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Optimizing Infrastructure Enhancements for Evacuation Planning

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Abstract

With rapid population growth and urbanization, emergency services in various cities around the world worry that the current transportation infrastructure is no longer adequate for large-scale evacuations. This paper considers how to mitigate this issue through infrastructure upgrades, such as the additions of lanes to road segments and the raising of bridges and roads. The paper proposes a MIP model for deciding the most effective infrastructure upgrades as well as a Benders decomposition approach where the master problem jointly plans the upgrades and evacuation routes and the subproblem schedules the evacuation itself. Experimental results demonstrate the practicability of the approach on a real case study, filling a significant need for emergencies services.

Introduction

With rapid population growth and increased urbanization, emergency services in various cities around the world worry that the current transportation infrastructure is no longer adequate for large-scale evacuations. In some cities, the infrastructure, and in particular the road network, has not kept up with population growth (Feneley 2015). This is especially worrisome given that existing infrastructure capacity is often well below the level what would be required for effective large-scale evacuations (Litman 2006). Yet little research on evacuation planning includes the possibility of improving road infrastructure. Some studies use contraflow in order to increase road capacities (Wolshon 2001; Theodoulou and Wolshon 2004; Even, Pillac, and Van Hentenryck 2014; Kim, Shekhar, and Min 2010). However, Wolshon (2001) warns that the presence of contraflow lanes can lead to congestion due to drivers' unfamiliarity with lane reversal. Peeta et al. (2010) considered structural upgrades that would strengthen roads against earthquake damage. In their model, the upgrades increase the probability that a road will withstand an earthquake.

In this paper, we attempt to fill this gap in the literature. We study how to upgrade the road network in order to maximize the number of evacuees reaching safety given an infrastructure budget. We consider zone-based evacuation planning over convergent plans and two types of upgrades: adding lanes to selected road segments and raising road segments so that they survive the flood. Zone-based evacuations assign a unique path to safety to each residential area, which makes mobilization and evacuation operations simpler to execute. Convergent plans ensure that there are no forks in the evacuation graph, making plan compliance simpler to enforce. Convergent plans also eliminate forks in the evacuation graph, which may lead to congestion due to driver hesitation (Wolshon, Catarella-Michel, and Lambert 2006). Even, Pillac, and Van Hentenryck (2015) showed that convergent plans can bring significant benefits on a flood case study in West Sydney, Australia.

To decide which infrastructure enhancements to perform, we present two approaches: 1) A MIP model whose decision variables are the infrastructure investment, the evacuation paths, and the evacuation schedule; 2) A Benders decomposition whose master variables are the investment decisions and the evacuation paths and the subproblem variables denote the evacuation schedule.

Experimental results evaluate the practicability of the approaches on the case study presented by Even, Pillac, and Van Hentenryck (2015) concerning a flood event in the Hawkesbury-Nepean region in the West of Sydney. Our results indicate the practibility of our Benders decomposition approach which significantly outperforms the MIP model and exbihits small optimality gaps in reasonable time. Our results also report on the tradeoff between the quality of the evacuation plans and the budget.

The rest of this paper is organized as follows. We first review the literature on the relevant evacuation planning algorithms. We then present the problem modeling, the MIP model, and the Benders decomposition approach. Finally, we present the experimental results and conclude the paper. Note that, although the results are presented for convergent plans, the approaches can be generalized to arbitrary zone-based evacuations.

Literature Review

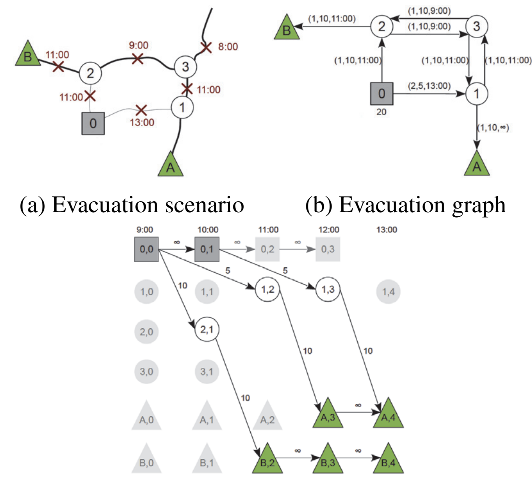
In this paper, we consider a macroscopic approach to evacuation planning, where evacuees are not modeled individually but as flows in the space-time representation of the road network (Hamacher and Tjandra 2002). Almost all the macroscopic evacuations consider dynamic network flow problems following the pioneering work of (Ford and Fulkerson

