for wff ϕ. The following proofs show equivalences for occurrence of 'next' excluding intertemporal relations, and make use of the semantics presented in Definitions 2 and 3.

Theorem 1 (Next and Negation)**.**

$$\models \forall x[\forall y[\neg \bigcirc R(x,y) \leftrightarrow \bigcirc\neg R(x,y)]]$$

Proof. Decomposing bi-implication into cases:

($\Rightarrow$) Assume $\mathcal{M}, t \models \neg \bigcirc R(x,y)$ holds for some arbitrary $\mathcal{M}$ and t. From the semantics of negation this means $\mathcal{M}, t \not\models \bigcirc R(x,y)$. According to the semantics of $\bigcirc$, this is equivalent to $\mathcal{M}, suc(t) \not\models R(x,y)$, thus $\mathcal{M}, suc(t) \models \neg R(x,y)$. Reintroducing $\bigcirc$ then yields $\mathcal{M}, t \models \bigcirc\neg R(x,y)$.

($\Leftarrow$) Analogous to the above in reverse order. □

Theorem 2 (Next and Always)**.**

$$\models \forall x[\forall y[\Box_{[t_1,t_2]} \bigcirc R(x,y) \leftrightarrow \Box_{[suc(t_1),suc(t_2)]} R(x,y)]]$$

Proof. Decomposing bi-implication into cases:

($\Rightarrow$) Assume $\mathcal{M}, t \models \Box_{[t_1,t_2]} \bigcirc R(x,y)$ holds for some arbitrary $\mathcal{M}$ and t. From the semantics of $\Box$, this means $\forall t_1 \leq t' \leq t_2 : \mathcal{M}, t' \models \bigcirc R(x,y)$ holds. By definition of $\bigcirc$, for every t' we get $\mathcal{M}, suc(t') \models R(x,y)$. Reintroducing the universal quantifier, we get $\forall suc(t_1) \leq t' \leq suc(t_2) : \mathcal{M}, t' \models R(x,y)$. Reintroducing $\Box$, this yields $\mathcal{M}, t' \models \Box_{[suc(t_1),suc(t_2)]} R(x,y)$.

($\Leftarrow$) Analogous to the above in reverse order. □

Theorem 3 (Next and Eventually)**.**

$$\models \forall x[\forall y[\Diamond_{[t_1,t_2]} \bigcirc R(x,y) \leftrightarrow \Diamond_{[suc(t_1),suc(t_2)]} R(x,y)]]$$

Proof. Analogous to the proof of Theorem 2, replacing symbols $\forall$ and $\Box$ by $\exists$ and $\Diamond$ respectively. □

Progression of Intertemporal Relations

The 'next' operator can also occur inside intertemporal relations $R(x, \bigcirc y)$. In this case, it is not possible to evaluate $R(x, \bigcirc y)$ at the current time-point, because the relation depends on a future state of y. To work around this problem, we make use of the 'previous' operator $\bigcirc^-$, which is the inverse of the 'next' operator and which follows trivially from Definition 3. The following proofs show equivalences for 'next' involving intertemporal relations, and make use of the 'previous' operator.

Theorem 4 (Extract Next)**.**

$$\models \forall x[\forall y[\bigcirc R(x,y) \leftrightarrow R(\bigcirc x, \bigcirc y)]]$$

Proof. Decomposing bi-implication into cases:

($\Rightarrow$) Assume $\mathcal{M}, t \models \bigcirc R(x,y)$ holds for some arbitrary $\mathcal{M}$ and t. From the semantics of $\bigcirc$, this means $\mathcal{M}, suc(t) \models R(x,y)$. Further, we have $\alpha(z, suc(t)) = \alpha(\bigcirc z, t)$ for any region z, so we get $\mathcal{M}, t \models R(\bigcirc x, \bigcirc y)$.

($\Leftarrow$) Analogous to the above in reverse order. □

Theorem 5 (Partially Extract Next)**.**

$$\models \forall x[\forall y[R(x, \bigcirc y) \leftrightarrow \bigcirc R(\bigcirc^- x, y)]]$$

Proof. Decomposing bi-implication into cases:

($\Rightarrow$) Assume $\mathcal{M}, t \models R(x, \bigcirc y)$ holds for some arbitrary $\mathcal{M}$ and t. From the semantics of $\bigcirc$ over regions, we have $\alpha(z,t) = \alpha(\bigcirc^- z, suc(t))$ and $\alpha(\bigcirc z, t) = \alpha(z, suc(t))$ for any region z. Therefore this is equivalent to $\mathcal{M}, suc(t) \models R(\bigcirc^- x, y)$ when applied to regions x and y respectively. Introducing $\bigcirc$ then yields $\mathcal{M}, t \models \bigcirc R(\bigcirc^- x, y)$.

($\Leftarrow$) Analogous to the above in reverse order. □

The ability to rewrite MSTL formulas such that occurences of 'next' over regions are either removed or replaced by 'previous' is vital for stream reasoning, because it allows for the delayed evaluation of formulas so that, at the time of evaluation, they only refer to the current and previous state(s) of the world. This makes the earlier-presented landmark approach applicable in a stream reasoning context.

Experiments and Results

In order to empirically evaluate MSTL with landmarks we ran experiments to test the effectiveness and the scalability of the land-mark based approach compared to the case where no landmarks were used. In these experiments, we were only interested in consistent scenarios, to capture the operational real-world domain. In particular, we are interested in the effects of landmarks on the resulting intertemporal disjunction size for non-landmark to non-landmark relations.

