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Power System Restoration With Transient Stability

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Abstract

We address the problem of power system restoration after a significant blackout. Prior work focus on optimization methods for finding high-quality restoration plans. Optimal solutions consist in a sequence of grid repairs and corresponding steady states. However, such approaches lack formal guarantees on the transient stability of restoration actions, a key property to avoid additional grid damage and cascading failures. In this paper, we show how to integrate transient stability in the optimization procedure by capturing the rotor dynamics of power generators. Our approach reasons about the differential equations describing the dynamics and their underlying transient states. The key contribution lies in modeling and solving optimization problems that return stable generators dispatch minimizing the difference with respect to steady states solutions. Computational efficiency is increased using preprocessing procedures along with traditional reduction techniques. Experimental results on existing benchmarks confirm the feasibility of the new approach.

Introduction

Power system restoration is a key ingredient in defining the self-healing property of the future smart grid. Restoration plans are generated automatically using smart optimization procedures, reducing the cost and the inconvenience caused by blackouts. A restoration plan consists in a set of repair actions, such as connecting or disconnecting a line. The primary objective is to isolate faulty equipments while maximizing the percentage of load recovery. The problem has a time dimension related to the ordering of the repair actions. This ordering has a direct influence on the load recovery over time. It is, for instance, natural to favor early load recoveries. This captures the essence behind the Restoration Ordering Problem which is at the core of this paper. Traditionally, this problem is addressed with simplifying assumptions, e.g., ignoring ramping rates of generators but, most importantly, disregarding transient stability during line switching. Incorporating operational limits on transient states is computationally challenging, as they are defined using partial differential equations (PDEs). Existing discrete optimization tools cannot deal with such non-algebraic expressions. In our context, this refers to the ability to reason about the power system transient states over a short period of time (e.g., a few seconds) after the line closing.

In this work, we focus on capturing the dynamics of generator rotor angles as a measure of transient stability related to switching operations in transmission networks. Our contributions are based on a few key ideas:

1. We use rotor angle stability, a traditional metric to assess the transient stability of a system, and the classical model of generator dynamics based on the Swing equations (Kundur, Balu, and Lauby 1994).
2. We capture transient states through a discretization of the dynamics model, including a nonlinear formulation of the power flow equations on these transient states.
3. For each restoration step, we define an optimization model that finds a transient-stable generator dispatch minimizing the difference with respect to the steady state solution. This ensures that the size of the blackouts increase as little as possible while ensuring transient stability.

More generally, this paper can be viewed as a novel, hierarchical approach to the optimization of complex hybrid dynamic systems. Given a static version of the models, the key idea relies in adjusting optimal steady-state solutions taking into account the system dynamics. The overall computational tractability is thus improved.

Related Work

Our work is closely related to the transient-stable optimal power flow problem, which was first proposed by Gan et al. (Gan, Thomas, and Zimmerman 2000). The problem was later extended to multi-contingency settings in (Yuan, Kubokawa, and Sasaki 2003). Both approaches utilize the swing equation to reason on transient stability. Our work is also related to techniques improving stability during transmission loop closures in normal operating conditions, e.g., techniques on reducing rotor shaft impacts and standing phase angles (Martins et al. 2008; Ye and Liu 2013; Ketabi, Ranjbar, and Feuillet 2002; Hazarika and Sinha 1999).

To the best of our knowledge, this work represents the first attempt to incorporate transient stability into full power restoration schemes and to optimize generator dispatches and load pickups as additional degrees of freedom.

Background

In this section, we introduce our notations and discuss the Restoration Ordering Problem (Van Hentenryck, Coffrin, and Bent 2011; Coffrin and Van Hentenryck 2014b).

