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Benders Decomposition for Large-Scale Prescriptive Evacuations

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Abstract

This paper considers prescriptive evacuation planning for a region threatened by a natural disaster such a flood, a wildfire, or a hurricane. It proposes a Benders decomposition that generalizes the two-stage approach proposed in earlier work for convergent evacuation plans. Experimental results show that Benders decomposition provides significant improvements in solution quality in reasonable time: It finds provably optimal solutions to scenarios considered in prior work, closing these instances, and increases the number of evacuees by 10 to 15% on average on more complex flood scenarios.

Introduction

Emergency evacuation is becoming increasingly critical with the rising number of natural disasters (e.g., floods, fires, hurricanes) and rapid urbanization across the globe. Evacuations, once rare events, are becoming increasingly common: On average, there is an evacuation of 1,000 or more people every two or three weeks in the United States alone (Murray-Tuite and Wolshon 2013). Moreover, the global trend towards urbanization makes planning for city-level evacuations imperative. For example, in Australia and Chile, 85% of the population lives in cities (Kates 2010).

This paper considers prescriptive evacuation planning (e.g., (Bish and Sherali 2013; Bretschneider and Kimms 2012; Cova and Johnson 2003; Dish, Sherali, and Hobeika 2013; Miller-Hooks and Sorrel 2008; Lim et al. 2012; Xie, Lin, and Waller 2010)) which has been gaining traction in the last decade. In prescriptive evacuations, the goal is to produce an operationally viable set of instructions for the authorities, who will close roads and manage traffic, and clear directions for evacuees about when to leave their homes and where to go. This task is computationally challenging because many factors must be taken into account, including the nature of the disaster, the layout of the road network, the locations of evacuees, and human behavior. Prescriptive evacuation planning contrasts with self-evacuations where people can choose how to evacuate and when, often leading to congestion as the demand exceeds the network capacity.

This paper reconsiders the Convergent Evacuation Planning Problem (CEPP) introduced by Even, Pillac, and Van Hentenryck (2015) to eliminate delays associated with forks in evacuation plans. Indeed, it was observed during Hurricane Ivan that forks used in contraflow operations induce driver hesitation and may become a significant source of delays (Wolshon, Catarella-Michel, and Lambert 2006). Convergent paths are easy to enforce because, once the necessary roads are blocked, minimal vehicle guidance is required. Even, Pillac, and Van Hentenryck (2015) proposed a two-stage approach to solve the CEPP. Their first stage is a tree design problem which chooses a convergent plan by aggregating the arc capacities, while the second stage is a flow scheduling problem on the time-expanded graph. They showed that the Tree Design Problem (TDP) is a relaxation of the CEPP. This two-stage approach produces high-quality results very quickly whenever the disaster affects all the road segments uniformly. However, on more complex disaster scenarios, the quality of the two-stage approach degrades and the duality gap is more substantial.

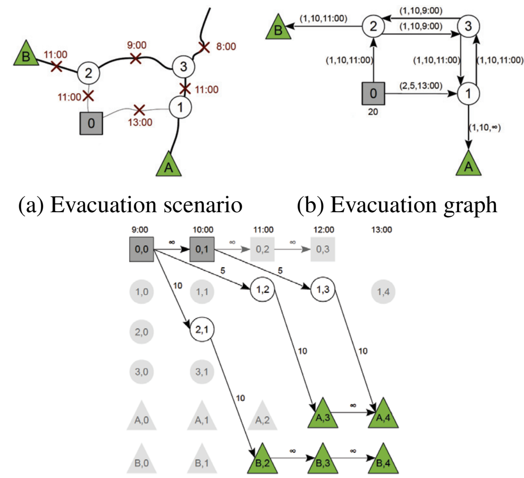
In this paper, we address this limitation and propose a Benders decomposition exploiting the insights of the two-stage approach. In particular, the master problem in the Benders decomposition enhances the TDP with Benders cuts obtained from the second stage. Since the second stage is a flow problem which typically has many optimal solution for specific values of the first-stage variables, the Benders decomposition may be extremely slow to converge. To remedy this limitation, we use Pareto-optimal Benders cuts (Magnati and Wong 1981). The Benders decomposition was evaluated on the real-case study from (Even, Pillac, and Van Hentenryck 2015). For the original case where the evacuation must conclude by a certain deadline, the Benders decomposition closes all the instances. For the more complex case of a 1/100 year flood scenario, the Benders decomposition is shown to produce significant improvements over the two-stage approach, with benefits exceeding 25% in the best case and 10% on average. The Benders decomposition is also shown to find high-quality solutions quickly. Note also that the Benders decomposition proposed here would also apply to zone-based evacuations plans that are not convergent.

