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False-Name-Proof Locations of Two Facilities: Economic and Algorithmic Approaches

Akihisa Sonoda and **Taiki Todo** and **Makoto Yokoo**
Department of Informatics, Kyushu University,
Motooka 744, Fukuoka, Japan
{sonoda@agent., todo@, yokoo@}inf.kyushu-u.ac.jp

Abstract

This paper considers a mechanism design problem for locating two identical facilities on an interval, in which an agent can pretend to be multiple agents. A mechanism selects a pair of locations on the interval according to the declared single-peaked preferences of agents. An agent's utility is determined by the location of the better one (typically the closer to her ideal point). This model can represent various application domains. For example, assume a company is going to release two models of its product line and performs a questionnaire survey in an online forum to determine their detailed specs. Typically, a customer will buy only one model, but she can answer multiple times by logging onto the forum under several email accounts. We first characterize possible outcomes of mechanisms that satisfy false-name-proofness, as well as some mild conditions. By extending the result, we completely characterize the class of false-name-proof mechanisms when locating two facilities on a circle. We then clarify the approximation ratios of the false-name-proof mechanisms on a line metric for the social and maximum costs.

1 Introduction

Facility location problems are traditional economic models that represent several social choice situations, such as elections and committee voting. One of their main objectives is to design social choice rules, or *mechanisms*, that satisfy several desirable properties. In this paper, we consider locating two identical facilities on an interval in which each agent only accesses the one closer to her ideal point, or *peak*. Such a preference structure is described as *single-peaked* and has several realistic applications, even for the case of two facilities. For example, in September 2015, Apple Inc. released two new models of its iPhone series; customers who prefer a smaller size/weight can choose iPhone 6s, while others who prefer a larger/higher display resolution can choose iPhone 6s Plus.

From the perspective of mechanism design, a mechanism is expected to satisfy an incentive property called *strategy-proofness*, which requires that reporting a true preference over the interval is a dominant strategy for each agent. Otherwise, agents may have an incentive to misreport their preferences and cheat on outcomes of mechanisms, which would result in undesirable locations. For the problem of locating a single facility, Moulin (1980) characterized a class of strategy-proof mechanisms known as *generalized median mechanisms*. Heo (2013) extended this idea for locating two or more facilities and proposed a class of mechanisms called *double median mechanisms*. Sui, Boutilier, and Sandholm (2013) suggested *percentile mechanisms* for multi-dimensional and multi-facility location problems.

Another type of cheating has also been studied in mechanism design for open and anonymous environments. If a mechanism is anonymous, i.e., ignoring who reports which preference, an agent may report more than one preference by creating and using fake identifiers (e.g., different email addresses or repeatedly logging onto an online forum). Such unfair practices are called *false-name manipulations*. There are several discussions on mechanisms that are robust against false-name manipulations (i.e., *false-name-proof*), traditionally in combinatorial auctions (Yokoo, Sakurai, and Matsubara 2004), more recently in such social choice situations as voting (Conitzer 2008; Wagman and Conitzer 2014), two-sided matching (Todo and Conitzer 2013), and single-facility locations (Todo, Iwasaki, and Yokoo 2011). However, to the best of our knowledge, no work has considered the case of two facilities.

In addition to these incentive issues, the quality of a mechanism's outcomes is also important for evaluating it. In this paper, we focus on two evaluation criteria. *Pareto efficiency*, which is one of the most standard economic efficiency criteria, requires that under any outcome from the mechanism, it is not possible to make any agent better off without making at least one agent worse off. *Approximation analysis* is a very popular approach in algorithm design for evaluating worst-case performance. To define an approximation factor of mechanisms, we use the two well-studied cost functions: the *social* and *maximum* costs.

Our contribution presented in this paper is two-fold. From an economic perspective, we characterize possible outcomes by false-name-proof mechanisms that also satisfy Pareto efficiency and another mild condition called *peak-onlyness*. For any given preferences of agents, (i) any two cyclically-adjacent peaks (i.e., two adjacent peaks and two extreme peaks) can be an outcome from one such mechanism and (ii) any such mechanism must return one of such two cyclically-adjacent peaks as the outcome. We also clarify what locations are possible, besides two cyclically-adjacent peaks, without peak-onlyness. Furthermore, we consider a two-facility location problem on a circle and completely characterize the class of mechanisms by means of false-name-proofness, Pareto efficiency, and peak-onlyness. From an algorithmic perspective, we clarify the approximation ratios of deterministic/randomized false-name-proof mechanisms on the line metric under the two well-studied cost functions. For the social cost, deterministically locating two facilities at the leftmost and rightmost peaks is asymptotically optimal among false-name-proof mechanisms. For the maximum cost, the mechanism proposed by Procaccia and Tennenholtz (2013) is false-name-proof and has a constant approximation ratio.

2 Related Works

The classical facility location problems, in which agents have single-peaked preferences and a single facility is located on a real line, were originally studied by Moulin (1980). Miyagawa (2001) and Heo (2013) extended this model for locating two facilities on a line and gave axiomatic characterizations of the rules, or mechanisms. Another extension of the problem is locating a facility on graphs, e.g., a circle and trees (Schummer and Vohra 2002). For the single facility on the circle, Gordon (2007) clarified that no Pareto efficient solution satisfies replacement-domination and population monotonicity, which is closely related to strategy-proofness and false-name-proofness, respectively.

Moreover, many researches including (Procaccia and Tennenholtz 2013) and (Alon et al. 2010) are interested in analyzing the approximation ratios of strategy-proof mechanisms for one or more facilities on a line, a circle, and trees. More recently, locating heterogeneous facilities, i.e., each facility is served for different purpose (Serafino and Ventre 2015), and a single facility location problem with *dual preferences*, where some agents prefer to stay close to the facility and the others prefer to stay away from (Zou and Li 2015), and with *double-peaked preferences*, i.e., staying too close to the facility also reduces agents' utility (Filos-Ratsikas et al. 2015), are much interested.

