Query Abduction for $\mathcal{ELH}_\bot$ Ontologies

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Introduction
With the current upward trend in semantically annotated data, ontology-based data access (OBDA) was formulated to tackle the problem of data integration and query answering, where an ontology is formalized as a description logic (DL) TBox. In order to meet usability requirements set by users, efforts have been made to equip OBDA system with explanation facilities. One important explanation tool for DL ontologies, referred to as query abduction, can be formalised as abductive reasoning. In particular, given an ontology and an observation (i.e., a query with an answer that called boolean query abduction with general CQs as observations. We implemented a prototypical system using the highly optimized Prolog engine XSB\(^1\). We evaluated our algorithm over some realistic ontologies with reasonably large data sets such as extended university benchmark ontology LSTW\((n)\) that has approximately $10^5n$ ABox assertions. Our experimental results show that the algorithm is capable of handling query abduction problems for LSTW\((n)\) where \(n\) is up to 100.

Query Abduction Problem
We consider countably infinite and mutually disjoint sets \(N_C, N_R, N_I\) and \(N_V\) of concept names, role names, individuals, and variables respectively. For details of $\mathcal{ELH}_\bot$ readers can refer to (Baader, Brandt, and Lutz 2005). A conjunctive query $Q(\vec{x})$ is of the form $\exists \vec{y}. \phi(\vec{x}, \vec{y})$, where $\phi$ is a conjunction of atoms with predicates and terms from $N_C \cup N_R$ and $N_I \cup N_V$, respectively. A boolean conjunctive query (BCQ) is a CQ that answer variables $\vec{x}$ are empty. In Definition 1, we formally introduce a query abduction problem. In contrast to the definition in (Calvanese et al. 2013), our definition has a finite domain $\Delta$ as an additional attribute, and contains all the constants occurring in a solution in order to avoid infinitely many (minimal) solutions for a QAP.

Definition 1 (QAP) An instance of a query abduction problem (QAP) in $\mathcal{ELH}_\bot$ is a tuple $(K, Q(\vec{a}), \Sigma, \Delta)$, where $K$ is an $\mathcal{ELH}_\bot$ KB, $Q(\vec{a})$ is a BCQ called the observation, $\Sigma \subseteq N_C \cup N_R$ is a finite set of predicates called abducibles, and $\Delta \subseteq N_I$ is a finite set of constants called the domain. A solution $E$ to the QAP is a set of facts over $\Sigma$ such that (1) $\text{pred}(E) \subseteq \Sigma$, (2) $const(E) \subseteq \Delta$, (3) $K \cup E$ is consistent, and (4) $K \cup E \models Q(\vec{a})$. Let $sol(K, Q(\vec{a}), \Sigma, \Delta)$ be the set of solutions to the QAP. Moreover, we call $E$ set minimal iff there is no solution $E' \in sol(K, Q(\vec{a}), \Sigma, \Delta)$ s.t. $E' \subset E$. For a datalog program $D$, a QAP in datalog and its (minimal) solutions are defined in the same way, so the set of solutions are denoted as $sol(D, Q(\vec{a}), \Sigma, \Delta)$.

Computing Minimal Solutions
For an $\mathcal{ELH}_\bot$ KB $K = (T, A)$, we will first transform $T$ into a datalog program $D_T$ in a similar way to (Stefanoni, Motik, and Horrocks 2013). The program $D_K = D_T \cup A$ is referred to as its datalog approximation. By this approximation, each solution to the QAP w.r.t. KB $K$ is a solution to the corresponding QAP w.r.t. datalog program $D_K$ which formally presents as follows.

Proposition 1 Let $K$ be an $\mathcal{ELH}_\bot$ KB, $Q(\vec{a})$ be a BCQ, $\Sigma \subseteq N_C \cup N_R$, and $\Delta \subseteq N_I$. Then, $sol(K, Q(\vec{a}), \Sigma, \Delta) \subseteq sol(D_K, Q(\vec{a}), \Sigma, \Delta)$.

\(^1\)http://xsbs.sourceforge.net/
However, the solutions of QAP \( \langle D_K, Q(\vec{a}), \Sigma, \Delta \rangle \) are not necessarily sound. The following example illustrates it.

**Example 1** Let \( K \) consist of TBox \( \{ A \sqsubseteq \exists R.A \} \), and an empty ABox. Then, the corresponding datalog program \( D_P \) contains the following datalog rules \( A(x) \rightarrow r(x, c_1) \) and \( A(x) \rightarrow A(c_1) \). Let \( \Sigma \) and \( \Delta \) be \( \{ A \} \) and \( \{ a, b \} \) respectively. For the queries \( Q_1(x, y) = \exists z. [r(x, z) \lor r(y, z)] \) and \( Q_2(x) = \exists y.[r(x, y) \land r(y, x)] \), then \( \{ A(a), A(b) \} \in \text{sol}(D_K, Q_1(a, b), \Sigma, \Delta) \) and \( \{ A(a) \} \in \text{sol}(D_K, Q_2(a), \Sigma, \Delta) \). However, neither \( \langle K, Q_1(a, b), \Sigma, \Delta \rangle \) nor \( \langle K, Q_2(a), \Sigma, \Delta \rangle \) has a solution.