Literature Review

Hamacher and Tjandra (2002) classifies approaches to evacuation planning into microscopic and macroscopic categories. Microscopic evacuation planning seeks to accurately model individual evacuee behavior, while macroscopic approaches, such as the one used in this paper, treat evacuees collectively as flows. Objectives for evacuation planning algorithms include finding the maximum flow, the quickest flow, or the minimum cost flow. The work by Ford and Fulkerson (1958) on the Maximum Dynamic Network Flow Problem (MDFP) for shipping goods is the foundation for much of later work on maximum flow evacuation problems. The same authors also defined the notion of a time-expanded graph for solving dynamic network flow problems (Ford and Fulkerson 1962). Chen and Chin (1990) introduced the quickest (single) path problem for data transmission, which was generalized to include multiple paths by Burkard, Dlaska, and Klinz (1993) in the quickest flow problem (QFP). The QFP was extended by Hoppe and Tardos (1995) to include multiple sources and sinks in the quickest transshipment problem (QTP).

There are several key requirements for evacuation problems not shared by other dynamic network flow problems. Emergency services typically prefer zone-based evacuations in which all residents in a residential areas are assigned the same evacuation path time in order to avoid confusion and increase compliance. Similarly, traffic control measures should be simple for authorities to enact. Few papers have proposed plans consistent with these requirements. Huibregtse et al. (2011) developed a two-stage approach for giving instructions to evacuees, where the first stage creates possible paths and departure times and the second stage assigns them to evacuees. Even, Pillac, and Van Hentenryck (2015) proposed the two-stage approach described in the introduction. In the first stage, they solve a relaxation of the maximum flow problem with aggregated arc capacities (the TDP problem), generating an evacuation network with convergent paths. This tree is then passed into the second stage, the Flow Scheduling Problem (FSP), which schedules the flow of evacuees on the corresponding time-expanded network. They also use a dichotomic search to find the minimum clearance time. Pillac, Cebrian, and Van Hentenryck (2015) proposes a column-generation approach that solves a more general problem: the joint mobilization and evacuation planning problem that schedules both the evacuation and the mobilization resources needed to enforce the plan.

Few studies have applied Benders decomposition to evacuation planning. Chen and Miller-Hooks (2008) introduced the Building Evacuation Problem with Shared Information (BEPSI), using Benders decomposition to solve the quickest flow problem in a building evacuation. The BEPSI is not a zone-based evacuation: the evacuation plan chooses dynamically where to send evacuees at every node and time steps. Andreas and Smith (2009) solved a variant of the quickest flow problem, using arc traversal penalty functions to encourage earlier evacuation. The model includes a number of possible scenarios, each with a given probability, and the objective is to minimize the expected sum of arc traversal penalties. The master problem chooses an evacuation tree and the subproblem solves a flow problem. However, contrary to our model, the master problem does not explicitly consider the flow variables and generates subtour constraints on the fly, making the relaxation much weaker.



(a) Evacuation scenario (b) Evacuation graph

(c) Time-expanded graph

Figure 1: Modeling of an Evacuation Planning Problem. (from Even, Pillac, and Van Hentenryck (2015)).