3 Preliminaries

In this paper, we consider a two-facility location problem for locating two identical facilities on an interval. We first introduce our model, which basically follows Miyagawa (2001). Let $\mathcal{N}$ be a set of all potential agents, and let $N \subset \mathcal{N}$ be a set of attending agents. Each agent $i \in N$ has a continuous preference relation R_i over closed interval $\mathcal{I} = [0, 1]$. For any locations $x, y \in \mathcal{I}$, xR_iy denotes that agent i weakly prefers x to y. Let P_i and I_i be the strict and indifference relations associated with R_i. We further assume that each R_i is *single-peaked*, that is, has a unique location $p(R_i) \in \mathcal{I}$ such that $\forall x, y \in \mathcal{I}$, $y < x \le p(R_i)$ or $p(R_i) \le x < y$ implies xP_iy. We call this unique location $p(R_i)$ agent i's *peak* under preference R_i. Under a single-peaked preference of an agent, the closer the facility is to her peak, the more she prefers it. Note that each R_i is not necessarily symmetric around peak $p(R_i)$, except for Section 6 in which we assume the preferences are based on a Euclidean distance.

We now extend preference relation R_i to represent agent i's preference over the *pairs* of locations. Let $X = \langle x_1, x_2 \rangle$ denote a pair of locations in $\mathcal{I}^2$. Since we assume that the two facilities are identical, each agent's utility is solely determined by the better of the two.

Definition 1 (Preferences over Pairs of Locations)**.** *Given any two pairs $\langle x_1, x_2 \rangle, \langle y_1, y_2 \rangle \in \mathcal{I}^2$, single-peaked preference R_i satisfies $\langle x_1, x_2 \rangle R_i \langle y_1, y_2 \rangle$ if and only if either $x_1R_iy_1 \land x_1R_iy_2$ or $x_2R_iy_1 \land x_2R_iy_2$ holds.*

Let $\mathcal{R}$ be the set of all possible single-peaked preferences. Also, let $R_N = \langle R_i \rangle_{i \in N} \in \mathcal{R}^{|N|}$ denote the preference profile of attending agents N. For each $R_N \in \mathcal{R}^{|N|}$, let $p(R_N) = \langle p(R_i) \rangle_{i \in N}$ be the profile of the peaks of agents N. We also define minimum peak $\underline{p}(R_N) = \min_{i \in N} p(R_i)$ and maximum peak $\overline{p}(R_N) = \max_{i \in N} p(R_i)$. Furthermore, for a given R_N, let $q(R_N)$ be the profile of all *distinct* peaks in $p(R_N)$ sorted in ascending order: $q(R_N) = \langle x \in \mathcal{I} \mid \exists i \in N, x = p(R_i) \rangle$ such that $q_1(R_N) < q_2(R_N) < \cdots < q_m(R_N)$, where m indicates the number of distinct peaks. In the rest of this paper, each element in $q(R_N)$ is indicated by indices $a, b \in \{1, \dots, m\}$, such as $q_a(R_N)$ and $q_b(R_N)$. For both $p(R_N)$ and $q(R_N)$, we use symbols p_i and q_a instead of $p(R_i)$ and $q_a(R_N)$ if there is no ambiguity in the context. We also use symbols $\underline{p}$ and $\overline{p}$ instead of $\underline{p}(R_N)$ and $\overline{p}(R_N)$.

Deterministic mechanism $f : \bigcup_{N \subset \mathcal{N}} \mathcal{R}^{|N|} \to \mathcal{I}^2$ is a function that associates each $N \subset \mathcal{N}$ and each $R_N \in \mathcal{R}^{|N|}$ with a pair of distinct locations in $\mathcal{I}^2$. For a given $R_N \in \mathcal{R}^{|N|}$ and a deterministic mechanism f, we refer to $f_1(R_N)$ as the left location and $f_2(R_N)$ as the right location, i.e., $f_1(R_N) < f_2(R_N)$. We use f_1 and f_2 instead of $f_1(R_N)$ and $f_2(R_N)$ if there is no ambiguity in the context. Similarly, a randomized mechanism associates each preference profile with a probability distribution over the set of all possible pairs of locations. We assume in this entire paper that mechanisms are *anonymous*; $\forall N, N' \subset \mathcal{N}$ s.t. $|N| = |N'|$, $\forall R_N \in \mathcal{R}^{|N|}$, $\forall \pi : N \rightarrowtail\!\!\!\!\twoheadrightarrow N'$, $f(R'_{N'}) = f(R_N)$ holds, where $R'_{N'} \in \mathcal{R}^{|N'|}$ is such that $\forall i \in N$, $R'_{\pi(i)} = R_i$ and $\rightarrowtail\!\!\!\!\twoheadrightarrow$ indicates a bijection. Under an anonymous mechanism, its outcome depends only on declared preferences by agents and is unaffected by their names.

Now we are ready to formally define the three properties considered in this paper: *peak-onlyness (PO)*, *Pareto efficiency (PE)*, and *false-name-proofness (FNP)* (a stronger notion of *strategy-proofness (SP)*). The main purpose of this paper is to understand the mechanisms that simultaneously satisfy PO, PE, and FNP.

PO requires that the outcome of a mechanism only depends on the peaks of agents (Heo 2013). Since it is difficult for agents to report their exact preferences all over an interval in practical situations, we introduce this property to focus on mechanisms whose communication cost is low.

Definition 2 (Peak-onlyness)**.** *Mechanism f is* peak-only *if $\forall N, N' \subset \mathcal{N}$ s.t. $|N| = |N'|$, $\forall R_N \in \mathcal{R}^{|N|}$, $\forall R_{N'} \in \mathcal{R}^{|N'|}$, $p(R_N) = p(R_{N'})$ implies $f(R_N) = f(R_{N'})$.*

PE enables mechanisms to avoid outcomes in which we can improve at least one agent's situation without worsening another agent's situation by choosing another outcome.

Definition 3 (Pareto Efficiency)**.** *Pair $X \in \mathcal{I}^2$ dominates another pair $Y \in \mathcal{I}^2$ at preference profile R_N if XR_iY for any $i \in N$ and XP_jY for some $j \in N$. Mechanism f is* Pareto efficient *if $\forall N \subset \mathcal{N}$, $\forall R_N \in \mathcal{R}^{|N|}$, there exists no pair $X \in \mathcal{I}^2$ that dominates $f(R_N)$ at R_N.*

As an incentive property, SP, which has attracted much attention in the literature, guarantees that reporting a true preference is a dominant strategy for every agent. FNP, which is a stronger notion of SP, requires that for each agent, reporting her true preference using only one identifier is a dominant strategy, even if she can use multiple identifiers.

