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Counter-Transitivity in Argument Ranking Semantics*

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Abstract

The principle of counter-transitivity plays a vital role in argumentation. It states that an argument is strong when its attackers are weak, but weak when its attackers are strong. In this work, we develop a formal theory about the argument ranking semantics based on this principle. Three approaches, i.e., quantity-based, quality-based and the unity of them, are defined to implement the principle. Then, we show an iterative refinement algorithm for capturing the ranking on arguments based on the recursive nature of the principle.

Introduction

Argumentation theory has gained significant interest in the field of artificial intelligence as it provides the basis for computational models inspired by the way humans reason. The most widely used framework for exploring general issues of argumentation is Dung's abstract argumentation (Dung 1995). It is represented as a pair $AF = \langle \mathcal{X}, \mathcal{R} \rangle$, where $\mathcal{X}$ is a finite set of arguments and $\mathcal{R} \subseteq \mathcal{X} \times \mathcal{X}$ is a binary *attack relation* on $\mathcal{X}$. $(a, b) \in \mathcal{R}$ means that a attacks b, or a is an attacker of b. We denote by $\mathcal{R}^-(x)$ the subset of $\mathcal{X}$ containing those arguments that attack the argument $x \in \mathcal{X}$.

Dung provides a series of extension-based semantics for solving an *AF* by selecting acceptable subsets. Extensions are evaluated on the basis of the principle that an argument is accepted if all its attackers are rejected; an argument is rejected if at least one of its attackers is accepted. To generalize Dung's two extreme views on reasoning, a trend has emerged toward considering and exploring rankings on arguments induced by a larger number of categories or continuous numerical scales (e.g., (Cayrol and Lagasquie-Schiex 2005; Grossi and Modgil 2015)). Even though numerous works have been done, one basic principle behind them is: the more acceptable the arguments that attacks argument x are, the more unacceptable x is, and the more unacceptable these arguments are, the better x's acceptability is. Of course, if no argument attacks x, then it is indeed very acceptable. In short, an argument x is ranked higher than argument y, if the attackers of x are ranked lower than those of y. Borrowing concepts from (Amgoud and Ben-Naim 2013), we call this principle as *counter-transitivity*. In this work, we mainly concern on how to utilize this principle to compare and rank arguments. A formal theory about the argument ranking semantics is developed based on this principle.

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The principle of counter-transitivity

Let $\mathcal{T}$ be some set. An *ordering* $\succeq$ on $\mathcal{T}$ is a binary relation on $\mathcal{T}$ (i.e., $\succeq \subseteq \mathcal{T} \times \mathcal{T}$) such that: (1) $\succeq$ is reflective; (2) $\succeq$ is transitive. Here, $x \succeq y$ means that x is at least as acceptable as y. We denote $x \simeq y$ iff $x \succeq y$ and $y \succeq x$, which means x and y are equally acceptable. Moreover, $x \succ y$, means x is strictly more acceptable than y, iff $x \succeq y$ but not $y \succeq x$. x is incomparable with y if both $x \succeq y$ and $y \succeq x$ are not in $\succeq$.

The *ranking semantics* defined for an $AF = \langle \mathcal{X}, \mathcal{R} \rangle$ is a function Γ which will transform *AF* into an ordering on $\mathcal{X}$, denoted by $\succeq_\Gamma$. Now, we formally define the principle of counter-transitivity as below:

Definition 1 (Counter-transitivity)**.** *Let* $AF = \langle \mathcal{X}, \mathcal{R} \rangle$*, and* Γ *a ranking semantics.* Γ *satisfies* counter-transitivity *iff for every* $x, y \in \mathcal{X}$*, if* $\mathcal{R}^-(y) \succeq_\Gamma \mathcal{R}^-(x)$*, then* $x \succeq_\Gamma y$.

Note that Definition 1 involves the concept of *set comparison*, i.e., the comparison between $\mathcal{R}^-(x)$ and $\mathcal{R}^-(y)$. The definition of the order between two sets is similar with that of the order between two elements in previous definitions. In this work, we mainly consider three approaches to instantiate the concept of set comparison. The first one is the *cardinality-based comparison*: S_1 precedes S_2 if the elements of S_1 are more numerous than those of S_2, i.e., $S_1 \succeq_{\mathbb{C}} S_2$ iff $|S_1| \geq |S_2|$. The cardinality-based comparison can be seen as a quantitative approach. The second approach, called *quality-based comparison*, states that S_1 is better than S_2 iff there exists one element in S_1 satisfying that it is ranked higher than all elements in S_2, formally, $S_1 \succeq_{\mathbb{Q}} S_2$ iff $\exists x \in S_1$ such that $x \succeq y$ for all $y \in S_2$. To compare arguments by considering both quantity and quality in their unity, thus we introduce *unity-based comparison*: $S_1 \succeq_{\mathbb{U}} S_2$ iff $S_1 \succeq_{\mathbb{C}} S_2$ and $S_1 \succeq_{\mathbb{Q}} S_2$.