1958) on the Maximum Dynamic Network Flow Problem (MDFP) for shipping goods. See also (Chen and Chin 1990; Burkard, Dlaska, and Klinz 1993; Hoppe and Tardos 1995; Choi, Hamacher, and Tufekci 1988; Chalmet, Francis, and Saunders 1982; Yamada 1996). However, emergency services often prefer zone-based evacuations, in which all residents in a residential area are assigned the same evacuation path in order to avoid confusion and increase compliance. Zone-based evacuations were studied in a number of papers including, for instance, (Bish and Sherali 2013; Huibregtse et al. 2011; Even, Pillac, and Van Hentenryck 2015). Zone-based evacuation over convergent plans was studied in (Andreas and Smith 2009; Even, Pillac, and Van Hentenryck 2015). The use of Benders decomposition in evacuation planning was considered in (Chen and Miller-Hooks 2008; Andreas and Smith 2009). Chen and Miller-Hooks (2008) used Benders decomposition for quickest flow problem in a building evacuation problem with shared Information. Andreas and Smith (2009) solved a variant of the quickest flow problem, using arc traversal penalty functions in order to encourage earlier evacuation. The model includes a number of possible scenarios, each with a given probability of occurring, and the objective is to minimize the expected sum of arc traversal penalties. The work by Even, Pillac, and Van Hentenryck (2015) is particularly relevant to this paper. They propose a two-stage approach for zone-based evacuation planning, where the first stage is a tree design problem which gives an upper bound to the number of evacuees reaching safety by aggregating the arc capacities. The tree design problem chooses the evacuation paths for the zone-based evacuation planning, while the second stage schedules the evacuation over these paths. The Benders decomposition presented in this paper transforms this two-stage approach into a Benders decomposition approach and generalizes the tree design problem into a restricted master problem which also includes decisions for infrastructure enhancement. Our initial master also gives an upper bound to the objective of the original problem and Pareto-optimal Benders cuts are generated at each iteration.

Problem Description

Following Even, Pillac, and Van Hentenryck (2015), we model the evacuation scenario with an *evacuation graph* $\mathcal{G} = (\mathcal{N} = \mathcal{E} \cup \mathcal{T} \cup \mathcal{S}, \mathcal{A})$, where $\mathcal{E}$, $\mathcal{T}$, and $\mathcal{S}$ are the set of evacuation, transit, and safe nodes respectively, and $\mathcal{A}$ is the set of arcs. Each evacuation node i is associated with a demand d_i. Each arc e is labeled with its travel time s_e, its capacity u_e, and the time f_e when it becomes flooded over. Figure 1 gives an example of such a model. Figure 1a illustrates an evacuation scenario that has one evacuation node labeled “0”, and two safe nodes labelled “A” and “B”. The times on each arc indicate when the flood would arrive if the road were not elevated. Figure 1b is an evacuation graph based on the evacuation scenario. There is a demand of 20 vehicles from the evacuation node. Arc $(0, 1)$ has a travel time of 2 minutes and a capacity of 5 vehicles/minute, and will be flooded at 13:00 if it is not elevated.

We express the spatio-temporal aspect of the problem through a *time-expanded graph* $\mathcal{G}^x = (\mathcal{N}^x = \mathcal{E}^x \cup \mathcal{T}^x \cup \mathcal{S}^x, \mathcal{A}^x)$. In the time-expanded graph, there is a copy of each static node for each discrete time step within the horizon. The set $\mathcal{A}^x$ contains time edges $e_t = (i_t, j_{t+s_{(i,j)}})$ corresponding to static edges $e = (i, j)$, for each such pair of times within the horizon. Figure 1c is the time-expanded graph for this scenario.



(a) Evacuation scenario (b) Evacuation graph

(c) Time-expanded graph

Figure 1: Modeling of an Evacuation Planning Problem. (Source: Even, Pillac, and Van Hentenryck (2015)).

