

## tBurton: A Divide and Conquer Temporal Planner

David Wang and Brian Williams

Computer Science and Artificial Intelligence Laboratory  
 Massachusetts Institute of Technology  
 Cambridge, Massachusetts 02139

### Abstract

Planning for and controlling a network of interacting devices requires a planner that accounts for the automatic timed transitions of devices, while meeting deadlines and achieving durative goals. Consider a planner for an imaging satellite with a camera that cannot tolerate exhaust. The planner would need to determine that opening a valve causes a chain reaction that ignites the engine, and thus needs to shield the camera. While planners exist that support deadlines and durative goals, currently, no planners can handle automatic timed transitions. We present tBurton, a temporal planner that supports these features, while additionally producing a temporally least-commitment plan. tBurton uses a divide and conquer approach: dividing the problem using causal-graph decomposition and conquering each factor with heuristic forward search. The ‘sub-plans’ from each factor are then unified in a conflict directed search, guided by the causal graph structure. We describe why this approach is fast and efficient, and demonstrate its ability to improve the performance of existing planners on factorable problems through benchmarks from the International Planning Competition.

### Introduction

Embedded machines are being composed into ever more complex networked systems, including earth observing systems and transportation networks. The complexity of these systems require automated coordination, but controlling these systems pose unique challenges: timed transitions – after turning a projector off a cool-down period must be obeyed before the projector will turn on again; periodic transitions – a satellite’s orbit provides predictable time-windows over when it will be able to observe a phenomenon; required concurrency (Cushing et al. 2007), – a communication channel must be open for the duration of a transmission. Furthermore, a user may wish to specify when different states need to be achieved (time-evolved goals) and may expect a plan that allows flexibility in execution time (a temporally least-commitment plan).

While there has been a long history of planners developed for these systems, no single planner supports this com-

plete set of features. Model-based planners, such as Burton (Williams and Nayak 1997; Chung and Williams 2003), the namesake of our planner, have exploited the causal structure of the problem in order to be fast and generative, but lack the ability to reason over time. Timeline-based planners, such as EUROPA (Frank and Jónsson 2003) and ASPEN (Chien et al. 2000), can express rich notions of metric time and resources, but have traditionally depended on domain-specific heuristics to efficiently guide backtracking search. Finally, metric-time heuristic planners (Coles et al. 2010; Benton, Coles, and Coles 2012; Röger, Eyerich, and Mattmüller 2008) have been developed that are domain-independent, fast and scalable, but lack support for important problem features, such as timed and periodic transitions.

tBurton is a fast and efficient partial-order, temporal planner designed for networked devices. Our overall approach is divide and conquer, a.k.a factored planning (Amir and Engelhardt 2003), but we leverage insights from model, timeline, and heuristic-based planning. Like Burton, tBurton factors the planning problem into an acyclic causal-graph and uses this structure to impose a search ordering from child to parent. Associated with each factor is a timeline on which the plan will be constructed. Timelines help maintain locality of the causal information, thereby reducing the need for time-consuming threat-detection steps common in partial-order planning. To find a plan, we use a conflict directed search that leverages the efficiency of a heuristic-based sub-planner to completely populate the timeline of a factor, before regressing the sub-goals of that plan to the timeline of its parent.

The contributions of this paper are three fold: first, we introduce a planner for networked devices that supports a set of features never before found in one planner. Second, we introduce a new approach to factored planning based on timeline-based regression and heuristic forward search. Third, we demonstrate the effectiveness of this approach on planning benchmarks.

We start by elaborating upon tBurton’s approach in the context of prior work and introduce a running example. We then define our notation, before presenting the algorithms underlying tBurton. Finally, we close with an empirical validation on International Planning Competition (IPC) benchmarks.

## Background

Divide and conquer is the basic principal behind factored planning, but alone tells only part of the story. A factored planner must decide how to factor (divide) the problem, how to plan for (conquer) each factor, *and* how to unify those plans.

**Divide** Key to tBurton’s approach to factored planning is exploiting the benefits of causal-graph based factoring in partial-order, temporal planning.

tBurton inherits its causal reasoning strategy from name-sake, Burton (Williams and Nayak 1997; Chung and Williams 2003), a reactive, model-based planner developed for NASA spacecraft. Burton, exploits the near-DAG structure of its domain and grouped cyclically-dependent factors to maintain an acyclic causal graph. The causal graph is then used to quickly determine a serialization of sub-goals. Even though this strategy is not optimal in the general case, complexity analysis has shown it is difficult to do better in the domain independent case (Brafman and Domshlak 2006; 2003).

Despite the lack of optimality, the clustering of variables identified by an acyclic causal-graph has important ramifications. Goal-regression, partial-order planners (Penberthy and Weld 1992; Younes and Simmons 2003) traditionally suffer from computationally expensive threat detection and resolution, where interfering actions in a plan must be identified and ordered. Factoring inherently identifies the complicating shared variables, reducing the number of cases that must be checked for interference. Furthermore, threat resolution is equivalent to adding temporal constraints to order sub-plans during composition – reducing the number of threats under consideration also reduces the difficulty of temporal reasoning.

**Conquer** In order to plan for each factor, tBurton uses a heuristic-based temporal planner. Heuristic-based planners, and especially heuristic forward search planners have scaled well (Röger, Eyerich, and Mattmüller 2008; Coles et al. 2010; Vidal 2011), and consistently win top places in the IPC. Using a heuristic forward search planner (henceforth sub-planner) allows tBurton to not only benefit from the state-of-the-art in planning, but to degrade gracefully. Even if a problem has a fully connected causal-graph, and therefore admits only one factor, the planning time will be that of the sub-planner plus some of tBurton’s processing overhead.

tBurton plans for each factor by first collecting and ordering all the goals for that factor along its timeline. The sub-planner is then used to populate the timeline by first planning from the initial state to the first goal, from that goal to the next, and so on. The problem presented to the sub-planner only contains variables relevant to that factor.