Definition 4 (False-name-proofness)**.** *Mechanism f is* false-name-proof *if $\forall N \subset \mathcal{N}$, $\forall R_N \in \mathcal{R}^{|N|}$, $\forall i \in N$, $\forall \Phi_i \subset (\mathcal{N} \setminus N) \cup \{i\}$, $\forall R'_{\Phi_i} \in \mathcal{R}^{|\Phi_i|}$, $f(R_N) R_i f(R'_{\Phi_i}, R_{N \setminus \{i\}})$ holds, where $\Phi_i (\neq \varnothing)$ denotes a set of identifiers used by agent i and R'_{Φ_i} denotes a preference profile reported by Φ_i.*

By setting $\Phi_i = \{i\}$, we obtain the definition of SP.

4 Characterization of Possible Locations

In this section, we characterize the possible outcomes by mechanisms that satisfy PO, PE, and FNP. Since we are locating two facilities, there is nothing interesting when the number of distinct peaks is one, i.e., $m = 1$. Our characterization therefore focuses on case $m \geq 2$. Before presenting the result, we also characterize possible locations by replacing FNP with SP.

Proposition 1. *Let $\mathcal{S}$ be the set of all mechanisms that satisfy PO, PE, and SP. For any $N \subset \mathcal{N}$ and any $R_N \in \mathcal{R}^{|N|}$ s.t. $m \geq 2$, it holds that*

$$\{f(R_N) | f \in \mathcal{S}\} = \{\langle q_a, q_b \rangle | a, b(\neq a) \in \{1, 2, \ldots, m\}\}.$$

Proof. (LHS $\supseteq$ RHS) To show that LHS weakly includes RHS, we consider the following mechanism. For given $\alpha, \beta(\neq \alpha) \in \{1, \ldots, |N|\}$, choosing the α-th and β-th peaks from the left side of the interval (if they coincide, we choose the peak next to them as a second location). We can easily see that this mechanism satisfies PO, PE, and SP.

(LHS $\subseteq$ RHS) Now we prove the other direction. More precisely, we derive a contradiction by assuming, without loss of generality, that for some $N \subset \mathcal{N}$ and some $R_N \in \mathcal{R}^{|N|}$ satisfying $m \geq 2$, there exists a mechanism $f \in \mathcal{S}$ s.t. for all $a \in \{1, \ldots, m\}$, $f_1 \neq q_a$ holds. Note that any Pareto efficient allocation must have the following property:

Observation 1 (Miyagawa, 2001)**.** *Given $N \subset \mathcal{N}$ and $R_N \in \mathcal{R}^{|N|}$ s.t. $m \geq 2$, pair $\langle x_1, x_2 \rangle$ with $x_1 < x_2$ is Pareto efficient if and only if $x_1, x_2 \in [\underline{p}, \overline{p}]$ and $\exists i, j \in N$ s.t. $(p_i, p_j \in [x_1, x_2]) \land (x_1 P_i x_2) \land (x_2 P_j x_1)$ hold.*

Since f is Pareto efficient, for profile R_N, $f_1, f_2 \in [\underline{p}, \overline{p}]$ and $\exists i, j \in N$ s.t. $p_i, p_j \in [f_1, f_2]$, $f_1 P_i f_2$, and $f_2 P_j f_1$ hold. We now assume without loss of generality that for $m \geq 3$ [1], $q_1 < f_1 < q_2 < f_2$ and that at least one agent whose peak belongs to $[q_2, f_2]$ strictly prefers f_1 to f_2. Here we consider a modified profile $R'_N \in \mathcal{R}^{|N|}$ s.t. $f_2 P'_k f_1$ and $p(R_k) = p(R'_k)$ hold for each agent k whose peak belongs to $[q_2, f_2]$ (all the other agents have identical preferences as in R_N). Since $q(R'_N) = q(R_N)$ obviously holds, PO implies $f_1(R'_N) = f_1(R_N)$. From PE, however, $f_1(R'_N) = q_1(R'_N) = q_1(R_N) < f_1(R_N)$ must hold, which derives a contradiction. □

[1] For any R_N s.t. $m = 2$, choosing both q_1 and q_2 is the only way to satisfy PE.

That is, for a given profile of preferences, any pair of distinct peaks can be realized by appropriately choosing one mechanism from $\mathcal{S}$ (i.e., LHS weakly includes RHS), and more surprisingly, no outcome exists that locates at least one facility at a point that differs from any agent's peak (i.e., RHS weakly includes LHS).

Note that the second part (LHS $\subseteq$ RHS) holds even without SP. The proposition is therefore important in the sense that, as we briefly mentioned, any pair of distinct peaks can be realized by a mechanism in $\mathcal{S}$. Actually, this point will be highlighted in the following theorem about possible locations by false-name-proof mechanisms, which is one of our main contributions.

Theorem 1. *Let $\mathcal{F}$ be the set of all mechanisms that satisfy PO, PE, and FNP. For any $N \subset \mathcal{N}$ and any $R_N \in \mathcal{R}^{|N|}$ s.t. $m \geq 2$, it holds that*

$$\{f(R_N) \mid f \in \mathcal{F}\} = \{\langle q_a, q_{(a \bmod m)+1} \rangle \mid a \in \{1, 2, \ldots, m\}\}. \quad (1)$$

Although this theorem does not characterize the set of *mechanisms* that satisfy all three properties, its strong implication restricts the possible behavior of such mechanisms. The RHS of Eq. (1) indicates the set of pairs of cyclically-adjacent peaks, i.e., two adjacent peaks and two extreme peaks. This theorem states that, if we require all three properties, the two locations must be cyclically-adjacent peaks. Compared with Proposition 1, by strengthening the incentive property from SP to FNP, the number of possible outcomes for a given profile of preferences is reduced from m-combinations of two to m.