There are two possible infrastructure upgrades: adding new lanes and elevating roads. Each lane has an existing number of lanes n_e, as well as a maximum number of lanes that can be added n_e^+. *We assume that capacity increases linearly with the number of lanes.* Each road segment can also be elevated, extending its availability by a given amount of time. The costs of the upgrades are given by $c_l(e)$ for adding a single lane to arc e and $c_e(e)$ for elevating arc e to extend its availability by a single time step. These costs are given per unit length. The *Convergent Evacuation Network Design Problem* can now be defined:

Definition 1. *The* Convergent Evacuation Network Design Problem (CENDP) *consists in finding a convergent evacuation plan that includes two kinds of infrastructure upgrades: lane additions and road elevations.*

The MIP Model

This section presents a MIP model for solving the CENDP. The decision variables in the model are as follows: Variable x_e is binary and represents whether arc e is selected, variable φ_{e_t} is continuous and denotes the flow on arc $e_t \in \mathcal{A}^x$, variable z_e is integer and indicates the number of lanes added to arc e, and variable v_{e_t} is binary and indicates whether arc e is available at time t according to its road elevation. The objective (1) maximizes the total flow of evacuees, with $\delta^-(k)$ and $\delta^+(k)$ respectively denoting the set of incoming and outgoing edges of node k. For simplicity, we assume that all roads have the same limit n^+ on the number of additional lanes and that the upgrade costs per unit distance are the same for all edges (c_l and c_e). It is easy to generalize the model to remove these assumptions. The MIP model operates on the expanded graph and is given by

$$\max \sum_{k \in \mathcal{E}} \sum_{e_t \in \delta^+(k)} \varphi_{e_t} \quad (1)$$

s.t.

$$\sum_{e_t \in \delta^-(i)} \varphi_{e_t} - \sum_{e_t \in \delta^+(i)} \varphi_{e_t} = 0 \qquad \forall i \in \mathcal{T}^x \quad (2)$$

$$\sum_{e \in \delta^+(i)} x_e \leq 1 \qquad \forall i \in \mathcal{E} \cup \mathcal{T} \quad (3)$$

$$\varphi_{e_t} \leq x_e \left(1 + \frac{n^+}{n_e}\right) u_{e_t} \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \quad (4)$$

$$\varphi_{e_t} \leq v_{e_t} \left(1 + \frac{z_e}{n_e}\right) u_{e_t} \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \quad (5)$$

$$\sum_{e_t \in \delta^+(k)} \varphi_{e_t} \leq d_k \qquad \forall k \in \mathcal{E} \quad (6)$$

$$v_{e_t} \geq v_{e_{t+1}} \qquad \forall e_t, e_{t+1} \in \mathcal{A}^{\mathrm{x}} \quad (7)$$

$$v_{e_t} = 1 \qquad \forall e \in A, \forall t \in [0, f_e) \quad (8)$$

$$\sum_{t=f_e}^{h} v_{e_t} = w_e \qquad \forall e \in \mathcal{A} \quad (9)$$

$$\sum_{e \in \mathcal{A}} l_e \left(c_l \cdot z_e + c_e \cdot w_e\right) \leq \mathcal{B} \quad (10)$$

$$z_e \leq n^+ \qquad \forall e \in \mathcal{A} \quad (11)$$

$$\varphi_{e_t} \geq 0 \qquad \forall e \in \mathcal{A} \quad (12)$$

$$x_e \in \{0, 1\} \qquad \forall e \in \mathcal{A} \quad (13)$$

$$z_e, w_e \in \mathbb{Z}^+ \qquad \forall e \in \mathcal{A} \quad (14)$$

$$v_{e_t} \in \{0, 1\} \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \quad (15)$$

Constraints (2) enforce flow conservation at each transit node, constraints (3) impose that the evacuation plan satisfies a tree structure (thus producing a convergent plan), and constraints (4) ensure that evacuation flows only travel on selected edges. Constraints (5) limit the flows to the edge capacities and capture the fact that the road capacity increases linearly with the number of lanes. Constraints (6) make sure that the total outflow of evacuees of each evacuation node does not exceed the node demand. Constraints (7) ensure that road blockages due to the flood persists until the time horizon and constraints (8) express the road availability before the onset of the flood. Constraints (9) capture the possibility of raising a road for a number of time steps. Constraint (10) is the budget constraint, where l_e is the length of arc e and w_e is number of units of elevation upgrades on e.