**Unify** Sub-plans are unified by regressing the subgoals required by a factor’s plan, to parent factors. Early work in factored planning obviated the need for unification by planning bottom-up in the causal-graph. Plans were generated for each factor by recursively composing the plans of its children, treating them as macro-actions (Amir and Engelhardt 2003; Brafman and Domshlak 2006). This approach

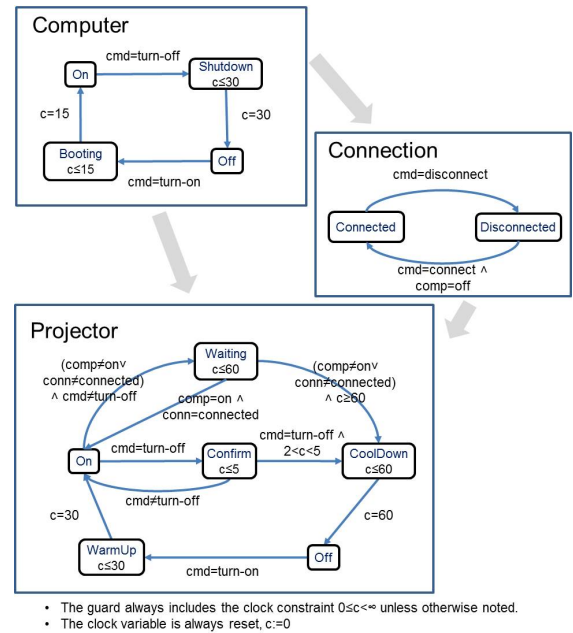


Figure 1: A simple computer-projector system represented as Timed Concurrent Automata.

obviated the need for unifying plans at the cost of storing all plans. Subsequent work sought to reduce this memory overhead by using backtracking search through a hierarchical clustering of factors called a dtree (Kelareva et al. 2007). While tBurton does not use a dtree, we do extend backtracking search with plan caching and conflict learning in order to more efficiently unify plans.

**Projector Example** A running example we will use in the remainder of this paper involves a computer, projector, and connection between them, which are needed to give a presentation. The computer exhibits boot-up and shutdown times. The projector exhibits similar warm-up and cool-down periods, but will also shutdown automatically when disconnected from the computer. Figure 1 depicts this example in tBurton’s native, automata formulation.

## Problem Formulation

Timed concurrent automata, the representation tBurton uses for the planning problem inherits from prior work on extending Concurrent Constraint Automata (CCA) (Williams and Nayak 1996) to time (Ingham 2003). Our representation can be viewed as a variation on timed-automata theory (Alur and Dill 1994), where transitions are guarded by expressions in propositional state-logic rather than symbols from an alphabet.

Formally, the planning problem tBurton solves is the tuple,  $\langle TCA, SP_{part} \rangle$ , where  $TCA$  is a model of the system expressed as a set of interacting automata called *Timed Concurrent Automata*, and  $SP_{part}$  is our goal and initial state representation, which captures the timing and duration of desired states as a *Partial State Plan*. We represent our plan as a Total State Plan  $SP_{total}$ , which is an elaboration of  $SP_{part}$

that contains no open goals, and expresses not only the control commands needed, but the resulting state evolution of the system under control.

## Timed Concurrent Automata

**Definition 1.** a *TCA*, is a tuple  $\langle L, \mathcal{C}, \mathcal{U}, \mathcal{A} \rangle$ , where:

- $\mathcal{L}$  is a set of variables,  $l \in \mathcal{L}$ , with finite domain  $D(l)$ , representing locations within the automata. An assignment to a variable is the pair  $(l, v)$ ,  $v \in D(l)$ .
- $\mathcal{C}$  is a set of positive, real-valued clock variables. Each clock variable,  $c \in \mathcal{C}$ , represents a resettable counter that increments in real-time, at the same rate as all other clock variables. We allow the comparison of clock variables to real valued constants,  $c \text{ op } r$ , where  $\text{op} \in \{\leq, <, =, >, \geq\}$ ,  $r \in \mathbb{R}$ , and assignments of real-valued constants to clock variables  $c := r$ , but do not allow clock arithmetic.
- $\mathcal{U}$  is a set of control variables,  $u \in \mathcal{U}$ , with finite domain  $D(u) \cup \perp$ .  $D(u)$  represents *commands* users ‘outside’ the *TCA* can use to control the *TCA*.  $\perp$  is a nullary command, indicating no command is given.
- $\mathcal{A}$  is a set of timed-automata,  $A \in \mathcal{A}$ .

A *TCA* can be thought of as a system where the location and clock variables maintain internal state while the control variables provide external inputs. The automata that compose the *TCA* are the devices of our system.

**Definition 2.** A single timed automaton  $A$  is the 5-tuple  $\langle l, c, u, \mathbb{T}, \mathbb{I} \rangle$ .

- $l \in \mathcal{L}$  is a location variable, whose values represent the locations over which this automaton transitions.
- $c \in \mathcal{C}$  is the unique clock variable for this automaton.
- $u \in \mathcal{U}$  is the unique control variable for this automaton.
- $\mathcal{T}$  is a set of *transitions* of the form,  $T = \langle l_s, l_e, g, c := 0 \rangle$ , that associates with a start and end location  $l_s, l_e \in D(l)$ , a guard  $g$  and a reset value for clock  $c$ ,  $c := 0$ . The guard is expressed in terms of propositional formulas with equality,  $\varphi$ , where:  $\varphi ::= \text{true} \mid \text{false} \mid (l^o = v) \mid (u = v) \mid (c \text{ op } r) \mid \neg\varphi_1 \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2$ . The guard can be expressed only in terms of location variables not belonging to this automaton,  $l^o \in \mathcal{L} \setminus l$ , and the control and clock variable of this automaton. For brevity, we will sometimes use the expression  $l \neq v$  in place of  $\neg(l = v)$ . The automaton is said to instantaneously transition from  $l_s$  to  $l_e$  and reset its clock variable when the guard is first evaluated true.
- $\mathbb{I}$  is a function that associates an invariant with each location. The invariant takes the form of a clock comparison  $c < r$  or  $c \leq r$  that bounds the maximum amount of time an automata can stay in that location. The invariant  $c \leq \infty$  expresses that an automaton can dwell in the associated location indefinitely.