Convergent Evacuation Planning

Following (Even, Pillac, and Van Hentenryck 2015), an evacuation scenario is represented by an *evacuation graph* $\mathcal{G} = (\mathcal{N} = \mathcal{E} \cup \mathcal{T} \cup \mathcal{S}, \mathcal{A})$, where $\mathcal{E}$, $\mathcal{T}$, and $\mathcal{S}$ are respectively the set of evacuation, transit, and safe nodes, and $\mathcal{A}$ is the set of arcs. Each evacuation node i has a demand d_i, and each arc e is characterized by its travel time s_e, its capacity u_e, and the time f_e at which it becomes unavailable due to flooding. Figure 1 offers an example of how evacuation instances are modeled. Figure 1a shows an evacuation scenario with one evacuation node, labeled “0”, and two safe nodes, labeled “A” and “B.” The times on each arc indicate when that arc will be flooded. Figure 1b is the corresponding evacuation graph. The evacuation node has a demand of 20 vehicles. Arc $(0, 1)$ has a travel time of 2 min and a capacity of 5 vehicles/min, and is flooded at 13:00.

In order to model the evolution of the evacuation over time, we discretize the time horizon and use a *time-expanded graph* $\mathcal{G}^{\mathrm{x}} = (\mathcal{N}^{\mathrm{x}} = \mathcal{E}^{\mathrm{x}} \cup \mathcal{T}^{\mathrm{x}} \cup \mathcal{S}^{\mathrm{x}}, \mathcal{A}^{\mathrm{x}})$. To construct the time-expanded graph from the static graph, we create copies of nodes over time and replace each arc $e = (i, j)$ with arcs $e_t = (i_t, j_{t+s_{(i,j)}})$, for each time that e is available. Figure 1c illustrates the corresponding time-expanded graph.

Definition 1. *A graph* $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ *is* connected *if for all* $k \in \mathcal{E}$*, there exists a path from* k *to a safe node.*

Definition 2. *A graph* $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ *is* convergent *if for all* $i \in \mathcal{E} \cup \mathcal{T}$*, the outdegree of* i *is* 1*.*

As stated by Even, Pillac, and Van Hentenryck (2015), any connected evacuation graph $\mathcal{G}$ contains a connected and convergent subgraph $\mathcal{G}'$. If an evacuation graph is connected and convergent, each evacuation node will have a unique path to a safe node. The Convergent Evacuation Planning Problem (CEPP) is defined as follows:

Definition 3. *Given a connected evacuation graph* $\mathcal{G}$*, the Convergent Evacuation Planning Problem (CEPP) consists* *of finding a convergent subgraph $\mathcal{G}'$ of $\mathcal{G}$ and a set of evacuee departure times that maximize the flow from evacuation nodes to safe nodes.*

The MIP Model

This section presents a Mixed-Integer Programming (MIP) model for solving the CEPP. The model is adapted from (Even, Pillac, and Van Hentenryck 2015): Variable x_e is binary and indicates whether arc e is selected and variable φ_{e_t} is continuous and represents the flow on arc $e_t \in \mathcal{A}^{\text{x}}$. The evacuation is scheduled over a discretized time horizon $\mathcal{H}$.

$$\max \sum_{k \in \mathcal{E}} \sum_{e_t \in \delta^+(k)} \varphi_{e_t} \quad (1)$$

s.t.

$$\sum_{e_t \in \delta^-(i)} \varphi_{e_t} - \sum_{e_t \in \delta^+(i)} \varphi_{e_t} = 0 \qquad \forall i \in \mathcal{T}^x \quad (2)$$

$$\sum_{e \in \delta^+(i)} x_e \leq 1 \qquad \forall i \in \mathcal{E} \cup \mathcal{T} \quad (3)$$

$$\varphi_{e_t} \leq x_e \cdot u_{e_t} \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \quad (4)$$

$$\sum_{e_t \in \delta^+(k)} \varphi_{e_t} \leq d_k \qquad \forall k \in \mathcal{E} \quad (5)$$

$$\varphi_{e_t} \geq 0 \qquad \forall e_t \in \mathcal{A}^x \quad (6)$$

$$x_e \in \{0, 1\} \qquad \forall e \in \mathcal{A} \quad (7)$$

In the models, $\delta^-(i)$ and $\delta^+(i)$ denote the set of incoming and outgoing edges of node i respectively. Constraints (2) require flow conservation at each of the transit nodes, constraints (3) ensure that the output paths are convergent, constraints (4) are the capacity constraints, constraints (5) state that no more than the demand can be evacuated for each residential area, and the objective (1) maximizes the total evacuee flow. Even, Pillac, and Van Hentenryck (2015) showed that this MIP model does not scale to the Hawkesbury-Nepean evacuation instances: After 24 hours of running time, the number of people evacueed in the MIP model was substantially smaller than in their two-stage approach.