It is important to note that constraints (5) are nonlinear as they contain products of two decision variables. However, these products can be linearized since variables v_{e_t} are binary. The next section, which presents the Benders decomposition, will illustrate this in more detail.

Benders Decomposition

The second approach considered in this paper is a Benders decomposition which separate the decision variables in two stages. The Restricted Master Problem (RMP) selects a set of convergent paths and infrastructure upgrades. The Subproblem (SP) schedules the departure times of evacuees. The RMP remains a MIP model but on the evacuation graph and not its time-expanded counterpart. The SP is a linear program on the expanded graph. A Benders decomposition proceeds as follows:

1. It first solves the RMP obtaining optimal values for the RMP decision variables;
2. It then solves the SP with the RMP variables fixed to their optimal values;
3. If the objective values of the RMP and SP coincide, the solution is optimal. Otherwise, a Benders cut is generated from the optimal solution of the SP and added to the RMP and the process is iterated.

In the Benders decomposition considered here, the SP is always feasible and the Benders constraints are optimality-based cuts. The RMP always returns an upper bound to the optimal number of evacuaees reaching safety, while the SP returns a feasible solution. We now go into the details of the Benders decomposition which uses Pareto-optimal cuts.

The Restricted Master Problem

We first present the RMP without the Benders cuts which will be derived from the SP. As mentioned previously, the RMP operates on the evacuation graph, not its time expansion. However, to produce reasonable evacuation plans and infrastructure improvements to seed the Benders decomposition, we use an idea from Even, Pillac, and Van Hentenryck (2015) and aggregate capacities over time for each arc in the graph. This makes it possible to have a RMP which provides an upper bound to the optimal value, while still working on the evacuation graph. Besides the decision variables x_e, z_e, and v_{e_t}, the RMP also uses a variable ψ_e to represent the aggregate flow of evacuees over time along arc e.[1] The objective of the RMP is to maximize the aggregate flow from evacuation nodes to safe nodes within the time horizon, given the infrastructure upgrade budget. The RMP can thus be specified as follows and can really be seen as an aggregation of the MIP model:

[1]The RMP can be simplified by aggregating the decision variables v_{e_t}, but we kept this formulation for simplicity.

$$\max \sum_{k \in \mathcal{E}} \sum_{e \in \delta^+(k)} \psi_e \quad (16)$$

s.t.

$$\sum_{e \in \delta^-(i)} \psi_e - \sum_{e \in \delta^+(i)} \psi_e = 0 \quad \forall i \in \mathcal{T} \quad (17)$$

$$\sum_{e \in \delta^+(i)} x_e \leq 1 \quad \forall i \in \mathcal{E} \cup \mathcal{T} \quad (18)$$

$$\psi_e \leq x_e \left(1 + \frac{n^+}{n_e}\right) \sum_{t \in \mathcal{H}} u_{e_t} \quad \forall e \in \mathcal{A} \quad (19)$$

$$\psi_e \leq \left(1 + \frac{z_e}{n_e}\right) \sum_{t \in \mathcal{H}} v_{e_t} \cdot u_{e_t} \quad \forall e \in \mathcal{A} \quad (20)$$

$$\sum_{e \in \delta^+(k)} \psi_e \leq d_k \quad \forall k \in \mathcal{E} \quad (21)$$

$$v_{e_t} \geq v_{e_{t+1}} \quad \forall e_t, e_{t+1} \in \mathcal{A}^{\text{x}} \quad (22)$$

$$v_{e_t} = 1 \quad \forall e \in A, \forall t \in [0, f_e) \quad (23)$$

$$\sum_{t=f_e}^{h} v_{e_t} = w_e \quad \forall e \in \mathcal{A} \quad (24)$$

$$\sum_{e \in \mathcal{A}} l_e \left(c_l \cdot z_e + c_e \cdot w_e\right) \leq \mathcal{B} \quad (25)$$

$$z_e \leq n^+ \quad \forall e \in \mathcal{A} \quad (26)$$

$$\psi_e \geq 0 \quad \forall e \in \mathcal{A} \quad (27)$$

$$x_e \in \{0,1\} \quad \forall e \in \mathcal{A} \quad (28)$$

$$z_e, w_e \in \mathbb{Z}^+ \quad \forall e \in \mathcal{A} \quad (29)$$

$$v_{e_t} \in \{0,1\} \quad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \quad (30)$$