In the projector example (Figure 1), the projector has locations *Off*, *WarmUp*, *On*, *Waiting*, *Confirm*, and *CoolDown*. The transitions, represented by directed edges between the locations, are labeled by guards that are a function of other location variables, clock variables (denoted by  $c$ ), and control variables (denoted by  $\text{cmd}$ ). Invariants label

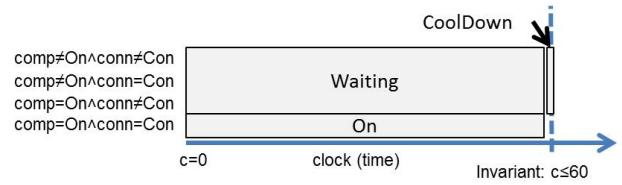


Figure 2: A graphical representation of the possible set of next-states from the *Waiting* state of the well-formed and deterministic Projector automaton.

some states, such as *WarmUp*, which allows us to encode the bounded-time requirement that the projector must transition from *WarmUp* to *On* in 30.

An automaton is *well formed* if there exists a unique next-state for all possible combinations of assignments to location, control, and clock variables. With regards to the transitions, an automaton is said to be *deterministic* if for any  $l_s$  only the guard  $g$  of one transition can be true at any time. Figure 2 depicts the set of next-states from the *Waiting* state of the well-formed, deterministic Projector automaton. Note that there is exactly one possible next-state for each combination of inputs, and a unique next-state at the upper-bound of the invariant. In order to produce plans with predictable behavior, tBurton must plan over *TCA* models consisting of well-formed, deterministic automata.

*TCA*s make representing problem features such as indirect effects, automatic-timed transitions, and periodic transitions straight forward. An *indirect effect* can occur when a transition in automaton  $A_c$  is *activated by* (the guard of the transition is evaluated true based on the assignments to) the locations of other automaton  $A_p$  without the need to assign the control variable of  $A_c$ . *Automatic-timed transitions* can occur within one automaton when a transition is activated by the clock variable. *Periodic transitions* can occur when a series of automated-timed transitions starts and ends with the same location (forming a cycle). A combination of these features can produce more complex interactions: several indirect effects could produce a chain-reaction, and indirect effects with automatic-timed transitions can result in periodic transitions spanning several automaton.

## State Plans

To control *TCA*, we need to specify and reason about executions (runs) of the system in terms of state trajectories; we represent these as state plans. A state plan specifies both the desired (goal) and required (plan) evolution of the location and command variables using a set of timelines we refer to as state *histories*. The temporal aspect of these histories and the relationship between events within them are expressed through simple temporal constraints (Dechter 2003). The flexibility afforded by simple temporal constraints allows the state plan to be a temporally least commitment specification, which is important in enabling plan executives (Levine 2012; Muscettola, Morris, and Tsamardinos 1998) to adapt to a range of disturbances without the need to re-plan. tBurton generalizes upon the value and justification episode and history representation employed in TCP (Williams 1986),

which introduced a least-commitment approach to representing interactions between timelines. The use differs in that TCP performs progression on histories through state constraints, as a form of least-commitment simulation, while tBurton performs regression on goal histories through timed concurrent automata, as a form of least-commitment planning.

Conceptually, a state-plan can be thought of as one large temporal network which we divide into histories and justifications. Each history describes the evolution of a particular location or control variable, while justifications are temporal constraints that relate the timing of one history to another.

**Definition 3.** a history  $H$  is a tuple  $\langle EV, EP \rangle$  where:  
 $EV = \{e_1, e_2, \dots, e_n\}$ , is a set of events  $e_i$  (that represent positive, real-valued time points), and  
 $EP = \{ep_1, ep_2, \dots, ep_m\}$ . is a set of episodes, where each episode,  $ep = \langle e_i, e_j, lb, ub, sc \rangle$ , associates with a temporal constraint  $lb \leq e_j - e_i \leq ub$ , a state constraint  $sc$  expressed as a propositional formula over location and control variable assignments. We refer to  $e_i$  and  $e_j$  as the *start* and *end events/times* of the episode, respectively, even though their actual temporal ordering depends on whether  $lb$  is positive.

**Definition 4.** a state plan  $SP$  is a tuple  $\langle GH, \mathcal{VH}, \mathcal{J} \rangle$  where:

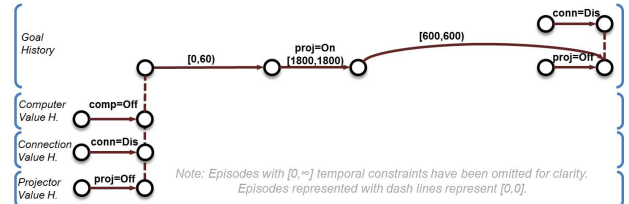
- $GH$  is a *goal history*, a type of history in which the state-constraints of episodes represent the *desired* evolution of location and control variables.
- $\mathcal{VH}$  is a set of value histories,  $VH \in \mathcal{VH}$ . Each *value history*,  $VH$ , is a type of history that represents the planned evolution of locations or control variables. The state constraints in a value history are restricted to contain only variable assignments and only assignments to the variables to which the value history is associated. As such, a value history represents the trace of the corresponding variables.
- $\mathcal{J}$  is a justification, a type of episode with a state constraint of value `true`, which is used to relate the temporal occurrence of events in goal histories to events in value histories.