Nomenclature

N	Set of buses in the power grid
$N(n)$	Set of buses connected to bus n
$L \subseteq N \times N$	Set of lines
$G \subseteq N$	Set of generators in the power grid
$G(n)$	Set of generators at bus n
$D \subseteq N$	Set of demands in the power grid
$D(n)$	Set of demands at bus n
R	Set of damaged components to repair
$S_{ij} = p_{ij} + iq_{ij}$	Power flow for line $< i, j >$
$y_{ij} = g_{ij} + ib_{ij}$	Line admittance for line $< i, j >$
$Y = G + iB$	The Y-bus admittance matrix
$V_i = v_i \angle \theta_i$	Voltage magnitude and phase angle at bus i
θ_{ij}, v_{ij}	Shorthand for $\theta_i - \theta_j$ and $v_i - v_j$
$\overline{x}, \underline{x}$	Upper and Lower bound on x

The Restoration Ordering Problem

This section introduces the main notations used in the paper and formalizes the Restoration Ordering Problem (ROP) (Van Hentenryck, Coffrin, and Bent 2011; Coffrin and Van Hentenryck 2014b). The ROP takes as inputs the topology of the network $\mathcal{PN}$ and the set R of damaged components to be repaired. Its main goal is to find the best ordering of the repairs in R in order to recover the loads in the network as quickly as possible. The intuition underlying the ROP is depicted in Figure 1. The ROP only reasons about steady states, each of which corresponds to a restoration action. Moreover, at each step, the ROP searches for a steady state that maximizes the served load, which can be expressed as a power flow sub-problem. In other words, the ROP searches over sequences of $|R|$ steady states and aims at finding the sequence maximizing the served load over time. This paper focuses on transmission line restorations for simplicity (i.e., $R \subseteq L$) but the ROP can be generalized easily to other components (Van Hentenryck, Coffrin, and Bent 2011).

Model 1 shows a simplified Mixed-Integer Nonlinear Program for the ROP. Its two main set of decision variables are the variables z_{rl} and l_{ri}: Binary variable z_{rl} specifies whether line l is operational at step r and variable l_{ri} represents the percentage of served load at bus i for step r. The remaining variables are the traditional power flow variables. There are $|R|$ copies of these variables (and the associated constraints), one for each restoration step.

The objective function (O.1) maximizes the load restored over time. Constraints (C.1.1–C.1.2) ensures that exactly only one line is repaired at each step. Constraints (C.2.1–C.2.2) enforce power conservation on nodes while constraints (C.2.3–C.2.8) implement the AC steady-state power flow equations and the thermal limits on transmission lines (i.e., constraints (C.2.5) and (C.2.8)). Constraints (C.2.3–C.2.5) are conditional to the appropriate line being operational.

Model 1 AC Restoration Ordering Problem

Inputs:

$\mathcal{PN} = \langle N, L, G, D \rangle$	- Power network
$R \subseteq L$	- Damaged items
p^l_i, q^l_i	- Active/reactive power loads on bus i

Variables: $(0 \le r \le |R|)$

$z_{rl} \in \{0,1\}, \forall l \in R$	- Operating state of line l at step r
$l_{ri} \in [0,1], \forall i \in D$	- load percentage of load i at step r
$\theta_{ri} \in (-\infty, \infty), \forall i \in N$	- bus angle at step r
$v_{ri} \in (\underline{v}_i, \overline{v}_i), \forall i \in N$	- bus voltage magnitude at step r
$p^g_{ri} \in (0, \overline{p}^g_i), \forall i \in G$	- Active injection of generator i
$q^g_{ri} \in (\underline{q}^g_i, \overline{q}^g_i), \forall i \in G$	- Reactive injection of generator i
$p_{rij} \in (-\overline{S}_{ij}, \overline{S}_{ij})$	- Active flow from i to j
$q_{rij} \in (-\overline{S}_{ij}, \overline{S}_{ij})$	- Reactive flow from i to j

Maximize

$$\sum_{r=0}^{|R|} \sum_{i \in D} \overline{p}^l_i l_{ri} \tag{O.1}$$

Subject to: $(0 \le r \le |R|)$

$$\sum_{l \in R} z_{rl} = r, \tag{C.1.1}$$

$$z_{(r-1)l} \le z_{rl}, \quad \forall l \in R, r \neq 0 \tag{C.1.2}$$

$\forall i \in N$:

$$\sum_{j \in G(i)} p^g_{rj} - \sum_{j \in G(i)} p^l_j l_{rj} = \sum_{j \in N(i)} p_{rij} \tag{C.2.1}$$

$$\sum_{j \in G(i)} q^g_{rj} - \sum_{j \in G(i)} q^l_j l_{rj} = \sum_{j \in N(i)} q_{rij} \tag{C.2.2}$$