Constraints (17) impose the aggregate flow conservation at each transit node, constraints (18) enforce a tree structure, and constraints (19) ensure that flow will only be sent on selected arcs. Constraints (20) and (21) are the capacity and demand constraints, and constraint (25) is the budget constraint. The objective (16) maximizes the aggregate flow. Constraints (20) are nonlinear as they contain products of variables $z_e \cdot v_{e_t}$. These constraints can be linearized by replacing each product with a new variable p_{e_t} to represent such a product and adding the following constraints:

$$p_{e_t} \leq z_e \quad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \quad (31)$$

$$p_{e_t} \leq v_{e_t} \cdot n^+ \quad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \quad (32)$$

$$p_{e_t} \in \mathbb{Z}^+ \quad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \quad (33)$$

We now show that the RMP is an upper bound to the CENDP. The proof is an extension of the result by Even, Pillac, and Van Hentenryck (2015) for the simpler case with no infrastructure enhancements.

Theorem 1. *The optimal solution of the RMP is an upper bound to the CENDP.*

Proof. The proof relies on showing that any optimal solution to the CENDP is also a feasible solution to the RMP with the same objective value. Let $\Phi = (\{\varphi_{e_t}\}, \{x_e\}, \{z_e\}, \{v_{e_t}\})$ be an optimal solution to the CENDP, with an objective value of $z(\Phi)$. Clearly, constraints (18) and (22) through (30) in the RMP will be satisfied. Let

$$\psi_e = \sum_{t \in \mathcal{H}} \varphi_{e_t}$$

for each arc $e \in \mathcal{A}$. The objective value of the RMP will be the same as the CENDP because

$$\begin{aligned} z(\Phi) &= \sum_{k \in \mathcal{E}} \sum_{e_t \in \delta^+(k)} \varphi_{e_t} \\ &\equiv \sum_{k \in \mathcal{E}} \sum_{e \in \delta^+(k)} \sum_{t \in \mathcal{H}} \varphi_{e_t} \\ &= \sum_{k \in \mathcal{E}} \sum_{e \in \delta^+(k)} \psi_e \end{aligned}$$

Since Φ is a solution to the CENDP, we have

$$\begin{aligned} &\sum_{e_t \in \delta^-(i)} \varphi_{e_t} - \sum_{e_t \in \delta^+(i)} \varphi_{e_t} = 0 && \forall i \in \mathcal{T}^x \\ \Rightarrow &\sum_{e \in \delta^-(i)} \sum_{t \in \mathcal{H}} \varphi_{e_t} - \sum_{e \in \delta^+(i)} \sum_{t \in \mathcal{H}} \varphi_{e_t} = 0 && \forall i \in \mathcal{T} \\ \Rightarrow &\sum_{e \in \delta^-(i)} \psi_e - \sum_{e \in \delta^+(i)} \psi_e = 0 && \forall i \in \mathcal{T} \end{aligned}$$

so that constraints (17) are satisfied. Similarly,

$$\begin{aligned} &\varphi_{e_t} \leq x_e \left(1 + \frac{n^+}{n_e}\right) u_{e_t} && \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \\ \Rightarrow &\sum_{t \in \mathcal{H}} \varphi_{e_t} \leq \sum_{t \in \mathcal{H}} x_e \left(1 + \frac{n^+}{n_e}\right) u_{e_t} && \forall e \in \mathcal{A} \\ \Rightarrow &\psi_e \leq x_e \left(1 + \frac{n^+}{n_e}\right) \sum_{t \in \mathcal{H}} u_{e_t} && \forall e \in \mathcal{A} \end{aligned}$$

satisfying constraints (19). Also,

$$\begin{aligned} &\varphi_{e_t} \leq v_{e_t} \left(1 + \frac{z_e}{n_e}\right) u_{e_t} && \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \\ \Rightarrow &\sum_{t \in \mathcal{H}} \varphi_{e_t} \leq \sum_{t \in \mathcal{H}} v_{e_t} \left(1 + \frac{z_e}{n_e}\right) u_{e_t} && \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \\ \Rightarrow &\psi_e \leq \left(1 + \frac{z_e}{n_e}\right) \sum_{t \in \mathcal{H}} v_{e_t} \cdot u_{e_t} && \forall e \in \mathcal{A} \end{aligned}$$