Benders Decomposition

This section presents a Benders Decomposition approach for zone-based evacuation planning. The key idea behind Benders decomposition is to separate the choice of the converging paths (restricted master problem) from the flow scheduling (subproblem). The Restricted Master Problem (RMP) chooses convergent paths, while the Subproblem (SP) uses these paths to schedule the evacuees over time. If the objective values of the RMP and SP are the same, the solution of the SP is optimal. Otherwise, the Benders decomposition approach generates a cut that removes at least the current convergent paths and the process is repeated. This paper uses Pareto-optimal Benders cuts to improve convergence.

The Restricted Master Problem

The RMP is built on top of the TDP, which was shown to be a relaxation of the CEPP where the evacuation flows and arc capacities are aggregated over time (Even, Pillac, and Van Hentenryck 2015). The TDP is formulated as a MIP model with a binary arc selection variable x_e and a continuous flow variable ψ_e for each arc $e \in \mathcal{A}$:

$$\max \sum_{k \in \mathcal{E}} \sum_{e \in \delta^+(k)} \psi_e \quad (8)$$

subject to

$$\sum_{e \in \delta^-(i)} \psi_e - \sum_{e \in \delta^+(i)} \psi_e = 0 \qquad \forall i \in \mathcal{T} \quad (9)$$

$$\sum_{e \in \delta^+(i)} x_e \leq 1 \qquad \forall i \in \mathcal{E} \cup \mathcal{T} \quad (10)$$

$$\psi_e \leq x_e \sum_{t \in H} u_{e_t} \qquad \forall e \in \mathcal{A} \quad (11)$$

$$\sum_{e \in \delta^+(k)} \psi_e \leq d_k \qquad \forall k \in \mathcal{E} \quad (12)$$

$$\psi_e \geq 0 \qquad \forall e \in \mathcal{A} \quad (13)$$

$$x_e \in \{0, 1\} \qquad \forall e \in \mathcal{A} \quad (14)$$

Constraints (9) impose flow conservation at each transit node, constraints (10) ensure a convergent plan, constraints (11) and (12) enforce the capacity and demand constraints, and the objective (8) maximizes the total evacuee flow.

The Subproblem

The output of the TDP is a convergent evacuation graph $\mathcal{G}$. The SP is a Flow Scheduling Problem (FSP) which schedules the flow of evacuees on the associated time-expanded graph $\mathcal{G}^{\text{x}}$. The FSP can be formulated as follows:

$$\max \sum_{k \in \mathcal{E}} \sum_{e_t \in \delta^+(k)} \varphi_{e_t} \quad (15)$$

subject to

$$\sum_{e_t \in \delta^-(i)} \varphi_{e_t} - \sum_{e_t \in \delta^+(i)} \varphi_{e_t} = 0 \qquad \forall i \in \mathcal{T}^x \quad (16)$$

$$\varphi_{e_t} \leq x_e \cdot u_{e_t} \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \quad (17)$$

$$\sum_{e_t \in \delta^+(k)} \varphi_{e_t} \leq d_k \qquad \forall k \in \mathcal{E} \quad (18)$$

$$\varphi_{e_t} \geq 0 \qquad \forall e_t \in \mathcal{A}^x \quad (19)$$

Constraints (16) are the flow conservation constraints, constraints (17) and (18) are the capacity and demand constraints, and the objective (15) maximizes the flow.

The Benders Cuts

The dual of the FSP is given by:

$$\min \sum_{e \in \mathcal{A}} x_e \sum_{t \in \mathcal{H}} u_{e_t} \cdot y_{e_t} + \sum_{k \in \mathcal{E}} d_k \cdot y_k$$

subject to

$$\sum_{i \in \mathcal{T}^x} \left(\mathbb{1}_{\delta^-(i)}(e_t) - \mathbb{1}_{\delta^+(i)}(e_t) \right) y_i$$

$$+ y_{e_t} + \sum_{k \in \mathcal{E}} \mathbb{1}_{\delta^+(k)}(e_t) y_k \geq \mathbb{1}_{\mathcal{E}}(e) \qquad \forall e \in \mathcal{E}, \forall t \in \mathcal{H}$$