For simplicity, we will use the term *goal episode* to refer to an episode occurring in the goal history, and the term *value episode* for an episode occurring in the value history.

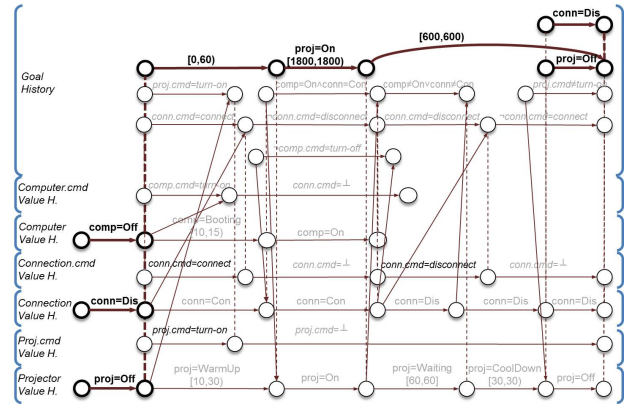
As with many partial-order planners, tBurton searches over a space of partial plans, starting from the partial state-plan  $SP_{part}$  and elaborating the plan until a valid total plan  $SP_{total}$ , in which all goals are closed, is reached. State plans allow tBurton to not only keep track of the plan (through value histories), but also keep track of why the plan was created (goal histories and justifications). This is useful for post-planning validation, but also allows tBurton to keep track of open-goals during the planning process.

Figure 3 shows a progression between the initial state plan and the total state plan for the projector example. Figure 3(a) depicts the partial state plan from which tBurton starts planning. The presenter wishes to project for 30 minutes and leave the room 10 minutes later with projector off and his computer disconnected. The goal history of the state-plan,

as indicated by the brackets, describes this desired state trajectory. The value histories represent the initial state of the computer, connection, and projector. Figure 3(a) shows the output of tBurton, a total state plan, where the histories track the evolution of each variable’s value. The emphasized commands of ‘connecting the computer and projector’, ‘turning on the projector’, and then ‘disconnecting the computer and projector’ represent the actionable control-variable assignments needed to achieve the presenter’s goals.



(a) The initial partial state plan for the projector example. Events are represented as vertices. Episodes as edges. The lower and upper time bounds for episodes are labeled using interval notation.



(b) The total state plan for the projector example. The control variables important to the execution of the state plan as well as the initial state plan are darkened.

Figure 3: Initial and total state plan for the projector example.

### Semantics of TCAs and State Plans

Until now, we have used the words *run*, *trace*, *closed*, *valid*, *least-commitment*, and *causal graph* in a general sense. We now return to formalize these definitions, starting with the semantics of *TCAs* and their relation to state plans.

**Definition 5. run.** The *run* of a single timed-automaton,  $A_a$ , can be described by its timed state trajectory,  $S_a = ((l_{a0}, 0), (l_{a1}, t_{a1}), \dots, (l_{am}, t_{am}))$ , a sequence of pairs describing the location and time at which the automaton first entered each state.  $l_{a0}$  is the initial assignment to the location variable of  $A_a$ . We say a run  $((l_{ai}, t_{ai}), (l_{aj}, t_{aj}))$  is *legal* if two conditions are met. First, if  $l_{ai}$  has invariant guard  $c < r$ , then  $t_{aj} - t_{ai} < r$  must be true. Second, if there exists a transition between locations  $l_{ai}$  to  $l_{aj}$ , guarded by  $g$ , the first time  $g$  became true in the interval from  $t_{ai}$  to  $t_{aj}$  must be at time  $t_{aj}$ .

**Definition 6. trace.** A *trace* in timed automata theory is usually defined as the timed-trajectory of symbols, a word. Relative to TCAs, this corresponds to the timed trajectory of the guard variables. For *TCAs*, where automata asynchronously guard each other through their location variables, a trace and run are almost identical, with the exception that a trace for a *TCA* would also include the timed state trajectory of control variables used in guards. A trace for a *TCA* therefore captures the evolution of all the variables.

**Definition 7. closed.** A goal-episode with constraint  $sc_g$  is considered trivially closed if  $sc_g$  evaluates to true, and trivially un-closable if it evaluates to false. Otherwise, a goal-episode is considered *closed* if there is a co-occurring episode in a value-history whose constraint entails the goal's constraint. Formally, a goal-episode  $\langle e_{gs}, e_{ge}, lb_g, ub_g, sc_g \rangle$  is closed by a value-episode  $\langle e_{vs}, e_{ve}, lb_v, ub_v, sc_v \rangle$ , if  $sc_v \models sc_g$ , and the events are constrained such that  $e_{vs} = e_{gs}$  and  $e_{ve} = e_{ge}$ . A goal appearing in the goal-history which is not closed is *open*.

In the context of a state-plan,  $\langle GH, \mathcal{VH}, \mathcal{J} \rangle$ : The goal-episode occurs in the the goal history  $GH$ . The value-episode occurs in a value history,  $VH \in \mathcal{VH}$ . And, we represent closed by adding two justifications to  $\mathcal{J}$ , which constrain the two episodes to start and end at the same time.

**Definition 8. valid.** In general,  $SP_{total}$  is a *valid* plan for the problem  $\langle TCA, SP_{part} \rangle$ , if  $SP_{total}$  has no *open* goal-episodes, *closes* all the goal-episodes in  $SP_{part}$ , and has value-histories that both contain the value history of  $SP_{part}$  and is a legal trace of the *TCA*.