Proof of Theorem 1. (LHS $\supseteq$ RHS) To show that LHS weakly includes RHS, we now consider a parameterized class of mechanisms. For a given parameter $\alpha \in \mathcal{I}$, mechanism ϕ locates two facilities as follows: $\forall N \subset \mathcal{N}$, $\forall R_N \in \mathcal{R}^{|N|}$ s.t. $m \geq 2$,

$$\phi(R_N; \alpha) = \begin{cases} \langle \overline{p}_{\leq \alpha}(R_N), \underline{p}_{>\alpha}(R_N) \rangle & \text{if } \underline{p} \leq \alpha < \overline{p}, \\ \langle \underline{p}(R_N), \overline{p}(R_N) \rangle & \text{otherwise,} \end{cases}$$

where $\overline{p}_{\leq \alpha}(R_N) = \max_{i \in N}\{p_i \mid p_i \leq \alpha\}$ and $\underline{p}_{>\alpha}(R_N) = \min_{i \in N}\{p_i \mid p_i > \alpha\}$. It basically chooses two distinct peaks around parameter α, except for the case of $\alpha \notin [\underline{p}, \overline{p})$, in which it is located at two extreme peaks. In other words, mechanism ϕ regards the line as a circle such that the two endpoints (i.e., 0 and 1) of the interval are connected and selects two distinct peaks around α.

For any R_N, any two cyclically-adjacent peaks in $q(R_N)$ can be realized by choosing appropriate α. The mechanism also obviously satisfies PO and PE by definition. Furthermore, by any manipulation with multiple fake identifiers, the two locations by the mechanism never go beyond the two cyclically-adjacent peaks originally around α, which suffices to guarantee FNP.

(LHS $\subseteq$ RHS) Since FNP implies SP and thus Proposition 1 holds, for any $N \subset \mathcal{N}$, any $R_N \in \mathcal{R}^{|N|}$, and any $f \in \mathcal{F}$, there exist $a, b(\neq a) \in \{1, 2, \ldots, m\}$ such that $f_1 = q_a$ and $f_2 = q_b$. Therefore, for the sake of contradiction, we assume without loss of generality that for some N and R_N satisfying $m \geq 4$ [2], $q_1 < f_1 = q_2 < q_3 < f_2 = q_4$.

Let $M = \{k \in N \mid p_k = q_2\}$ and $R_{N\setminus M} = \langle R_j \rangle_{j \in N \setminus M}$. Then either $f_1(R_{N\setminus M}) = q_4$ or $f_2(R_{N\setminus M}) = q_4$ holds; otherwise an agent with peak q_4 would be better off by adding M as her fake identifiers. Now let x be the other location chosen by f for $R_{N\setminus M}$, i.e., $x \in f(R_{N\setminus M})$ and $x \neq q_4$. From Proposition 1, x must also be located at an agent's peak in $R_{N\setminus M}$. However, any choice from $R_{N\setminus M}$ is vulnerable against false-name manipulations; if $x = q_1$, then an agent with peak q_3 has an incentive to manipulate. If $x = q_a(R_{N\setminus M})$ for any $a > 1$, an agent with peak q_1 also has an incentive to manipulate. $\square$

5 Further Discussions

In this section, we more deeply discuss the possible outcomes by false-name-proof mechanisms for two-facility location problems. We first investigate how the space of the possible outcomes changes when PO is ignored. We then consider a different problem where a mechanism locates two facilities on a circle, and show a complete characterization of our proposed mechanisms.

5.1 Without Peak-Onlyness

Although we introduced the property of PO as an important restriction in practical situations, it is still worth clarifying what other locations are possible if we can eliminate PO, e.g., when agents have enough knowledge as well as more informative bidding language, to represent their complete preferences. Here we show the following characterization of the set of possible locations by mechanisms that satisfy PE and FNP. For a given profile of preferences and a mechanism f that satisfies PE and FNP, if two locations are not cyclically-adjacent, i.e., there exist peaks between f_1 and f_2, then either f_1 is the left peak or f_2 is the right peak.

Proposition 2. *Assume f satisfies PE and FNP. For any $N \subset \mathcal{N}$ and any $R_N \in \mathcal{R}^{|N|}$ s.t. $m \geq 2$, if there exists some $i \in N$ s.t. $p_i \in (f_1, f_2)$, then either $f_1 = \underline{p}$ or $f_2 = \overline{p}$ holds.*

Proof. For the sake of contradiction, we assume that $\exists N \subset \mathcal{N}$, $\exists R_N \in \mathcal{R}^{|N|}$ s.t. $m \geq 4$ [3], mechanism f exists, which satisfies PE and FNP, that satisfies $p_i \in (f_1, f_2)$ for some $i \in N$ and both $f_1 \neq \underline{p}$ and $f_2 \neq \overline{p}$. Now we partition the set of agents N into the following four subsets:

$$
\begin{aligned}
A_1 &= \{i \in N \mid p_i \in [0, f_1)\}, \\
A_2 &= \{i \in N \mid p_i \in (f_2, 1]\}, \\
B_1 &= \{i \in N \mid p_i \in [f_1, f_2], f_1 P_i f_2\}, \\
B_2 &= \{i \in N \mid p_i \in [f_1, f_2], f_2 P_i f_1\}.
\end{aligned}
$$

By definition, $A_1, A_2, B_1, B_2 \neq \varnothing$.

Step 1: We first add agent $k \in \mathcal{N} \setminus N$ who is indifferent between f_1 and f_2 (i.e., $f_1 I_k f_2$). Such an agent must have peak $p_k \in (f_1, f_2)$. We can see that $f(R_N, R_k) = f(R_N)$; otherwise at least one agent benefits by adding such a k as her fake identifier.