Finally,

$$\sum_{e_t \in \delta^+(k)} \varphi_{e_t} \leq d_k \qquad \forall k \in \mathcal{E}$$

$$\equiv \sum_{e \in \delta^+(k)} \sum_{t \in \mathcal{H}} \varphi_{e_t} \leq d_k \qquad \forall k \in \mathcal{E}$$

$$\Rightarrow \sum_{e \in \delta^+(k)} \psi_e \leq d_k \qquad \forall k \in \mathcal{E}$$

□

The Subproblem

The RMP produces a convergent evacuation graph $\mathcal{G}$ with infrastructure upgrades, specified by the optimal values $\overline{x}_e$, $\overline{z}_e$, and $\overline{v}_{e_t}$ for the RMP decision variables x_e, z_e, and v_{e_t}. The SP uses these optimal values to determine the departure times of evacuees in the expanded graphs maximizing the number of evacuees reaching safety. The SP can be formulated as follows:

$$\max \sum_{k \in \mathcal{E}} \sum_{e_t \in \delta^+(k)} \varphi_{e_t} \tag{34}$$

s.t.

$$\sum_{e_t \in \delta^-(i)} \varphi_{e_t} - \sum_{e_t \in \delta^+(i)} \varphi_{e_t} = 0 \qquad \forall i \in \mathcal{T}^x \tag{35}$$

$$\varphi_{e_t} \leq \overline{x}_e \cdot u_{e_t} \left(1 + \frac{n^+}{n_e}\right) \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{36}$$

$$\varphi_{e_t} \leq \overline{v}_{e_t} \cdot u_{e_t} \left(1 + \overline{z}_e n_e\right) \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{37}$$

$$\sum_{e_t \in \delta^+(k)} \varphi_{e_t} \leq d_k \qquad \forall k \in \mathcal{E} \tag{38}$$

$$\varphi_{e_t} \geq 0 \qquad \forall e_t \in \mathcal{A}^x \tag{39}$$

Constraints (35) are the flow conservation constraints. Constraints (36) ensure that flow will only be sent on edges in the network. Constraints (37) and (38) are the capacity and demand constraints. Note that the right-hand sides of constraints (36), (37), and (38) are constants. The objective (34) maximizes the flow.

The Restricted Master Problem Revisited

After each iteration of the RMP and SP, and whenever the objective value of the SP is smaller than the objective value of the RMP, the Benders decomposition algorithm generates a cut of the form:

$$z \leq \sum_{e \in \mathcal{A}} x_e \left(1 + \frac{n^+}{n_e}\right) \sum_{t \in \mathcal{H}} u_{e_t} \cdot y_{e_t} + \sum_{e \in \mathcal{A}} \sum_{t \in \mathcal{H}} \left(v_{e_t} \cdot u_{e_t} + \frac{p_{e_t} \cdot u_{e_t}}{n_e}\right) y'_{e_t} + \sum_{k \in \mathcal{E}} d_k \cdot y_k \tag{40}$$

where $\{y_{e_t}\}$, $\{y'_{e_t}\}$, and $\{y_k\}$ are the dual variables associated with SP constraints (36), (37), and (38) respectively. These constraints accumulate in the RMP, which remains an upper bound to the CENDP since the Benders cuts are valid and do not remove optimal solutions.

Pareto-Optimal Cuts

It is well-known that a Benders decomposition with a flow-based subproblem may be slow to converge, as the SP may have many optimal solutions. Hence many Benders cuts may need to be generated. To strengthen the Benders cuts, we use the Magnanti-Wong method (Magnati and Wong 1981), which generates *Pareto-optimal cuts* for the restricted master problem. A Pareto-optimal cut is defined as a cut that is not dominated by any other cut for a given iteration of the decomposition. Denote by X the convex hull of the feasible solutions to the RMP, i.e., feasible assignments for variables x_e, z_e, and v_{e_t}. The Magnanti-Wong method uses a *core point* of X in order to generate a Pareto-optimal cut, i.e., a point x^0 in the interior of X. To generate a Pareto-optimal Benders cut, it is necessary to solve the dual of the Magnanti-Wong Problem:

$$\max \sum_{k \in \mathcal{E}} \Big(\sum_{e_t \in \delta^+(k)} \varphi_{e_t} + \xi \sum_{e_t \in \delta^+(k)} \bar{\varphi}_{e_t} \Big)$$

s.t.