For tBurton, we help ensure  $SP_{total}$  is a valid plan by introducing two additional requirements. First, we require that  $SP_{part}$  contains an episode in the value history of each location variable,  $l$ , whose end event must precede the start event of any goal on  $l$ , thus providing a complete 'initial state'. Second we require that  $SP_{part}$  be a subgraph of  $SP_{total}$ . These two additional requirements allow us to simplify the definition of valid:  $SP_{total}$  is a valid plan if it has no open goals and is a trace of the *TCA*. Figure 3(b), which depicts the total state plan for the projector example, uses bold lines to emphasize the subset of episodes that came from the partial plan.

To ensure  $SP_{total}$  is temporally, *least commitment*, the state plan must be consistent, complete, and minimal with respect to the planning problem. A valid state plan is already *consistent* and *complete* in that it expresses legal behavior for the model and closes all goals. We consider a state plan to be *minimal* if relaxing any of the episodes (decreasing the lower-bound, or increasing upper-bound) in the histories admit traces of the *TCA* that are also legal. Structurally, the definition of a state plan (def. 4) ensures tBurton cannot return plans containing disjunctive state constraints in the value histories, nor can the plan contain disjunctive temporal constraints.

**Definition 9. causal graph.** The causal graph of a *TCA* is a digraph  $\langle \mathcal{A}, E \rangle$  consisting of a set of automata embedded as vertices and a set of causal edges. A causal edge  $(A, B)$  is a member of  $E$  iff  $A \neq B$  and there exists a transition in

automaton  $B$  guarded by the location variable,  $l$  of automaton  $A$ . We say  $A$  is the *parent* of  $B$ , and equivalently,  $B$  is the child of  $A$ .

## tBurton Planner

tBurton's fundamental approach is to plan in a factored space by performing regression over histories; this is supported through several algorithms. Practically, tBurton's search involves: 1. Deciding which factor to plan for first. This scopes the remaining decisions by selecting the value history we must populate and the goal-episodes we need to close. 2. Choosing how to order the goal-episodes that factor can close. 3. Choosing a set of value-episodes that should be used to close those goal-episodes. 4. Choosing the plans that should be used to achieve the value-episodes. 5. And finally, extracting the subgoals of the value history (guards) required to make the value-history a legal trace of the automata and adding corresponding goal-episodes to the goal history. These steps repeat until a plan is found, or no choices are available.

As a partial order planner, tBurton searches over variations of the state plan. Since we use the causal graph to define a search order, and subgoal extraction requires no search, tBurton only has three different choices with which to modify  $SP_{part}$ :

1. **Choose a goal ordering.** Since actions are not reversible and reachability checking is hard, the order in which goals are achieved matters. tBurton must impose a total ordering on the goals involving the location of a single automaton. Recall that since an automaton can have no concurrent transitions, a total order does not restrict the space of possible plans for any automaton. Relative to  $SP_{part}$ , imposing a total order involves adding episodes to the goal history of the form  $ep = \langle e_i, e_j, 0, \infty, true \rangle$ , for events  $e_i$  and  $e_j$  that must be ordered.
2. **Choose a value to close a goal.** Since goals can have constraints expressed as propositional state-logic, it is possible we may need to achieve disjunctive subgoals. In this case, tBurton must select a value that entails the goal. To properly close the goal, tBurton must also represent this value selection as an episode added to the value history of the appropriate automata or control variable, and introduce justifications to ensure the new episode starts and ends with goal episode (as described in definition 7).
3. **Choose a sub-plan to achieve a value.** The sub-plan tBurton must select need only consider the transitions in a single automaton,  $A$ . Therefore, the sub-plan must be selected based on two sequential episodes,  $ep_s, ep_g$ , in the value history of  $A$  (which will be the initial state and goal for the sub-plan), and the set-bounded temporal constraint that separates them. tBurton can use any black box approach to select this sub-plan, but we will use a heuristic forward search, temporal planner. To add this sub-plan to  $SP_{part}$ , tBurton must represent the sub-plan in the value history and introduce any new subgoals this sub-plan requires of parent automata as additional goal episodes.

Adding a sub-plan to a value history is a straightforward process of representing each state in the plan as a sequential chain of episodes  $ep_1, ep_2, \dots, ep_m$  reaching from the end event of  $ep_s$  to the start event of  $ep_g$ . Introducing subgoals is a bit trickier. Subgoals need to be computed from all value episodes added to represent the sub-plan,  $ep_i$ , as well as  $ep_g$  (for which we did not introduce goals as a result of the previous type of plan-space action, choosing a value to close a goal). The purpose of these subgoals is to ensure the parent automata and control variables have traces consistent with the trace (value-history) of  $A$ . There are two types of subgoals we can introduce: One is a maintenance related subgoal, that ensures  $A$  is not forced by any variables it depends on to transition early out of  $ep_i$ . The other expresses the subgoal required to effect the transition into  $ep_i$ .

To formalize the modifications to  $SP_{part}$ , let  $ep1 = \langle e_{s1}, e_{e1}, lb_1, ub_1, sc_1 \rangle$  be an episode  $ep_s, ep_i$ , or  $ep_g$  in the value history of the location variable  $l$  of  $A$ , for which we need to introduce goals. Let  $ep2$  be the episode that immediately follows  $ep1$ . Since  $ep1$  (and similarly for  $ep2$ ) is in the value history, we know from definition 4 that  $sc_1$  and  $sc_2$  are assignments to  $l$ . From the  $TCA$  model, lets also identify the set of transitions of the form,  $T = \langle l_s, l_e, g, c := 0 \rangle$  [def. 2], for which  $sc_1$  is the assignment  $l = l_s$ .