$$y_{e_t} \geq 0 \qquad \forall e_t \in \mathcal{A}^x$$

$$y_k \geq 0 \qquad \forall k \in \mathcal{E}$$

where $\{y_i\}$, $\{y_{e_t}\}$, and $\{y_k\}$ are the dual variables associated with constraints (16), (17), and (18) respectively and the indicator function $\mathbb{I}_A(x)$ for a set A is defined as

$$\mathbb{I}_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

The Benders cuts are thus of the form:

$$z \leq \sum_{e \in \mathcal{A}} x_e \sum_{t \in \mathcal{H}} u_{e_t} \cdot y_{e_t} + \sum_{k \in \mathcal{E}} d_k \cdot y_k. \tag{20}$$

The RMP then becomes an extension of the TDP with the set of Benders cuts $\mathcal{C}$, i.e.,

$$\begin{aligned}
&\max z \\
&\text{subject to} \\
&z \leq \sum_{k \in \mathcal{E}} \sum_{e \in \delta^+(k)} \psi_e \\
&z \leq \sum_{e \in \mathcal{A}} x_e \sum_{t \in \mathcal{H}} u_{e_t} \cdot y^c_{e_t} + \sum_{k \in \mathcal{E}} d_k \cdot y^c_k && \forall c \in \mathcal{C} \\
&\sum_{e \in \delta^-(i)} \psi_e - \sum_{e \in \delta^+(i)} \psi_e = 0 && \forall i \in \mathcal{T} \\
&\sum_{e \in \delta^+(i)} x_e \leq 1 && \forall i \in \mathcal{E} \cup \mathcal{T} \\
&\psi_e \leq x_e \sum_{t \in H} u_{e_t} && \forall e \in \mathcal{A} \\
&\sum_{e \in \delta^+(k)} \psi_e \leq d_k && \forall k \in \mathcal{E} \\
&\psi_e \geq 0 && \forall e \in \mathcal{A} \\
&x_e \in \{0, 1\} && \forall e \in \mathcal{A}
\end{aligned}$$

It is important to note that the two-stage approach of (Even, Pillac, and Van Hentenryck 2015) can be viewed as the first iteration of this Benders decomposition. As a result, the Benders decomposition inherits directly a high-quality starting point contrary to (Andreas and Smith 2009) whose master problem only reasons about the flow variables through the Benders cuts, and not through aggregation.[1]

Pareto-Optimal Benders Cuts

The Benders decomposition presented so far is guaranteed to converge in a finite number of iterations. However, in practice, the algorithm rarely converged within a reasonable amount of time. Convergence can be accelerated by using stronger Benders cuts through the Magnanti-Wong method (Magnati and Wong 1981). At each iteration of the Benders decomposition, the Magnanti-Wong method generates a *Pareto-optimal cut* which is not dominated by any other Benders cut. The method requires a *core point*, i.e., a point within the relative interior of the convex hull of the feasibility domain of the first-stage variables. Finding such a core point is easy for this application, since the SP is always feasible and thus never generates feasibility cuts: it suffices to assign, say, $x^o_e = \frac{1}{2}$ to all the binary variables x_e. Note also that only one core point is needed for all iterations. To obtain a Pareto-optimal Benders cuts, we solve the dual of the Magnanti-Wong Problem, which is given by:

$$\max \sum_{k \in \mathcal{E}} \sum_{e_t \in \delta^+(k)} \varphi_{e_t} + \xi \sum_{k \in \mathcal{E}} \sum_{e_t \in \delta^+(k)} \bar{\varphi}_{e_t} \tag{21}$$

subject to

$$\sum_{e_t \in \delta^-(i)} \varphi_{e_t} - \sum_{e_t \in \delta^+(i)} \varphi_{e_t} = 0 \qquad \forall i \in \mathcal{T}^x \tag{22}$$

$$\varphi_{e_t} + \bar{x}_e \cdot u_{e_t} \cdot \xi \leq \frac{1}{2} u_{e_t} \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{23}$$

$$\sum_{e_t \in \delta^+(k)} \varphi_{e_t} + d_k \cdot \xi \leq d_k \qquad \forall k \in \mathcal{E} \tag{24}$$

$$\varphi_{e_t} \geq 0 \tag{25}$$

where $\{\bar{x}_e\}$ are from the optimal solution of the RMP and $\{\bar{\varphi}_{e_t}\}$ are from the optimal solution of the Benders subproblem. In order to generate a Pareto-optimal Benders cut, the coefficients $\{y_{e_t}\}$ and $\{y_k\}$ in the cut come from constraints (23) and (24) respectively.