$$\sum_{e_t \in \delta^-(i)} \varphi_{e_t} - \sum_{e_t \in \delta^+(i)} \varphi_{e_t} = 0 \qquad \forall i \in \mathcal{T}^x$$

$$\varphi_{e_t} + \bar{x}_e \cdot u_{e_t} \left(1 + \frac{n^+}{n_e}\right) \cdot \xi \leq x_e^0 \cdot u_{e_t} \left(1 + \frac{n^+}{n_e}\right) \qquad \forall e \in \mathcal{A}, t \in \mathcal{H}$$

$$\varphi_{e_t} + \bar{v}_{e_t} \cdot u_{e_t} \left(1 + \frac{\bar{z}_e}{n_e}\right) \cdot \xi \leq v_{e_t}^0 \cdot u_{e_t} \left(1 + \frac{z_e^o}{n_e}\right) \qquad \forall e \in \mathcal{A}, t \in \mathcal{H}$$

$$\sum_{e_t \in \delta^+(k)} \varphi_{e_t} + d_k \cdot \xi \leq d_k \qquad \forall k \in \mathcal{E}$$

$$\varphi_{e_t} \geq 0 \qquad \forall e_t \in \mathcal{A}^x$$

where $\{\bar{x}_e\}$, $\{\bar{z}_e\}$ and $\{\bar{v}_{e_t}\}$ are from the optimal solution to the RMP and $\{\bar{\varphi}_{e_t}\}$ are from the optimal solution to the SP. We set the core point to be the center point of the convex hull of the domain: $x_e^0 = \frac{1}{2} \forall e \in \mathcal{A}$, $z_e^0 = \frac{n^+}{2} \forall e \in \mathcal{A}$, $v_{e_t}^0 = 1 \forall e \in \mathcal{A}, t \in [0, f_e)$, and $v_{e_t}^0 = \frac{1}{2} \forall e \in \mathcal{A}, t \in [f_e, h]$. The Benders cuts use dual variables from the Magnanti-Wong Problem, which means $\{y_{e_t}\}$, $\{y'_{e_t}\}$, and $\{y_k\}$ are the dual variables of the three inequality constraints.

Experimental Results

The MIP and the Benders decomposition approaches were applied to the evacuation of the Hawkesbury-Nepean (HN) floodplain, located near Sydney. This region was also considered by Even, Pillac, and Van Hentenryck (2015) but, since their work, there have been increasing concerns that the road infrastructure has not kept up with population growth (see, for instance, (Feneley 2015)). This region is a large floodplain protected by the Warrangaba Dam from the Blue Montains where precipitation accumulates. The dam spills over every year and the authorities are worried

Instance	Benders Decomposition CPU (s)	LRMP (%)	BD (%)	BD10 (%)	Gap (%)	MIP Perc. Evac.	%Imp
HN-1.7							
300 min	779.7	100	98.2	94.5	1.8	87.5	12.2
360 min	885.6	100	99.2	97.7	0.8	99.2	0.0
420 min	386.9	100	100	100	0	99.0	1.0
HN-2.0							
300 min	763.1	100	96.1	92.9	4.0	89.4	7.5
360 min	3585.3	100	98.6	97.5	1.4	91.7	7.5
420 min	2559.5	100	99.5	99.2	0.5	87.9	13.1
HN-2.5							
300 min	29.8	100	88.9	88.9	12.5	78.6	13.1
360 min	524.4	100	96.9	96.9	3.2	82.5	17.4
420 min	1069.9	100	97.0	96.3	3.1	80.7	20.2
HN-3.0							
300 min	616.7	100	81.5	80.2	22.6	65.6	24.3
360 min	1884.1	100	86.5	84.2	15.6	67.9	27.5
420 min	931.3	100	90.9	87.1	10.0	70.6	28.9

Table 1: Results for the HN-1.7, 2.0, 2.5, and 3.0 Instances.