Adding a maintenance subgoal for  $ep_2$  requires the following additions: A new goal episode for each  $T$  where  $sc_2$  is not the assignment  $l = l_e$ , of the form  $ep_{new} = \langle e_{s1}, e_{e1}, 0, \infty, l \neq l_e \rangle$ . Adding a subgoal to effect the transition from  $ep_1$  to  $ep_2$  requires both a new goal episode and a justification. We add one goal episode for  $T$  where  $sc_2$  is the assignment  $l = l_e$ , of the form  $ep_{new} = \langle e_{e1}, e_{new}, 0, \infty, g \rangle$ . We also add a justification of the form  $J = \langle e_{e1}, e_{new}, 0, \infty \rangle$ . These last two additions ensure that as soon as the guard of the transition is satisfied,  $A$  will transition to its next state.

## Making Choices with Backtracking Search

At this point we have described the choices and changes tBurton must make to traverse the space of partial plans. We implement tBurton as a backtracking search, but use several additional algorithms to make this search more efficient: a subplanner implemented as a wrapper around an ‘off-the-shelf’ temporal planner, Incremental Total Order (ITO) to efficiently choose goal orderings, Incremental Temporal Consistency (ITC) to efficiently check whether the temporal constraints used in the episodes are temporally consistent, and Causal Graph Synthesis (CGS) to simplify traversing the causal graph by reducing the parent-child relation of the causal graph to a sequence of numbers. We summarize the tBurton algorithm in this subsection and then sketch out each of the component algorithms.

In order to maintain search state, tBurton uses a queue to keep track of the partial plans,  $SP_{part}$  that it needs to explore. For simplicity, one can assume this queue is LIFO, although in practice a heuristic could be used to sort the queue. To make search more efficient, we make two additional modifications to the  $SP_{part}$  we store on the queue. First, we annotate  $SP_{part}$  with both the automaton we are currently making choices on, as well as which of the three

choices we need to make next. The second involves the use of ITO. When tBurton needs to choose a goal ordering for a given partial plan, it could populate the queue with partial plans representing all the variations on goal ordering. Since storing all of these partial plans would be very memory intensive, we add to the partial plan a data structure from ITO, which allows us to store one partial plan, and poll it for the next consistent variation in goal ordering.

Algorithm 1 provides the pseudo code for tBurton’s high-level search algorithm. The algorithm reflects the three choices tBurton makes. In general, backtracking search traverses a decision tree and populates a search queue with all the children nodes of the expanded node. Since this can be very memory intensive for plan-space search, we take an alternate approach of populating the queue with the next child node to explore as well as a single child node representing ‘what not to do’ for the remaining children. Line 17 takes advantage of the incremental nature of ITO, to queue the next possible set of goal orderings. Lines 29-30, queue a partial state-plan with a guard representing which value assignments have already been tried. Lines 38-38, queue a state-plan with new goal episodes which remove already tried subplans. The behavior of the algorithm with a LIFO queue is to plan for the children automata in the causal graph before their parents, and to plan in a temporally forward manner for each automata.

## Subplanner

tBurton’s backtracking search exploits the structure of the causal graph, but finding a sub-plan for a factor, an automaton in the causal graph, is still hard. In general, we can leverage the state-of-the-art in temporal planners to find these sub-plans. However, there are a few special sub-plans we can compute without search. When asked to plan for a control variable  $u$ , the sub-plan will involve a single value episode where the state constraint is  $\perp$  with temporal constraints  $lb = 0, ub = \infty$ . This sub-plan represents our assumption that there is no transition behavior for commands. When asked to plan from one location to another for an automaton  $A$ , we can exploit the locality of information from having automata, and check the set of transitions of  $A$ . If there is a sequence of automatic timed transitions, we can return the subplan without search. Finally, the subplanner’s functional behavior, in which it returns a plan, given an automaton, the initial variable assignment, a goal variable assignment, and a  $[lb, ub]$  duration, is easily memoized.

## Incremental Total Order: Unifying Goal Histories

One of the choices that tBurton needs to make is how to order the goals required of an automaton. More specifically, given a set of goal-episodes, we want to add temporal constraints relating the events of those goal-episodes, so any goal-episodes that co-occur have logically consistent guards. By computing the total order over all the goal-episodes we can discover any inconsistencies, where, for example two goal-episodes could not possibly be simultaneously closed. In practice, computing the total order of goal-episodes is faster than computing a plan, so we can discover inconsistencies inherent in the set of goal-episodes faster than we

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**Algorithm 1:**  $tBurton(TCA, SP_{part})$ 