$\forall i, j : \exists\, l \in R$ between buses i and j

$$p_{rij} = z_{rl}(g_{ij} v^2_{ri} - v_{ri} v_{rj}(g_{ij} \cos(\theta_{rij}) - b_{ij} \sin(\theta_{rij}))) \tag{C.2.3}$$

$$q_{rij} = z_{rl}(-b_{ij} v^2_{ri} - v_{ri} v_{rj}(g_{ij} \sin(\theta_{rij}) - b_{ij} \cos(\theta_{rij}))) \tag{C.2.4}$$

$$p^2_{rij} + q^2_{rij} \le \overline{S}_{ij} z_{rij} \tag{C.2.5}$$

$\forall i, j : \exists\, l \in L - R$ between buses i and j

$$p_{rij} = g_{ij} v^2_{ri} - v_{ri} v_{rj}(g_{ij} \cos(\theta_{rij}) - b_{ij} \sin(\theta_{rij})) \tag{C.2.6}$$

$$q_{rij} = -b_{ij} v^2_{ri} - v_{ri} v_{rj}(g_{ij} \sin(\theta_{rij}) - b_{ij} \cos(\theta_{rij})) \tag{C.2.7}$$

$$p^2_{rij} + q^2_{rij} \le \overline{S}_{ij} \tag{C.2.8}$$

The ROP is computationally challenging. On the one hand, it suffers from the combinatorial explosion of ordering problems. On the other hand, the non-convex power flow equations represent a major computational challenge in cold-start contexts (Stott, Jardim, and Alsac 2009; Overbye, Cheng, and Sun 2004). The first difficulty is typically addressed using greedy algorithms, large neighborhood search, and/or randomized adaptive decompositions (Bent and Van Hentenryck 2007; Van Hentenryck, Coffrin, and Bent 2011). The second difficulty is avoided by convexifying the nonlinear power flow equations, e.g., using linear approximations (Van Hentenryck, Coffrin, and Bent 2011; Coffrin and Van Hentenryck 2014a; 2014b), or quadratic relaxations (Coffrin et al. 2014; Hijazi, Coffrin, and Van Hentenryck 2013).

Transient Stability for Restoration Planning

Prior work on power restoration has largely focused on computing optimal restoration plans maximizing load recovery over steady states. However, in practice, these restorations

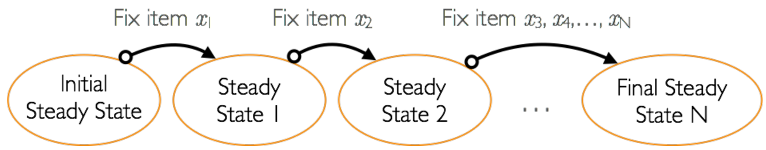

Figure 1: A diagram illustrating a sequence of steady states generated by Restoration Ordering Problem

may not always be feasible as there could be violations of operational limits, safety limits, and stability limits (Adibi and Kafka 1991). The ROP only captures such operational limits as algebraic constraints over steady states, e.g., thermal limits imposed by Constraint C.2.5 in Model 1. It does not capture limits on transient states. Hence, it is possible to derive restoration actions that are not transient stable (Kheradmandi and Ehsan 2004), thus increasing the size of the blackout. Adding constraints on the steady states (Mak et al. 2014) can reduce this possibility. This paper goes one step further by incorporating the system dynamics, which we now present the system dynamics before describing the overall approach.

Rotor Dynamics and Swing Equations

The general form for computing power system dynamic response (Kundur, Balu, and Lauby 1994; Yuan, Kubokawa, and Sasaki 2003) after a disturbance is usually written as follows:

$$\dot{x} = f(x, y)$$
$$0 = g(x, y)$$

where $g(\bullet)$ represents a set of algebraic equations describing the power transmission network and $f(\bullet)$ represents a set of first order differential equations describing the internal dynamics of network equipments. Vector x captures the short-term dynamic variables and y is a vector of algebraic state variables. Given an initial condition for variables x and y, we can compute a set of transient states over time by using equations $f(\bullet)$ and $g(\bullet)$. To incorporate rotor angle stability, we first introduce the "classical" model of generator dynamics corresponding to the Swing equations (Kundur, Balu, and Lauby 1994) (for implementing $f(\bullet)$):