Step 2: We then remove both B_1 and B_2. Let f_1' and f_2' be the two locations of the mechanism for set $A_1 \cup A_2 \cup \{k\}$. Now we show that either (i) $(f_1' \in [0, f_1)) \wedge (f_2' = f_2)$ or (ii) $(f_2' \in (f_2, 1]) \wedge (f_1' = f_1)$ holds. First, if $f_1', f_2' \in [0, p_k]$ (resp. $f_1', f_2' \in [p_k, 1]$), some agent in A_2 (resp. in A_1) has an incentive to add $B_1 \cup B_2$. Therefore, it must hold that $f_1' \leq p_k \leq f_2'$. Next, if $f_1' \in (f_1, p_k]$ (resp. $f_2' \in [p_k, f_2)$), some agent in A_1 (resp. in A_2) has an incentive to add $B_1 \cup B_2$. Therefore, it must hold that $f_1' \in [0, f_1]$ and $f_2' \in [f_2, 1]$. Finally, if both $f_1' = f_1$ and $f_2' = f_2$ hold, PE is violated. Similarly, if both $f_1' \in [0, f_1)$ and $f_2' \in (f_2, 1]$ hold, agent k has an incentive to add $B_1 \cup B_2$. Combining all the above, we have either (i) or (ii).

Step 3: For (i), return B_1 to the mechanism and let f_1'', f_2'' be the two locations for set $A_1 \cup A_2 \cup B_1 \cup \{k\}$. By the same argument from Step 2, $f_1'' \leq p_k \leq f_2''$, $f_1'' \in [0, f_1]$ and $f_2'' \in [f_2, 1]$ hold. If $f_1'' \in [0, f_1)$ (resp. $f_2'' \in (f_2, 1]$) holds, some agent in B_1 (resp. in A_2) has an incentive to add B_2 (resp. B_1 in the Step 2 situation). Thus, it must be the case that $f_1'' = f_1$ and $f_2'' = f_2$. However, no agent exists who strictly prefers f_2'' to f_1'' and whose peak belongs to $[f_1'', f_2'']$, which violates PE. A similar argument follows for (ii). $\square$

Actually, the following mechanism satisfies both PE and FNP, and can locate two facilities at two peaks that are not cyclically-adjacent. The main idea comes from a well-known class of mechanisms called *target rules*, which was originally designed as a parameterized class of mechanisms that respect population monotonicity (Ching and Thomson 1997). It is also a unique class of mechanisms that satisfy PE and FNP (Todo, Iwasaki, and Yokoo 2011) for single-facility location problems.

Mechanism 1. *For a given parameter $\alpha \in \mathcal{I}$, let $\tau : \bigcup_{N \subset \mathcal{N}} \mathcal{R}^{|N|} \to \mathcal{I}$ be a function s.t.*

$$
\tau(R_N; \alpha) = \begin{cases} \underline{p} & \text{if } \alpha \leq \underline{p}, \\ \overline{p} & \text{if } \alpha \geq \overline{p}, \\ \alpha & \text{otherwise.} \end{cases}
$$

For a given parameter $\alpha \in \mathcal{I}$, mechanism π locates two facilities as follows: $\forall N \subset \mathcal{N}$, $\forall R_N \in \mathcal{R}^{|N|}$ s.t. $m \geq 2$,

$$
\pi(R_N; \alpha) = \langle \underline{p}, \tau(\langle R_i \mid i \in N \text{ s.t. } \alpha' P_i \underline{p} \rangle; \alpha') \rangle,
$$

where $\alpha' = \tau(R_{\{i \in N \mid p(R_i) \neq \underline{p}\}}; \alpha)$.

[2] For any R_N s.t. $2 \leq m \leq 3$, any pair of distinct locations is automatically cyclically-adjacent.

[3] For any R_N s.t. $2 \leq m \leq 3$, from PE, we can easily see that Proposition 2 is satisfied.

This mechanism locates the first facility at $\underline{p}$. The second facility is located as follows. First, target point α is updated to $\alpha' = \tau(R_{\{i \in N | p(R_i) \neq \underline{p}\}}; \alpha)$. Then we apply function τ to this revised α' and the agents who strictly prefer α' over $\underline{p}$. Since PE and FNP of the mechanism can be straightforwardly proved, we omit the proof.

5.2 Characterization on a Circle

In this subsection, we consider the two-facility location problem on a circle $\mathcal{C} \subset \mathbb{R}^2$ with perimeter 1. The main idea here is to apply the mechanism introduced in the proof of Theorem 1. Here we completely characterize the mechanisms by means of FNP, PE, and PO.

To define the two-facility location problem on a circle, we first need to introduce several notions. For a given circle $\mathcal{C}$ with perimeter 1 and any two points $x, y (\neq x) \in \mathcal{C}$, we define closed interval $[x, y]$ as the set of (infinitely many) points from x to y along the circle in a counter-clockwise direction, i.e., $[x, y] \neq [y, x]$. We also define $[x, y)$ as $[x, y] \setminus \{y\}$, $(x, y]$ as $[x, y] \setminus \{x\}$, and (x, y) as $[x, y] \setminus \{x, y\}$. Notice that $[x, x] = \{x\}$, $[x, x) = (x, x] = \mathcal{C}$. For any $x \in \mathcal{C}$ and any $l \in [0, 1]$, let $x + l$ (resp. $x - l$) be the position that is length l away from position x in counter-clockwise (resp. clockwise) direction. Notice that $x + 1 = x - 1 = x$.

Now we re-define the preferences of agents. Each agent $i \in N$ is assumed to have a general single-peaked preference R_i over circle $\mathcal{C}$. More formally, each agent i with preference R_i has a peak $p(R_i) \in \mathcal{C}$ and a dip $d(R_i) \in \mathcal{C}$ such that $\forall x, y \in \mathcal{C}$, $d(R_i) \leq y < x \leq p(R_i)$ or $p(R_i) \leq x < y \leq d(R_i)$ implies $x P_i y$. Let $\mathcal{R}$ be the set of all general single-peaked preferences over the circle. According to this, a mechanism f is defined as $f : \bigcup_{N \subset \mathcal{N}} \mathcal{R}^{|N|} \to \mathcal{C}^2$, which maps a given preference profile to a location of two facilities on the circle. PE and FNP are straightforwardly extended. PO can also be extended so that the decision only depends on the agents' peaks, i.e., independent from their dips.

We are now ready to describe one of the main contributions of this paper. In contrast to the case of the line, any mechanism that satisfies all three properties must behave in the same manner as the mechanism introduced in the proof of Theorem 1, and no other mechanism simultaneously satisfies these properties. Here we first explicitly define the mechanism, which is followed by its characterization result.