[1] Even, Pillac, and Van Hentenryck (2015) counteracts the fact that a loose time horizon may reduce the quality of the TDP by finding the smallest time horizon that preserves the value of the first stage. We also include this insight to seed the decomposition.

Algorithm 1 The two-stage algorithm for clearance time minimization

Require: $\mathcal{G}$ the evacuation graph, h the time horizon
Ensure: A convergent evacuation plan of minimum clearance time
1: $h^\dagger \leftarrow \min\left\{t \in [0,h] \mid z(\texttt{TDP}(\mathcal{G}, t)) = \sum_{i \in \mathcal{E}} d_i\right\}$
2: $h^* \leftarrow \min\left\{t \in [h^\dagger,h] \mid z(\texttt{BD}(\mathcal{G}, t)) = \sum_{i \in \mathcal{E}} d_i\right\}$
3: **return** h^*, $\texttt{BD}(\mathcal{G}, h^*)$

Minimizing Clearance Time

Emergency services are often interested in determining the minimum clearance time, i.e., the smallest amount of time to evacuate the entire region. More precisely, the minimum clearance time h^* is defined as

$$h^* = \min\left\{h \in \mathbb{R} | z(\texttt{CEPP}(\mathcal{G}, [0,h])) = \sum_{k \in \mathcal{E}} d_k\right\}.$$

To compute the clearance time, we use a binary search over the time horizon, an idea already present in (Even, Pillac, and Van Hentenryck 2015). We first perform a dichotomic search on the TDP (TDP-DS), giving us a lower bound on the clearance time, followed by a dichotomic search on the Benders decomposition algorithm (BD-DS), seeded with the

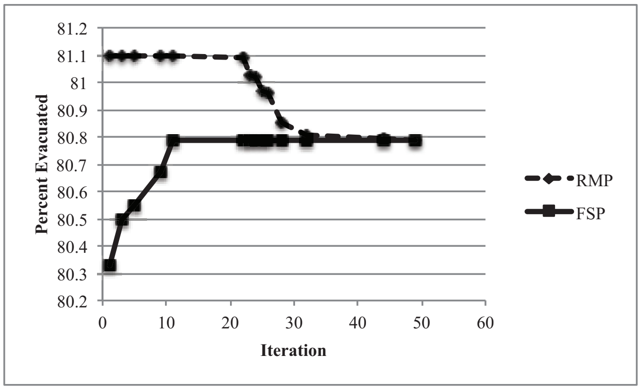


Figure 2: The Behavior of the Benders Decomposition in the Deadline Setting for Instance HN-2.5.

tree from the TDP-DS. Algorithm 1 outlines this approach and uses a binary search is used in Steps 1 and 2 to compute the minimum clearance time for the TDP and CEEP.

Experimental Results

This section presents experimental results for a case study of the evacuation of the Hawkesbury-Nepean (HN) floodplain, which is located near Sydney. The HN evacuation graph has 80 evacuation nodes, 184 transit nodes, 5 safe nodes, and 580 edges. We use time horizons of 600 min for scenarios without flooding and 1000 min for scenarios with flooding, discretized into 5 minute time-steps. Since the population in HN region is growing steadily, we consider several scenarios that scale the population by a factor $x \in [1.1, 3]$, with 38343 vehicles in the base instance. Each instance was run for one hour, unless the algorithm converged earlier. For the Benders decomposition, the reported CPU times are the times to the best solution. The algorithms were implemented using JAVA 8 and GUROBI 6.0 and the results were obtained on a 64 bit machine with a 1.4 GHz Intel Core i5 processor and 4 GB of RAM. Algorithms based upon the work by Even, Pillac, and Van Hentenryck (2015) were reimplemented.