$$\frac{d\delta_i}{dt} = \omega_i - \omega_0$$
$$\frac{2H_i}{\omega_0}\frac{d\omega_i}{dt} = p_i^m - p_i^e - D_i\omega_i$$

where H_i, δ_i, D_i, ω_i, and ω_0 respectively denote the inertia constant, the rotor angle, the damping coefficient, the current angular velocity, and the nominal angular velocity of a generator i (which is assumed constant for all generators at 50Hz/60Hz). On the right hand side of the equation, p_i^m and p_i^e are the mechanical and electrical powers acting on the rotor of generator i. In the Swing equations, δ_i and ω_i are short-term dynamic variables (i.e., the x-vector above) and p_i^m and p_i^e are algebraic state variables (i.e., the y-vector above). In steady states, the mechanical power is assumed to be equal to the electrical power and the rotor angle of all generators remains constant. Once a disturbance occurs, the electrical power changes and there is a difference between mechanical and electrical power. Hence, the rotor angle of generators changes according to the Swing equation. Figure 2 shows a the rotor swings after the closure of a transmission line for a 3-bus example. The y-axis shows the rotor angle (in degrees) of the generators and the x-axis shows the time in seconds after the line closing.

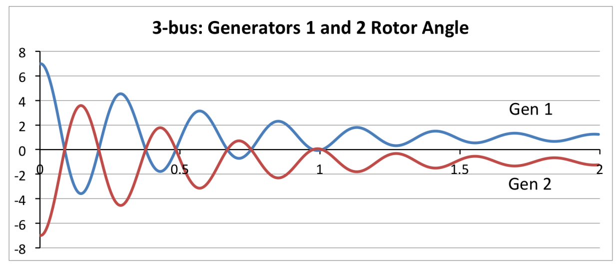

Figure 2: Rotor swings during transmission loop closures

To implement the Swing equations in optimization tools, we perform a trapezoidal discretization over a given time horizon ($1 \leq t \leq T$) (Gan, Thomas, and Zimmerman 2000). This leads to the following discretized equations,
$\forall i \in G, 1 \leq t \leq T - 1,$

$$\delta_{it+1} - \delta_{it} - \frac{\Delta}{2}(\omega_{it+1} + \omega_{it}) = 0 \quad (1)$$

$$\omega_{it+1} - \omega_{it} - \frac{\Delta}{2}(a_{it+1} + a_{it}) = 0 \quad (2)$$

$$a_{it+1} - \frac{\omega_0}{2H_i}(p_{i1}^e - p_{it+1}^e - D_i\omega_{it}) = 0 \quad (3)$$

with the following initial conditions at $t = 1$,

$$\omega_{i1} = 0, a_{i1} = 0 \quad (4)$$

where δ_{it}, ω_{it}, a_{it}, and p_{it}^e are the rotor angle, the angular velocity, the angular acceleration, and the electrical power of generator i at time step t, $1 \leq t \leq T$. Δ is a constant representing the time interval between every time step.

To increase rotor stability, (Gan, Thomas, and Zimmerman 2000) suggest to bound at all time steps the difference between the rotor angle of a generator δ_{it} and a reference angle δ_t^r representing the Center Of Inertia (COI):

$$\delta_t^r = \frac{\sum_{i \in G} H_i\delta_{it}}{\sum_{i \in G} H_i}, 1 \leq t \leq T \quad (5)$$

The stability constraints then become:

$$\forall i \in G, 1 \leq t \leq T : -\overline{\delta} \leq \delta_{it} - \delta_t^r \leq \overline{\delta} \quad (6)$$

where $\overline{\delta}$ represents the bound constant to be determined.

AC Power Flow Equations Over Time

The set of algebraic constraints $g(\bullet)$ include the AC power flow equations characterizing power transmission and passive network equipments (Liu and Thorp 2000). Two admittance matrices define the line properties and the load characteristics, before and after a line switch, resp. Y^o and Y^c. The AC power flow equations are then specified as follows, $\forall\, i \in N, 2 \le t \le T$,

$$\sum_{j \in N} [V_{it} V_{jt} (G^c_{ij} \cos(\theta_{ijt}) + B^c_{ij} \sin(\theta_{ijt}))] = \sum_{j \in G(i)} p^e_{jt} \quad (7)$$