Mechanism 2. *Consider a circle $\mathcal{C}$. For a given parameter $\alpha \in \mathcal{C}$, mechanism φ_{CCW} locates two facilities as follows: $\forall N \subset \mathcal{N}$, $\forall R_N \in \mathcal{R}^{|N|}$ s.t. $m \geq 2$,*

$$\varphi_{CCW}(R_N; \alpha) = \langle p(R_{i^*}), p(R_{j^*}) \rangle,$$

where $\langle i^*, j^* \rangle \in \arg\min_{\langle i, j \rangle \in N^2} |[p_i, p_j)|$ s.t. $\alpha \in [p_i, p_j)$.

If there exists some agent whose peak is at α, the mechanism described by Mechanism 2 locates one facility at α and the other at the closest peak to α in a *counter-clockwise* direction. By replacing $[p_i, p_j)$ of Mechanism 2 with $(p_i, p_j]$, we can obtain mechanism φ_{CW}, which locates one facility at α and the other at closest peak to α in a *clockwise* direction if some agent exists at α.

Theorem 2. *When $m \geq 2$, mechanism f satisfies PO, PE, and FNP on a circle if and only if $f = \varphi_{CCW}$ or $f = \varphi_{CW}$.*

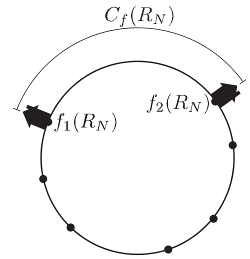

Figure 1: No other peak exists in $C_f(R_N)$.

Theorem 2 is proved by Lemmas 1, 2, 3, and 4.

Lemma 1. *Assume f satisfies PO, PE, and FNP. Then $\forall N \subset \mathcal{N}$, $\forall R_N \in \mathcal{R}^{|N|}$ s.t. $m \geq 2$, $\exists i, j \in N$ s.t. $f(R_N) = \langle p_i, p_j \rangle$ and $(\forall k \in N, p_k \in [f_1, f_2]) \vee (\forall k \in N, p_k \in [f_2, f_1])$ hold.*

Lemma 1 means that the two locations must be at adjacent peaks on the circle, assuming that a mechanism satisfies the three properties. We omit the proof since it closely resembles the proof of Theorem 1.

Here, for any $m \geq 3$, let $C_f(R_N)$ be the interval of the two locations, returned by mechanism f for given $N \subset \mathcal{N}$ and $R_N \in \mathcal{R}^{|N|}$, such that there exist no other peaks between them. Notice that when $m = 2$, we define $C_f(R_N) = \mathcal{C}$. From Lemma 1, such an interval $C_f(R_N)$ is uniquely determined (Fig. 1, for example).

Lemma 2. *Assume f satisfies PO, PE, and FNP. Then $\forall N \subset \mathcal{N}$, $\forall R_N \in \mathcal{R}^{|N|}$ s.t. $m \geq 2$, $\forall M \subset \mathcal{N} \setminus N$, $\forall R_M \in \mathcal{R}^{|M|}$, $C_f(R_N, R_M) \subseteq C_f(R_N)$ holds.*

Proof. For the sake of contradiction, we assume that $\exists N \subset \mathcal{N}$, $\exists R_N \in \mathcal{R}^{|N|}$ s.t. $m \geq 2$, $\exists M \subset \mathcal{N} \setminus N$, $\exists R_M \in \mathcal{R}^{|M|}$, $C_f(R_N, R_M) \not\subseteq C_f(R_N)$ holds. This implies that either $f_1(R_N, R_M) \notin C_f(R_N)$ or $f_2(R_N, R_M) \notin C_f(R_N)$ holds. From PO and the fact that $p(R_i) \notin C_f(R_N) \setminus \{f_1(R_N), f_2(R_N)\}$ holds for any $i \in N$, the existence of agent $i' \in N$ such that $f(R_N, R_M) P_{i'} f(R_N)$ is guaranteed, which obviously violates FNP. □

Lemma 3. *Assume f satisfies PO, PE, and FNP. Then there exists an $\alpha \in \mathcal{C}$ such that the following holds:*

$$\forall N \subset \mathcal{N}, \forall R_N \in \mathcal{R}^{|N|} \text{ s.t. } m \geq 2, \alpha \in C_f(R_N).$$

Proof. Since the set $\mathcal{N}$ is sufficiently large and any point in $\mathcal{C}$ can be the peak of a preference, $C_f(R_N)$ converges to a certain point $\alpha \in \mathcal{C}$ as N goes to $\mathcal{N}$; $\lim_{N \to \mathcal{N}} C_f(R_N) = \alpha$. Here, obviously from Lemma 2, such a parameter α must be in $C_f(R_N)$ for any N and R_N. □

Lemma 4. *Assume f satisfies PO, PE, and FNP. Then there exists an $\alpha \in \mathcal{C}$ such that either of the following holds:*

(i) $\forall N, \forall R_N$ *s.t.* $m \geq 2$, $\exists \delta > 0$, $[\alpha, \alpha + \delta] \subseteq C_f(R_N)$,

(ii) $\forall N, \forall R_N$ *s.t.* $m \geq 2$, $\exists \delta > 0$, $[\alpha - \delta, \alpha] \subseteq C_f(R_N)$.

Proof. For the sake of contradiction, we assume that the negation of the consequence holds for some $N \subset \mathcal{N}$ and

	Det.	Rand.
Social cost	UB: $\lvert N\rvert - 2$ LB: $\lvert N\rvert - 2$	**UB:** $\boldsymbol{\lvert N\rvert - 2}$ **LB:** $\boldsymbol{\Omega(\lvert N\rvert)}$
Maximum cost	UB: 2 LB: 2	**UB:** $\boldsymbol{5/3}$ LB: $3/2$

Table 1: Approx. ratios achieved by false-name-proof mechanisms for the two-facility location problem on the line. UB and LB stand for upper and lower bounds, respectively.

$R_N \in \mathcal{R}^{|N|}$. Combining it with Lemma 3, we obtain $C_f(R_N) = \{\alpha\}$ for such N and R_N. However, since $m \geq 2$ holds for R_N, $C_f(R_N)$ contains at least two distinct points, which derives a contradiction. $\square$

It is obvious that if case (i) (or case (ii)) of Lemma 4 holds, the only mechanism is φ_{CCW} (or φ_{CW}).