---

**Input:** Timed Concurrent Automata  $TCA = \langle L, \mathcal{C}, \mathcal{U}, A \rangle$ ,  
Partial State Plan  $SP_{part}$   
**Output:** Total State Plan  $SP_{total}$

```
1 // Factor the problem to create the causal graph
2  $\{A_1, A_2, \dots, A_n\} \leftarrow CausalGraphSynthesis(TCA)$ ;
3 // Add the partial plan to the queue.
4  $A_i$  is the lowest numbered automaton with a goal in  $SP_{part}$ 
5  $SP_{part} \leftarrow initITO(SP_{part}, A_i)$ ;
6  $Q.push(\langle SP_{part}, A_i, choose\_goal\_order \rangle)$ ;
7 while  $Q \neq \emptyset$  do
8   // remove a partial plan from the queue.
9    $\langle SP_{part}, A, choice \rangle \leftarrow Q.pop()$ ;
10  switch  $choice$  do
11    case  $choose\_goal\_order$ 
12      if  $SP_{part}$  has no open goals then
13        return  $SP_{part}$ ;
14      else
15         $SP_{part} \leftarrow ITO(SP_{part}, A)$ ;
16        if  $SP_{part}$  exists then
17           $Q.push(\langle SP_{part}, A, choose\_goal\_order \rangle)$ ;
18          if  $ITC(SP_{part})$  then
19             $Q.push(\langle SP_{part}, A, choose\_value \rangle)$ ;
20          else
21            continue;
22    case  $choose\_value$ 
23       $l$  or  $u$  of  $A$  is used in the guard  $g$  of open goal
24      episode  $ep_g$ 
25      if  $g = false$  then
26        continue;
27      else
28        Choose assignments  $\bigwedge(x = v)$  that entail  $g$ ,
29        where  $x$  can be  $l$  or  $u$  of  $A$ .
30        // update guard and re-queue.
31         $SP_{part\_up} = SP_{part}$  with new guard
32         $g \leftarrow g \wedge \neg \bigwedge(x = v)$ ;
33         $Q.push(\langle SP_{part\_up}, A, choose\_value \rangle)$ ;
34        // add chosen values to partial state plan
35        Add  $\bigwedge(x = v)$  as value episodes to  $SP_{plan}$ 
36         $Q.push(\langle SP_{part}, A, choose\_subplan \rangle)$ ;
37    case  $choose\_subplan$ 
38       $ep_s$  and  $ep_e$  are sequential value episodes of the
39      value history for  $l$  or  $u$  of  $A$  in  $SP_{part}$ , separated
40      by a temporal constraint  $dur$ .
41       $subplan \leftarrow subplanner(A, ep_s.sc, ep_e.sc, dur)$ ;
42      if  $subplan$  exists then
43        Add  $subplan$  and parent subgoals to  $SP_{part}$ 
44         $SP_{part\_up} = SP_{part}$  with negated  $subplan$ 
45        as goal-episodes.
46         $Q.push(\langle SP_{part\_up}, A, choose\_subplan \rangle)$ ;
47        if  $l$  or  $u$  of  $A$  is used in open goals of  $SP_{part}$  then
48           $Q.push(\langle SP_{part}, A, choose\_value \rangle)$ ;
49        else
50           $A_i$  is the lowest numbered automaton with a goal in
51           $SP_{part}$ 
52           $SP_{part} \leftarrow initITO(SP_{part}, A_i)$ ;
53           $Q.push(\langle SP_{part}, A_i, choose\_goal\_order \rangle)$ ;
54        else
55          continue;
56  return  $failure$ ;
```

---

can discover that no plan exists to get from one goal-episode to the next.

Our incremental total order algorithm builds upon work which traverses a tree to enumerate all total orders given a partial order (Ono and Nakano 2005). We modify their algorithm into an incremental one consisting of a data structure maintaining the position in the tree, an initialization function, `init`, and an incremental function, `next`, which returns the next total order.

Our ITO algorithm is similarly divided into two pieces. The `initITO` algorithm creates the ITO data structure by calling `init` with  $SP_{part}$  and a particular automaton  $A$ . `initITO` treats temporal constraints of the episodes as partial orders. ITO can then be called repeatedly to enumerate the next consistent ordering of goal episodes where the location variable  $l$  of  $A$  is involved.

## Causal Graph Synthesis and Temporal Consistency

We use existing algorithms for Causal Graph Synthesis and Incremental Temporal Consistency (ITC), so we briefly motivate their purpose and use.

The Causal Graph Synthesis (CGS) algorithm is based on the algorithm used for Burton (Williams and Nayak 1997), and is simple to describe. Given a TCA, CGS checks the dependencies of each automaton in TCA and creates a causal graph. If the causal graph contains cycles, the cycles are collapsed and the product of the participating automata are used to create a new compound automata. Finally, each automaton in the causal graph is numbered sequentially in a depth-first traversal of the causal graph starting from 1 at a leaf. The numbering imposes a search order (with 1 being first), which removes the need to choose which factor to plan for next.

The Incremental Temporal Consistency (ITC) algorithm is used to check whether the partial plan  $SP_{part}$  is temporally consistent, or that the temporal constraints in goal, value histories, and justifications are satisfied. Since `tBurton` will perform this check many times with small variations to the plan, it is important this check be done quickly. For this purpose we use the incremental temporal consistency algorithm defined in (Shu, Effinger, and Williams 2005).

## Mapping PDDL to TCA

Even though `tBurton` reasons over TCAs, it is still a capable PDDL planner. In order to run `tBurton` on PDDL problems, we developed a PDDL 2.1 (without numeric fluents) to TCA translator. Here, we provide only a sketch of this translator.

In order to maintain required concurrency, the translator first uses temporal invariant synthesis (Bernardini and Smith 2011) to compute a set of invariants. An instance of an invariant identifies a set of ground predicates for which at most one can be true at any given time. We select a subset of these invariant instances that provide a covering of the reachable state-space, and encode each invariant instance into an automaton. Each possible grounding of the invariant instance becomes a location in the automata.

Each ground durative action is also translated into an automaton. In Figure 4, three of the transitions are guarded

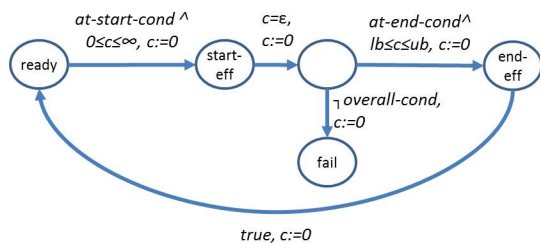


Figure 4: An example TCA automaton for a generic PDDL grounded action.

by conditions from the corresponding PDDL action, translated into propositional state logic over location variables. Another transition uses  $\epsilon$  to denote a small-amount of time to pass for the start-effects of the action to take effect, prior to checking for the invariant condition. A fifth transition is used to reset the action.

Finally, the transitions of each invariant-instance based automata is labeled with a disjunction of the states of the ground-action automata that affect its transition.