We consider two main settings: (1) The *deadline* setting used in (Even, Pillac, and Van Hentenryck 2015) that requires the evacuation to be completed by a deadline (10 hours); (2) A *flood* setting in which the flood affects the road network at various times. In the deadline setting, the road network is available for the duration of the evacuation. In contrast, the flood setting uses the flood extent, the timing, and the height of the water produced by an hydro-dynamic simulation for a 1 in 100 years event. We study various scenarios under which the flood reaches the road network after 8, 9, 10, and 11 hours.

Table 1 displays the results for the deadline setting. The table reports the number of evacuees reaching safety (in percentage) in the tree design problem (**TDP**), the two-stage approach (**2S**), the last restricted master problem (**LRMP**), and the Benders decompositions (**BD**). Both the TDP and the restricted master problems provide upper bounds on the number of evacuees reaching safety. The table also gives the CPU time and the duality gap. The duality gap is computed using the formula $\frac{z(LRMP))-z(\star)}{z(\star)}$ where $z(\star)$ is the total number of evacuees reaching safety in model $\star$.

The results show that the Benders decomposition closes all these instances in less than 10 minutes (2.5 minutes on average). The Benders decomposition improves the two-stage approach by an average of 0.4%. Observe that the TDP provides a very accurate upper bound in this setting. Finally, Figure 2 depicts the behavior of the Benders Decomposition over time for Instance HN-2.5.

Table 2 displays the results of the two-stage and Benders decomposition approaches for the flood setting. The table presents the same information as before, with the addition of column **BD10** which gives the best evacuation plans found after 10 minutes. The results indicate that these instances are significantly harder since the Benders decomposition cannot prove optimality in an hour and the duality gap can be as high as 35% (instance HN-2.0/ 8 h) initially. But the results also show that the Benders decomposition provides significant improvements in solution quality compared to the two-stage approach, bridging most of the initial duality gaps. The Benders decomposition may improve the two-stage approach by more than 25% (instance HN-2.0/ 8 h). For the HN-1.7, HN-2.0, and HN-2.5 instances, the average improvements are 9.6%, 15.2%, and 9.4%. This is substantial in the context of evacuation planning for the HN region, since this corresponds to the evacuation of thousands more people. The duality gaps produced by the Benders decomposition are reasonably small: For the HN-1.7, HN-2.0, and HN-2.5 instances, they decrease from 10.7%, 19.1%, and 15.0% initially to 1.0%, 3.1%, and 5.0%. Finally, observe that the improvements provided by the Benders decomposition remain strong even with a time limits of 10 minutes, including a 24% improvement on instance HN-2.0/ 8 h.

Table 3 presents the results for minimizing the clearance time: It compares the results of using a dichotomic search with and without Benders decomposition. Each TDP iteration in the initial dichotomic search is solved with a time limit of 300 s. For the Benders decomposition dichotomic search, each run is given for up to 20 iterations, using the tree from the TDP-DS as a seed. The results show that the Benders decomposition approach closes all instances but one within the time limit.

Conclusion

This paper applied Benders decomposition for finding zoned-based convergent evacuation plans. The restricted master problem uses the tree design problem proposed by Even, Pillac, and Van Hentenryck (2015) as its core and generates a Pareto-optimal Benders cut at each iteration. The last RMP value found is an upper bound to the Convergent Evacuation Planning Problem (CEPP), while the best FSP value encountered is a lower bound to the CEPP.

Experimental results demonstrated the benefits of the Benders decomposition approach on a real study. For deadline evacuations, where the evacuation must be completed before the flood reaches the road network, the Benders decomposition algorithm closes all the instances from (Even, Pillac, and Van Hentenryck 2015). It also closes all the clear-

Instance	2-Stage Approach CPU (s)	TDP (%)	2S (%)	Gap (%)	Benders Decomposition CPU (s)	LRMP(%)	BD (%)	Gap (%)
HN	1.6	100	99.1	1.0	33.6	100	100	0
HN-1.1	0.9	100	99.8	0.2	22.9	100	100	0
HN-1.2	1.5	100	100	0	1.5	100	100	0
HN-1.4	1.1	100	100	0	1.1	100	100	0
HN-1.7	1.4	100	100	0	1.4	100	100	0
HN-2.0	7.9	96.2	95.5	0.7	160.0	96.1	96.1	0
HN-2.5	3.6	81.1	80.3	1.0	372.5	80.8	80.8	0
HN-3.0	1.5	68.1	67.5	0.9	590.2	68.0	68.0	0
Average	*2.4*	*93.2*	*92.8*	*0.5*	*147.9*	*93.1*	*93.1*	*0*

Table 1: Results for the HN Instances in the Deadline Setting.