6 Two Facilities in the Line Metric Space

In this section, we evaluate the quality of the locations based on the approximation ratio for the *social cost* and the *maximum cost*, both of which have attracted much research attention in algorithmic game theory. The basic idea is that we quantify the cost of agents by assuming that each one has a cost function based on the distance over interval $\mathcal{I}$. A summary of the result in this section is presented in Table 1, where our contribution is emphasized in boldface.

Here we define the cost function of each agent. Given $N \subset \mathcal{N}$ and $R_N \in \mathcal{R}^{|N|}$, the *cost* of agent i with peak p_i under deterministic mechanism f is its distance to the nearest facility, i.e., $c(p_i, f(R_N)) = \min\{|p_i - f_1|, |p_i - f_2|\}$. The expected cost of agent i under randomized mechanism f is also defined as $c(p_i, f(R_N)) = \mathbb{E}_{X\sim f(R_N)} c(p_i, X)$, where $f(R_N)$ is a probability distribution over $\mathcal{I}^2$ and $X \in \mathcal{I}^2$ is a realization from $f(R_N)$. Given $N \subset \mathcal{N}$ and $R_N \in \mathcal{R}^{|N|}$, the *social cost* and the *maximum cost* under deterministic/randomized mechanism f are defined as follows:

$$\begin{aligned}\Gamma_{\text{SC}}(f, R_N) &= \sum_{i\in N} c(p_i, f(R_N)),\\ \Gamma_{\text{MC}}(f, R_N) &= \max_{i\in N} c(p_i, f(R_N)).\end{aligned}$$

Mechanism f has approximation ratio $\gamma \geq 1$ with regard to objective function Γ if for any $N \subseteq \mathcal{N}$ and $R_N \in \mathcal{R}^{|N|}$,

$$\Gamma(f, R_N) \leq \gamma \cdot \Gamma^*(R_N)$$

holds, where $\Gamma^*(R_N)$ is the optimal solution for R_N.

6.1 Social Cost

In deterministic mechanisms, the extreme peaks mechanism, which locates two facilities at the leftmost and rightmost peaks with probability one, achieves an approximation ratio of $|N| - 2$ for the social cost (Procaccia and Tennenholtz 2013). Moreover, the extreme peaks mechanism is the only deterministic, anonymous, strategy-proof mechanism that has a bounded approximation ratio for two-facility location problems with $|N| \geq 5$ agents (Fotakis and Tzamos 2013). Since FNP is more restrictive, any deterministic false-name-proof mechanism must have an approximation ratio of at least $|N| - 2$. Also, the extreme peaks mechanism satisfies FNP since it is a special instance of the mechanism described in the proof of Theorem 1. The bound of $|N| - 2$ is therefore tight for deterministic false-name-proof ones.

For randomized strategy-proof mechanisms, a lower bound of 1.045 and an upper bound of 4 have been obtained (Lu, Wang, and Zhou 2009; Lu et al. 2010). Here we show that any randomized false-name-proof mechanism has an approximation ratio of $\Omega(|N|)$ for the social cost. Thus, the extreme peaks mechanism is asymptotically optimal for the social cost among false-name-proof mechanisms.

Theorem 3. *Any randomized false-name-proof mechanism has an approximation ratio of* $\Omega(|N|)$ *for the social cost.*

Proof. First, consider a set of agents N' s.t. $|N'| = 3$ and a profile $R_{N'}$ s.t. $p(R_{N'}) = \langle 0, 0.5, 1\rangle$. Any randomized mechanism can be represented as a distribution over some deterministic mechanisms. Each deterministic mechanism obviously has the social cost of at least 0.5 for profile $R_{N'}$. Thus, any randomized mechanism f also has the social cost of at least 0.5 for $R_{N'}$. Here we assume without loss of generality that $c(0, f(R_{N'})) \geq 0.5/3$. Next, consider a new profile R_N of agents N s.t. $p(R_N) = \langle \underbrace{0, \ldots, 0}_{|N|-2}, 0.5, 1\rangle$. From FNP, it must be the case that $c(0, f(R_N)) \geq c(0, f(R_{N'})) \geq 0.5/3$. Thus, the social cost for profile R_N is at least $0.5 \cdot (|N| - 2)/3$. The optimal social cost for R_N is obviously 0.5, and the approximation ratio is at least $(|N| - 2)/3$. $\square$

6.2 Maximum Cost

We next discuss the maximum cost of false-name-proof mechanisms. For deterministic ones, the tight bound of 2 obtained by Procaccia and Tennenholtz (2013) for a strategy-proof mechanism is carried over to our setting. Furthermore, they got a lower bound of $3/2$ for randomized strategy-proof mechanisms and suggested a randomized mechanism that achieves an approximation ratio of $5/3$ for the maximum cost. Now we show that it satisfies FNP.

Mechanism 3. *Given a profile* $R_N \in \mathcal{R}^{|N|}$*, let us define*

$$\begin{aligned}\text{lb} &= \max_{i\in N}\{p_i \mid p_i \leq (\underline{p} + \overline{p})/2\},\\ \text{rb} &= \min_{i\in N}\{p_i \mid p_i \geq (\underline{p} + \overline{p})/2\},\\ \text{dist} &= \max\{\text{lb} - \underline{p}, \overline{p} - \text{rb}\}.\end{aligned}$$

Mechanism ψ *locates two facilities as follows:* $\forall N \subset \mathcal{N}$, $\forall R_N \in \mathcal{R}^{|N|}$ *s.t.* $m \geq 2$,

$$\psi(R_N) = \begin{cases}\langle \underline{p}, \overline{p}\rangle & \textit{with prob. } 1/2,\\ \langle \underline{p} + \text{dist}/2, \overline{p} - \text{dist}/2\rangle & \textit{with prob. } 1/3,\\ \langle \underline{p} + \text{dist}, \overline{p} - \text{dist}\rangle & \textit{with prob. } 1/6.\end{cases}$$

Theorem 4. *Mechanism 3 satisfies FNP.*

Theorem 4 is proved by Lemmas 5 and 6. Lemma 5 is derived from Theorem 1 of Bu (2013), who showed that a mechanism satisfying anonymity, SP, and population monotonicity satisfies FNP.