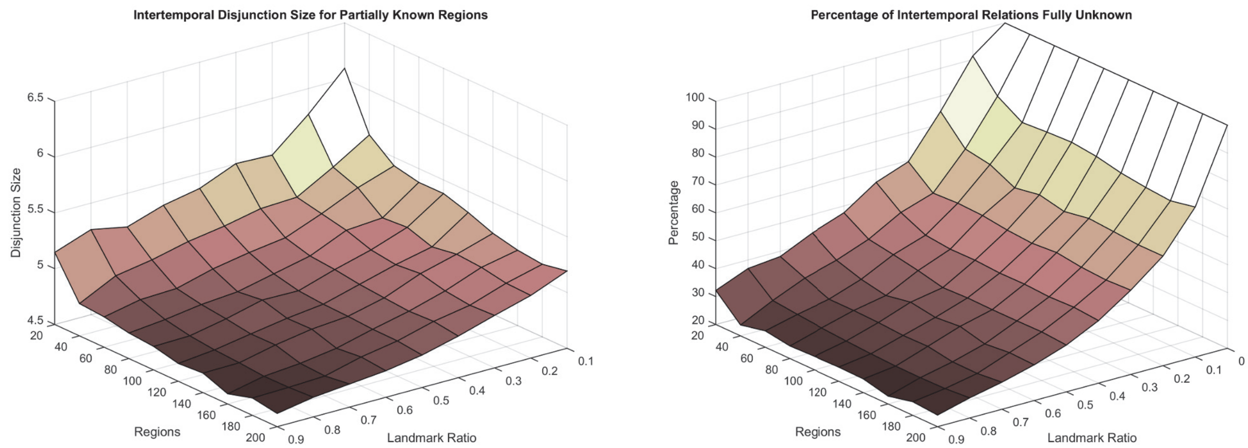

Figure 1: Left: Absolute disjunction size for varying number of regions and landmark ratio; smaller is better. Right: Percentage of such relations fully unknown. Note that a landmark ratio of 0 corresponds to the situation prior to our proposed solution.

Scenario Generation

When considering two time-points t_1 and t_2, the problem of generating scenarios is given a consistent scenario with landmarks for time-point t_1 generate a consistent scenario with those same landmarks for time-point t_2. To achieve this, we make use of a variation of the scenario generation method presented by (Renz and Nebel 2001), which was previously extended to handle static regions (Heintz and de Leng 2014). Scenarios for a single time-point are generated based on the number of (non-landmark) regions n and the average disjunction size l. We extend this by also considering the number of landmarks m such that $n + m = |\mathcal{R}|$, and fixing parameter $l = 4$. The reason for fixing $l = 4$ is that it provides a middle ground between fully known and fully unknown. Our parameter combinations consist of varying numbers of regions between 20 and 200 with step size 20, and varying landmark ratios relative to the number of regions (i.e. m/n) between 0 and 0.9 with step size 0.1.

The initial ‘seed’ for a scenario covers the landmark regions and their relations to each other. In our experiments we generated 30 such seeds per parameter combination. Here we are only interested in a consistent scenario with complete knowledge, so GQR is used to generate consistent interpretations of scenarios. These fully known seeds can then be used as the basis for a larger spatial relation matrix by adding further regions until we obtain the desired $|\mathcal{R}|$ regions. The number of CSPs generated from a seed was kept constant at 20. Note that these CSPs then all share a seed as a common component. We can therefore combine two CSPs that share a common seed. Excluding combinations that involve the same CSP twice, given 30 seeds and 20 CSPs per seed we get $30 \times (20 \times (20 - 1))/2 = 5700$ instances for each parameter set.

Results

The results of our experiments are shown in Figure 1, where every point represents the average over 5700 instances. On the left side, the number of regions and the landmark ratio are changed to see how they affect the disjunction size of non-landmark to non-landmark spatial relations. Here we limit ourselves to the average over the spatial relations that are not fully unknown. The results show that the more landmarks are added, the less uncertainty in terms of disjunction size is measured for these relations, reaching between disjunction sizes 4 and 5 for a landmark ratio of 0.9. The landmark approach is also scalable in terms of the number of regions.

This is also shown in the graph on the right, which illustrates the percentage of non-landmark to non-landmark intertemporal relations that remain fully unknown. Previously, we could not say anything about these relations, as illustrated by the percentage of fully unknown relations being 100%. Using landmarks, this is reduced to 30% for landmark ratio 0.9, but having a landmark ratio as low as 0.1 results in an improvement of roughly 20%.

Conclusions

We have presented a landmark-based approach to qualitative spatio-temporal stream reasoning to handle unobservable intertemporal spatial relations. Landmarks represent regions that do not change over time and can therefore serve as a qualitative frame of reference. The presented logic MSTL is a combination of MTL and RCC-8, and makes it possible to reason over spatio-temporal objects. This includes applying the ‘next’ operator to spatio-temporal objects and thereby allowing spatial relations between regions from different time-points to be described. To evaluate statements in MSTL, we presented an approach to handle intertemporal relations during progression with the help of rewriting rules. The landmark-based approach was tested for its scalability and effectiveness, showing an improvement in the disjunction sizes of non-landmark to non-landmark relations independent of the number of regions involved.

The presented work can serve as a starting point for interesting future efforts to further improve the ability to reason with uncertainty. Another interesting angle of research focuses on expanding the reasoning capabilities to include further temporal operators over regions, or to consider intertemporal relations across many time-points.

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