about the need to evacuate the flood plain which hosts about 80,000 people. We consider a number of worst-case scenarios where a significant (1 in 100 years) flood would reach the flood plain after 5, 6, or 7 hours respectively and uses the flood extent as computed by standard 2D hydro-dynamic flood simulation models. The road infrastructure consists of 80 evacuation nodes, 184 transit nodes, 5 safe nodes, and 580 arcs. The time horizon was discretized into 5 minute intervals. The upgrade costs were taken to be 5 units per kilometer of additional lanes built and 0.01 units per kilometer for elevating a road to extend its availability by one time step. Unless otherwise stated, the budget is 100 units. The population was scaled by a factor $x \in [1.7, 3]$ to model population growth in the Hawkesbury-Nepean region. Each instance was run for up to one hour. The algorithms were implemented using JAVA 8 with GUROBI 6.0 and run on a 64 bit machine with a 1.4 GHz Intel Core i5 processor and 4 GB of RAM under OSX 10.10.5.

Table 1 compares the percent evacuated by the Benders decomposition and the MIP model (1 h) on four population instances and three flood scenarios. The CPU times correspond to when the best FSP value was found by the Benders decomposition. The LRMP is the last Restricted Master Problem solution, and BD is the Benders decomposition solution. Column BD10 is the best Benders decomposition solution after 10 minutes. The gap is between the LRMP and best BD and is calculated as $\frac{z(\texttt{LRMP}(\mathcal{G},\mathcal{H},\mathcal{B}))-z(\texttt{BD}(\mathcal{G},\mathcal{H},\mathcal{B}))}{z(\texttt{BD}(\mathcal{G},\mathcal{H},\mathcal{B}))}$. Column %Imp is the improvement of the Benders approach over the MIP model in percentage.

The duality gaps are quite small for instance HN-1.7, but increase with the population growth. The Benders decomposition provides significant improvements compared to the MIP model: The difference in quality grows as the population increases and the Benders decomposition evacuates about 30% more people on the last instance. The Benders decomposition after 10 minutes also improves the MIP in all but one instance, evacuating up to 23% more people.

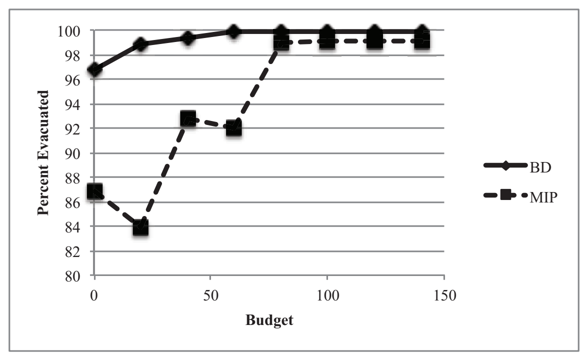


Figure 2: Perc. of People Evacuated for Given Budgets.

Figure 2 shows the effect of varying the budget parameter for Instance 1.7 with a flood arriving at the 360 minute mark, a profile emergency services are keen to study. The graph shows that the performance of MIP model degrades substantially when the budget is tight and performs reasonably when the budget is sufficiently large to evacuate everyone. In contrast, the Benders formulation produces excellent results for all budgets. This confirms the findings of Table 1, where the quality differences between the Benders decomposition and the MIP model increase with population growth. This is especially relevant, since infrastructure improvement projects traditionally operate under tight budgets.

Conclusion

In this paper, we introduced the Convergent Evacuation Network Design Problem (CENDP) for creating convergent evacuation plans with infrastructure upgrades. We proposed a MIP model and a Benders decomposition approach for solving the CENDP. The aprroaches were tested on a case study for a flood plain West of Sydney where the road infrastructure has not kept up with population growth, creating significant concerns from emergency services (Feneley 2015). Experimental results show that the Benders decomposition performed significantly better than the MIP model, evacuating as much as 28.9% more people on the instances with higher population growth. Varying the budget for the easiest instance revealed the large gap in solution quality between the Benders approach and the MIP model, especially when the budget is tight which is the typical case in infrastructure improvement studies.

Overall, our results show that Benders decomposition provides a novel tool for emergency services that seek to improve their road infrastructure to meet the evacuation needs coming from increased urbanization. Our current work focuses on generalizing the Benders decomposition to handle uncertainty by considering multiple disaster scenarios.

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