	2-Stage Approach				**Benders Decomposition**				
Instance	**CPU (s)**	**TDP (%)**	**2S (%)**	**Gap (%)**	**CPU (s)**	**LRMP%**	**BD (%)**	**BD10 (%)**	**Gap (%)**
HN-1.7									
8 h	1.3	100	88.4	13.1	3372.1	100	96.6	88.4	3.5
9 h	1.4	100	83.0	20.5	604.5	100	99.5	95.5	0.5
10 h	1.6	100	93.3	7.2	882.2	100	100	99.4	0
11 h	2.1	100	98.1	2.0	820.5	100	100	98.4	0
Average	*1.6*	*100*	*90.7*	*10.7*	*1419.8*	*100*	*99.0*	*95.4*	*1.0*
HN-2.0									
8 h	1.4	98.8	73.1	35.1	3096.9	98.8	91.5	90.1	8.0
9 h	1.6	99.6	81.5	22.3	895.1	99.6	96.2	94.2	3.5
10 h	1.4	100	88.6	12.8	3137.7	100	98.9	98.0	1.1
11 h	2.2	100	94.3	6.0	2532.0	100	100	98.7	0
Average	*1.6*	*99.6*	*84.4*	*19.1*	*2415.4*	*99.6*	*96.7*	*95.4*	*3.1*
HN-2.5									
8 h	1.7	97.4	78.6	23.9	3581.7	97.4	89.6	87.0	8.7
9 h	2.1	98.1	80.2	22.3	1061.3	98.1	90.6	89.5	8.3
10 h	1.4	98.8	89.5	10.4	2238.2	98.8	96.1	93.1	2.7
11 h	1.5	99.5	96.2	3.4	2720.6	99.5	99.2	97.4	0.3
Average	*1.6*	*98.4*	*86.1*	*15.0*	*2400.5*	*98.4*	*93.9*	*91.7*	*5.0*

Table 2: Results for the HN-1.7, 2.0, and 2.5 Instances on the Flooding Setting.

Inst.	**TDP-DS** CPU (s)	CT (min)	**2S-DS** CPU (s)	CT (min)	**BD-DS** CPU (s)	CT (min)
HN	558.7	340	567.1	365	691.7	340
HN-1.1	313.8	370	321.8	390	704.6	370
HN-1.2	313.2	400	321.3	405	420.0	400
HN-1.4	323.4	455	331.9	455	393.4	455
HN-1.7	618.1	550	627.7	580	889.6	550
HN-2.0	50.2	625	60.5	635	699.0	630
HN-2.5	663.5	795	675.9	815	939.2	795
HN-3.0	335.3	925	346.0	930	488.0	925

Table 3: Clearance Time on the HN Instances.

ance time instances but one. For the flood scenarios, the Benders decomposition approach provides substantial improvements compared to the two-stage approach and produces high-quality solutions quickly. In particular, it improves the two-stage approach by more than 25% in one instance and its average improvement over these instances is about 11.4%, which is considerable in an evacuation setting. These results contrasts with the MIP model developed in (Even, Pillac, and Van Hentenryck 2015) which were alrealy dominated by the two-stage approach even when run for 24 hours.

This paper shows that the Benders decomposition approach provides an excellent tradeoff between solution quality and runtime. It should apply to other evacuation scenarios involving road blockages such as bushfires, earthquakes, and volcanic eruptions. Future work will attempt to integrate various types of uncertainties, including accidents, breakdowns, and flood predictions. The main difficulty is that the probability of an accident increases with the flow on the road segments, creating significant optimization challenges.

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