Inherent in this mapping is an assumption that PDDL durative actions will not self-overlap in a valid plan. Planning with self-overlapping durative actions in PDDL is known to be EXPSPACE, while without such actions is PSPACE (Rintanen 2007). This suggests that tBurton may solve a simpler problem, but the actual complexity of TCA planning with autonomous transitions has yet to be addressed. In the meantime, if the number of duplicate actions can be known ahead of time, they can be added as additional ground-action automata.

## Results

We benchmarked tBurton on a combination of IPC 2011 and IPC 2014 domains. Temporal Fast Downward (TFD) from IPC 2014 was used as an ‘off-the-shelf’ sub-planner for tBurton because it was straight-forward to translate *TCA*s to the SAS representation used by TFD (Helmert 2006). For comparison, we also benchmarked against YAHSP3-MT from IPC 2014, POPF2 from IPC 2011, and TFD from IPC 2014, the winner or runners up in the 2011 and 2014 temporal satisficing track (Table 1). Each row represents a domain. Columns are grouped by planner and show the number of problems solved (out of 20 for each domain) and the IPC score (out of 20, based on minimizing make-span). Rows with interesting results between TFD and tBurton are italicized, and the best scores in each domain are in bold. The tests were run with scripts from IPC 2011 and were limited to 6GB memory and 30 minute runtime.

In general, tBurton is capable of solving approximately the same number of problems as TFD with the same quality, but for problems which admit some acyclic causal factoring (parcprinter, satellite, and TMS), tBurton performs particularly well. On domains which have no factoring, tBurton’s high-level search provides no benefit and thus degrades to

using the sub-planner. This often resulted in tBurton receiving the same score as its subplanner, TFD (elevators, parking, openstack). While the same score often occurs when TFD is already sufficiently fast enough to solve the problem, there are a few domains where tBurton’s processing overhead imposes a penalty (floortile, sokoban).

A feature of using tBurton is that it is capable of solving problems with required concurrency. For TMS, tBurton is able to consistently solve more problems than the other planners. However, the IPCScore of tBurton is lower than POPF, perhaps because the goal-ordering strategy used by tBurton does not inherently minimize the make-span like the temporal relaxed planning graph used by POPF.

While benchmarking on existing PDDL domains is a useful comparison, it is worth noting that these problems do not fully demonstrate tBurton’s capabilities. In particular, none of these domains have temporally extended goals, and all actions have a single duration value instead of an interval. We look forward to more comprehensive testing with both existing PDDL domains and developing our own benchmarks.

## Conclusion

This paper presents tBurton, a planner that uses a novel combination of causal-graph factoring, timeline-based regression, and heuristic forward search to plan for networked devices. It is capable of supporting a set of problem features not found before in one planner, and is capable of doing so competitively. Furthermore, tBurton can easily benefit from advancements in state-of-the art planning by replacing its sub-planner. The planning problem tBurton solves assumes devices with controllable durations, but we often have little control over the duration of those transitions. In future work we plan to add support for uncontrollable durations.

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Domain	YAHSP3-MT (2014)		POPF (2011)		TFD (2014)		tBurton+TFD	
	#Solved	IPCScore	#Solved	IPCScore	#Solved	IPCScore	#Solved	IPCScore
CREWPLANNING (2011)	<b>20</b>	<b>19.88</b>	<b>20</b>	<b>20.00</b>	<b>20</b>	<b>19.85</b>	<b>20</b>	<b>20.00</b>
DRIVERLOG (2014)	<b>3</b>	<b>1.77</b>	0	0.00	0	0.00	0	0.00
ELEVATORS (2011)	<b>20</b>	11.20	3	2.10	<b>20</b>	<b>18.95</b>	<b>20</b>	<b>18.95</b>
FLOORTILE (2011)	<b>11</b>	<b>9.29</b>	1	0.89	5	5.00	3	3.00
FLOORTILE (2014)	<b>6</b>	<b>5.83</b>	0	0.00	0	0.00	0	0.00
OPENSTACKS (2011)	<b>20</b>	14.47	<b>20</b>	16.59	<b>20</b>	<b>19.84</b>	<b>20</b>	<b>19.84</b>
PARCPRINTER (2011)	<b>1</b>	<b>1.00</b>	0	0.00	<b>10</b>	<b>9.67</b>	<b>13</b>	<b>11.98</b>
PARKING (2011)	<b>20</b>	15.74	<b>20</b>	17.42	<b>20</b>	<b>19.14</b>	<b>20</b>	<b>19.14</b>
PARKING (2014)	<b>20</b>	<b>17.96</b>	12	9.16	<b>20</b>	16.18	<b>20</b>	16.18
PEGSOL (2011)	<b>20</b>	18.52	19	<b>18.77</b>	19	18.42	18	17.38
SATELLITE (2014)	<b>20</b>	<b>17.46</b>	4	3.67	<b>17</b>	<b>12.57</b>	<b>19</b>	<b>16.26</b>
SOKOBAN (2011)	<b>10</b>	<b>8.69</b>	3	2.54	5	4.94	3.00	2.54
STORAGE (2011)	<b>7</b>	<b>6.54</b>	0	0.00	0	0.00	0	0.00
STORAGE (2014)	<b>9</b>	<b>8.41</b>	0	0.00	0	0.00	0	0.00
TMS (2011)	0	0.00	5	<b>5.00</b>	0	0.00	<b>6</b>	3.77
TMS (2014)	0	0.00	0	0.00	0	0.00	<b>1</b>	<b>1.00</b>
TURNANDOPEN (2011)	0	0.00	9	<b>8.47</b>	<b>19</b>	<b>16.53</b>	<b>20</b>	<b>17.03</b>
TURNANDOPEN (2014)	0	0.00	0	0.00	<b>6</b>	<b>6.00</b>	<b>6</b>	<b>6.00</b>
TOTALS:	187	156.74	116	104.61	181	167.09	<b>189</b>	<b>173.07</b>

Table 1: Benchmark results on IPC domains

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