The above three approaches can induce three ranking semantics, called *cardinality-*, *quality-* and *unity-based ranking semantics*, respectively, and denoted by $\mathbb{C}$, $\mathbb{Q}$ and $\mathbb{U}$, respectively. Let $\succeq_{\mathbb{C}}$ and $\succeq_{\mathbb{U}}$ be two orderings on $\mathcal{X}$ transformed from the semantics $\mathbb{C}$ and $\mathbb{U}$ w.r.t. $AF = \langle \mathcal{X}, \mathcal{R} \rangle$, respectively. Obviously, $\succeq_{\mathbb{U}} \subseteq \succeq_{\mathbb{C}}$. We can see that the unity-based ranking semantics inherits the quantitative nature of the cardinality-based ranking semantics. In the following section, we will show that it also inherits the recursive nature of the quality-based ranking semantics (i.e., the rank of an argument relies on the rank of its attackers, which in turn depends on the rank of their attackers, and so on).

The recursive nature of counter-transitivity

The recursive nature of $\mathbb{Q}$ and $\mathbb{U}$ provides a way of solving them by iteration. Here, we consider Algorithm **??** that defines a ranking semantics for $AF = \langle \mathcal{X}, \mathcal{R} \rangle$, which iteratively refines an ordering on $\mathcal{X}$ by applying the principle of counter-transitivity. Let $x \succeq^{(k)} y$ denote the fact that the rank of x is higher than or equal to the rank of y in iteration k of the algorithm. The algorithm begins with an initial ordering in which all arguments have equal ranks (see line 3). The ordering in stage $k+1$ is refined from the ordering in stage k by the function $\mathsf{Refining}(\succeq_\Gamma^{(k)})$ described as: (a) Let $\succeq_\Gamma^{(k+1)} = \emptyset$; (b) For all pairs of x and y in $\mathcal{X}$, if $\mathcal{R}^-(y) \succ_\Gamma^{(k)} \mathcal{R}^-(x)$, then add the order $x \succ_\Gamma^{(k+1)} y$ to the set $\succeq_\Gamma^{(k+1)}$. By iteratively applying the function $\mathsf{Refining}$, if there is no change between the current ordering and the previous ordering, the algorithm terminates, and output the current ordering as the chosen one (in line 7 and 8). The final ordering will be consistent with the principle of counter-transitivity since if there exist a pair of arguments whose relative order should be refined to realize counter-transitivity, then the algorithm will not terminate and will continue to make further refinements.

Theorem 1. *Algorithm **??** necessarily converges with respect to $\Gamma \in \{\mathbb{Q}, \mathbb{U}\}$.*

Since $\succeq_\Gamma^{(0)} \supseteq \succeq_\Gamma^{(1)} \supseteq \succeq_\Gamma^{(2)} \supseteq \cdots$ is a decreasing sequence of sets of orderings on arguments for $\Gamma \in \{\mathbb{Q}, \mathbb{U}\}$, and there are only finitely many arguments, thus Algorithm **??** necessarily converges. Algorithm **??** will terminate after at most $|\mathcal{X}|$ iterative calls to function $\mathsf{Refining}$ since each iteration is just a refinement that makes the ordering more strict. In the following, an example is presented to show how Algorithm **??** ranks arguments within the context of quality- and unity-based semantics.