Lemma 5. *Assume f satisfies anonymity and SP. f satisfies FNP if and only if $\forall N \subset \mathcal{N}$, $\forall R_N \in \mathcal{R}^{|N|}$, $\forall i \in N$, $\forall i' \in \mathcal{N} \setminus N$, $\forall R_{i'} \in \mathcal{R}$, $c(p_i, f(R_N)) \leq c(p_i, f(R_N, R_{i'}))$ holds.*

This means that to verify whether an anonymous and strategy-proof mechanism satisfies FNP, it suffices to focus only on one additional identifier. The following lemma shows that under Mechanism 3, no agent can be better off by just adding one identifier.

Lemma 6. *Let f be Mechanism 3. Then $\forall N \subset \mathcal{N}$, $\forall R_N \in \mathcal{R}^{|N|}$, $\forall i \in N$, $\forall i' \in \mathcal{N} \setminus N$, $\forall R_{i'} \in \mathcal{R}$, $c(p_i, f(R_N)) \leq c(p_i, f(R_N, R_{i'}))$ holds.*

Proof. Mechanism 3 determines the outcome depending only on the distinct peaks of the agents. Even if we add a preference $R^*_{i'}$ of agent i' s.t. $p(R^*_{i'}) = p(R_i)$, the outcome does not change, i.e., $f(R_N) = f(R_N, R^*_{i'})$, and thus $c(p_i, f(R_N)) = c(p_i, f(R_N, R^*_{i'}))$. Since it satisfies SP, agent i' has no beneficial misreport of preference; for any $R_{i'} \in \mathcal{R}$, $c(p_i, f(R_N, R^*_{i'})) \leq c(p_i, f(R_N, R_{i'}))$ holds. This coincides with the desired equation. □

7 Conclusion

In this paper, we investigated false-name-proof mechanisms for the two-facility location problem from two points of view. From an economic perspective, we characterized the possible outcomes by false-name-proof mechanisms on a line and fully characterized the class of false-name-proof mechanisms on a circle. From an algorithmic perspective, we investigated the approximation ratios of the deterministic/randomized false-name-proof mechanisms for both social and maximum costs on a line metric. One of our future directions is to consider the problem with three or more facilities. Also, on a circle, whether false-name-proof mechanisms exist with a bounded approximation ratio remains an open question. It would also be interesting to design false-name-proof mechanisms on trees and general multi-dimensional spaces.

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References

Alon, N.; Feldman, M.; Procaccia, A. D.; and Tennenholtz, M. 2010. Strategyproof approximation of the minimax on networks. *Mathematics of Operations Research* 35(3):513–526.

Bu, N. 2013. Unfolding the mystery of false-name-proofness. *Economics Letters* 120(3):559–561.

Ching, S., and Thomson, W. 1997. Population-monotonic solutions in public good economies with single-peaked preferences. *forthcoming in Social Choice and Welfare*.

Conitzer, V. 2008. Anonymity-proof voting rules. In *Proceedings of the 4th Workshop on Internet and Network Economics*, 295–306.

Filos-Ratsikas, A.; Li, M.; Zhang, J.; and Zhang, Q. 2015. Facility location with double-peaked preferences. In *Proceedings of the 29th AAAI Conference on Artificial Intelligence*, 893–899.

Fotakis, D., and Tzamos, C. 2013. On the power of deterministic mechanisms for facility location games. In *Automata, Languages, and Programming*. Springer. 449–460.

Gordon, S. 2007. Solidarity in choosing a location on a cycle. *Social Choice and Welfare* 29(1):125–147.

Heo, E. J. 2013. Strategy-proof rules for two public goods: double median rules. *Social Choice and Welfare* 41(4):895–922.

Lu, P.; Sun, X.; Wang, Y.; and Zhu, Z. A. 2010. Asymptotically optimal strategy-proof mechanisms for two-facility games. In *Proceedings of the 11th ACM Conference on Electronic Commerce*, 315–324.

Lu, P.; Wang, Y.; and Zhou, Y. 2009. Tighter bounds for facility games. In *Internet and Network Economics*. Springer. 137–148.

Miyagawa, E. 2001. Locating libraries on a street. *Social Choice and Welfare* 18(3):527–541.

Moulin, H. 1980. On strategy-proofness and single peakedness. *Public Choice* 35(4):437–455.

Procaccia, A. D., and Tennenholtz, M. 2013. Approximate mechanism design without money. *ACM Transactions on Economics and Computation* 1(4):18:1–18:26.

Schummer, J., and Vohra, R. V. 2002. Strategy-proof location on a network. *Journal of Economic Theory* 104(2):405–428.

Serafino, P., and Ventre, C. 2015. Truthful mechanisms without money for non-utilitarian heterogeneous facility location. In *Proceedings of the 29th AAAI Conference on Artificial Intelligence*.

Sui, X.; Boutilier, C.; and Sandholm, T. 2013. Analysis and optimization of multi-dimensional percentile mechanisms. In *Proceedings of the 23rd International Joint Conference on Artificial Intelligence*, 367–374.

Todo, T., and Conitzer, V. 2013. False-name-proof matching. In *Proceedings of the 12th International Conference on Autonomous Agents and Multiagent Systems*, 311–318.

Todo, T.; Iwasaki, A.; and Yokoo, M. 2011. False-name-proof mechanism design without money. In *Proceedings of the 10th International Conference on Autonomous Agents and Multiagent Systems*, 651–658.

Wagman, L., and Conitzer, V. 2014. False-name-proof voting with costs over two alternatives. *International Journal of Game Theory* 43(3):599–618.

Yokoo, M.; Sakurai, Y.; and Matsubara, S. 2004. The effect of false-name bids in combinatorial auctions: New fraud in internet auctions. *Games and Economic Behavior* 46(1):174–188.

Zou, S., and Li, M. 2015. Facility location games with dual preference. In *Proceedings of the 2015 International Conference on Autonomous Agents and Multiagent Systems*, 615–623.