To retain the soundness of our ontology transformation, we propose to rewrite observations in a QAP to filter out incorrect solutions. Our rewriting approach is adapted from (Lutz, Toman, and Wolter 2009) but it is simpler since our datalog approximation is tighter. Intuitively, the unsoundness problem is caused by fork-shaped or cyclic structures (e.g. respectively \( Q_1 \) and \( Q_2 \) in Example 1) in the models of approximation \( D_K \) but not necessarily in all models of \( K \). For a CQ \( Q \), such substructures can be identified in time polynomial to the size of \( Q \). (2009).

Then rewriting for CQ \( Q(\vec{x}) = \exists \vec{y}. \phi(\vec{x}, \vec{y}) \) is \( Q^*(\vec{x}) = \exists \vec{y}. [\phi_1 \land \phi_2] \). Intuitively, filter \( \phi_1 \) says that fork-shaped substructures in \( Q \) should be instantiated in a way that all the legs merge into one, and \( \phi_2 \) says that a cyclic substructure in \( Q \) can not be instantiated with fresh constants introduced in approximation phase. Now, we present a major result in the paper that shows the new rewriting is sufficient for the correctness of query abduction over the datalog approximation.

**Theorem 1** Let \( K \) be an \( \mathcal{EL}^H \perp \) KB, \( Q(\vec{a}) \) be a BCQ, \( \Sigma \subseteq N_C \cup N_R \) and \( \Delta \subseteq N_I \). Then \( \text{sol}(K, Q(\vec{a}), \Sigma, \Delta) = \text{sol}(D_K, Q^*(\vec{a}), \Sigma, \Delta) \).

Like many other procedures of computing abductive solutions, a resolution-based algorithm is needed for our procedure. We implemented our new procedure of computing QAP solutions using the highly optimized Prolog engine XSB. To compute all minimal solutions to a QAP \( \langle D_K, Q^*(\vec{a}), \Sigma, \Delta \rangle \), we encode the QAP into Prolog rules, and use the list structure to store the solutions generated during the resolution. We have also employed the database feature of XSB to allow the initial ABox to be stored in a database, which significantly improves the efficiency.

<table>
<thead>
<tr>
<th>Ontologies</th>
<th>Atomic queries</th>
<th>Conjunctive queries</th>
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<tbody>
<tr>
<td></td>
<td># Succ. Avg. Max.</td>
<td># 56% 2.5 28.3</td>
</tr>
<tr>
<td>LSTW1</td>
<td>13</td>
<td>100% 2.4 10.4</td>
</tr>
<tr>
<td>LSTW10</td>
<td>100% 25.6 134.7</td>
<td>56% 2.5 28.3</td>
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<tr>
<td>LSTW50</td>
<td>100% 139.8 720.7</td>
<td>56% 2.5 28.3</td>
</tr>
<tr>
<td>LSTW100</td>
<td>100% 488.0 2589.0</td>
<td>56% 2.5 28.3</td>
</tr>
</tbody>
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Table 1: Evaluate ABEL for CQs.

**Experimental Results**

We have evaluated our system on the LSTW(\( n \)), an extended version of the university ontology LUBM, which has axioms with existentially quantified variables on the right-hand sides. We used a query generator to generate 13 atomic and 27 conjunctive queries. The abducibles are all the concept names in the TBox, and the domain consists of all individuals in the ABox. The sizes of ABoxes range from 100 thousands (LSTW(1)) to 10 millions assertions (LSTW(100)), the sizes of domain are between 17 thousands (LSTW(1)) and 1.7 million (LSTW(100)) and the abducible size is 132. Overall, we ran the first experiment for 200 test cases. Table 1 shows the average and maximum execution times as well as success rates. All times are in seconds, and the time limit is one hour. The overall success rates of ABEL for both atomic and conjunctive queries over LSTW(1) is 70%, and for the rest of ontologies is 67%. ABEL failed to return all explanations for general CQs that have atoms from the recursive axiom \( \exists \text{advisor}. \text{Professor} \sqsubseteq \text{Professor} \). However, it was successful to compute all explanations for the atomic query \( Q(x) \leftarrow \text{Professor}(x) \).

Since the query generator could not generate fork-shaped or cyclic CQs, we conducted another set of experiments to evaluate the performance of ABEL as shown in Table 2 over manually crafted queries. Q1 is a cyclic CQ and the rest is fork-shaped. OM means that XSB ran out of memory.

Despite the complexity of the query abduction problem with general CQs, our prototype system can efficiently handle more than half of the CQs over large data sets (10 millions assertions), which shows that our approach can provide efficient query explanation services in real-life applications with realistic ontologies and large data sets.

**References**


Stefanoni, G.; Motik, B.; and Horrocks, I. 2013. Introducing nominals to the combined query answering approaches for EL. In Proc. of the 27th AAAI.