Algorithm 1: Solving ranking semantics by iteration

Input: $\langle \mathcal{X}, \mathcal{R} \rangle$: argumentation framework;
Output: $\succeq_\Gamma^{(k)}$: the (partial or complete) ordering on $\mathcal{X}$;
1 **begin**
2 $\quad k := 0$;
3 $\quad \succeq_\Gamma^{(0)} := \{\forall x, y \in \mathcal{X}, x \simeq^{(0)} y\}$;
4 $\quad$ **repeat**
5 $\quad\quad \succeq_\Gamma^{(k+1)} := \mathsf{Refining}(\succeq_\Gamma^{(k)})$;
6 $\quad\quad k := k+1$;
7 $\quad$ **until** $\succeq_\Gamma^{(k)} = \succeq_\Gamma^{(k-1)}$;
8 $\quad$ **return** $\succeq_\Gamma^{(k)}$;
9 **end**

Proposition 1. *Let $\succeq_\mathbb{Q}$ and $\succeq_\mathbb{U}$ be the orderings on $\mathcal{X}$ computed by Algorithm **??** w.r.t. $\mathbb{Q}$ and $\mathbb{U}$ for $AF = \langle \mathcal{X}, \mathcal{R} \rangle$, respectively. It holds that $\succeq_\mathbb{U} \subseteq \succeq_\mathbb{Q}$.*

Example 1. *Consider the AF with $\mathcal{X} = \{x, y_1, y_2, y_3, z\}$ and $\mathcal{R} = \{x\mathcal{R}y_i, y_i\mathcal{R}z\}, i = 1, 2, 3$. The cardinality-based ranking semantics gives $\succeq_\mathbb{C} = \{x \succ y_1 \simeq y_2 \simeq y_3 \succ z\}$. Now, we calculate the other semantics by using Algorithm **??**.*

For the quality-based ranking semantics $\mathbb{Q}$, the initial ordering is $\succeq_\mathbb{Q}^{(0)} = \{x \simeq y_1 \simeq y_2 \simeq y_3 \simeq z\}$. Applying $\mathsf{Refining}$, we obtain $\succeq_\mathbb{Q}^{(1)} = \{x \succ y_1 \simeq y_2 \simeq y_3 \simeq z\}$. Here, x is ranked above all other arguments as it has no attacker; $y_i \simeq z$ due to $\mathcal{R}^-(y_i) \simeq_\mathbb{Q}^{(0)} \mathcal{R}^-(z)$. Applying $\mathsf{Refining}$ again, we have $\succeq_\mathbb{Q}^{(2)} = \{x \succ z \succ y_1 \simeq y_2 \simeq y_3\}$, where z is ranked higher than y_i due to $\mathcal{R}^-(y_i) \succ_\mathbb{Q}^{(1)} \mathcal{R}^-(z)$. Applying $\mathsf{Refining}$ again, we also have $\succeq_\mathbb{Q}^{(3)} = \{x \succ z \succ y_1 \simeq y_2 \simeq y_3\}$. Then, the iteration process terminates, and the ordering for this AF w.r.t. $\mathbb{Q}$ is $\succeq_\mathbb{Q} = \{x \succ z \succ y_1 \simeq y_2 \simeq y_3\}$.

For the unity-based ranking semantics $\mathbb{U}$, the initial ordering is also $\succeq_\mathbb{U}^{(0)} = \{x \simeq y_1 \simeq y_2 \simeq y_3 \simeq z\}$. After applying the first $\mathsf{Refining}$, we get $\succeq_\mathbb{U}^{(1)} = \{x \succ y_1 \simeq y_2 \simeq y_3 \succ z\}$, in which $y_i \succ z$ as $|\mathcal{R}^-(y_i)| < |\mathcal{R}^-(z)|$ and $\mathcal{R}^-(z) \succeq_\mathbb{U}^{(0)} \mathcal{R}^-(y_i)$. Actually, as it is assumed in the initial ordering that the quality of all arguments are the same, thus, the first refinement ranks arguments merely based on the quantity of their attackers, and we thus have $\succeq_\mathbb{C} = \succeq_\mathbb{U}^{(1)}$. After the second refinement, we then obtain $\succeq_\mathbb{U}^{(2)} = \{x \succ y_1 \simeq y_2 \simeq y_3, x \succ z\}$, where z is incomparable with y_i, since $\mathcal{R}^-(y_i) \succeq_\mathbb{U}^{(1)} \mathcal{R}^-(z)$ is inconsistent with $|\mathcal{R}^-(y_i)| < |\mathcal{R}^-(z)|$. The third refinement has the same result as the second one. Thus, $\succeq_\mathbb{U} = \{x \succ y_1 \simeq y_2 \simeq y_3, x \succ z\}$ for this AF. We can see that $\succeq_\mathbb{U} \subseteq \succeq_\mathbb{Q}$, which is compatible with Proposition 1.

Conclusion

The work discusses the principle of counter-transitivity in argumentation and addresses the issue of graded acceptability of arguments from a new perspective. It formalises three ranking semantics based on three comparison criteria and an iterative algorithm to compute two of those proposed semantics.

References

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