$$\sum_{j \in N} [V_{it} V_{jt} (G^c_{ij} \sin(\theta_{ijt}) - B^c_{ij} \cos(\theta_{ijt}))] = \sum_{j \in G(i)} q^e_{jt} \quad (8)$$

with the following initial conditions for $t = 1$:

$$\sum_{j \in N} [V_{i1} V_{j1} (G^o_{ij} \cos(\theta_{ij1}) + B^o_{ij} \sin(\theta_{ij1}))] = \sum_{j \in G(i)} p^e_{j1} \quad (9)$$

$$\sum_{j \in N} [V_{i1} V_{j1} (G^o_{ij} \sin(\theta_{ij1}) - B^o_{ij} \cos(\theta_{ij1}))] = \sum_{j \in G(i)} q^e_{j1} \quad (10)$$

In the classical generator model, every generator i is defined as an internal bus connected to a terminal node with transient reactance X_i and constant voltage V^g_i. The rotor angles δ_{it} are then treated as bus phase angles. Note that we ignore the reactive power flow constraints for transient states (i.e., when time $t > 1$) (Liu and Thorp 2000). For all $i \in G, 1 \le t \le T$,

$$p^e_{it} = \frac{V^g_i V_{it}}{X_i} \sin(\theta_{it} - \delta_{it}) \quad (11)$$

with the following extra condition on time step $t = 1$,

$$q^e_{i1} = \frac{V^2_{i1}}{X_i} - \frac{V^g_i V_{i1}}{X_i} \cos(\theta_{i1} - \delta_{i1}) \quad (12)$$

The last part corresponds to boundary conditions representing safety/operating limits. For the initial state at $t = 1$, for all $i \in G$ we have,

$$0 \le p^e_{i1} \le \overline{p}^e_i, \underline{q}^e_i \le q^e_{i1} \le \overline{q}^e_i, \underline{V}^g_i \le V^g_i \le \overline{V}^g_i \quad (13)$$

For all time steps t, we have,

$$\forall i \in N : \underline{V}_i \le V_{it} \le \overline{V}_i \quad (14)$$

$$\forall i \in G : -\overline{\theta} \le \delta_{it} - \theta_{it} \le \overline{\theta} \quad (15)$$

$$\forall \langle i, j \rangle \in L : -\overline{\theta} \le \theta_{it} - \theta_{jt} \le \overline{\theta} \quad (16)$$

Let us emphasize that this formulation allows voltage magnitudes to fluctuate at every transient time step t. This may naturally lead to other stability issues. In order to avoid drastic fluctuations, we add the following constraints bounding their magnitude for all time steps.
$\forall\, i \in N, 1 \le t \le T - 1$,

$$V_{it} - \overline{V^\Delta} \le V_{it+1} \le V_{it} + \overline{V^\Delta}. \quad (17)$$

A Non-Linear Program for Stability Enhancement

Given the set of constraints (1)-(17) defining the transient behavior of the system, we now introduce Model 2 for stability enhancement after a line-switching operation. An optimal solution in Model 2 guarantees a transient-stable switching with generator dispatches that minimize the distance to the original steady-state solution.

Model 2 Transient Stable Line Closing Model

Inputs:

$\mathcal{PN}$	- Power network
Y^o, Y^c	- Admittance matrix before and after line closing
p^T_i, q^T_i	- i^{th} generator active/reactive target dispatch,
X_i, H_i, D_i	- transient reactance, inertia, and damping constant
$f_q = \frac{\omega_0}{2\pi}, \Delta, T$	- Grid frequency, integration step, time horizon
$\overline{\delta}, \overline{V^\Delta}$	- Maximum rotor swing and voltage fluctuations

Variables: $(\forall t : 1 \le t \le T)$

$\forall i \in N$,

θ_{it}	- Terminal bus angle
V_{it}	- Bus voltage

$\forall i \in G$,

V^g_i	- Internal bus voltage
p^e_{it}, q^e_{it}	- Generator active and reactive injection
$\delta_{it}, \omega_{it}, a_{it}$	- Generator rotor angle, velocity, and acceleration
δ^r_t	- Rotor angle reference index

Minimize

$\sum_{i \in G} [(p^e_{i1} - p^T_i)^2 + (q^e_{i1} - q^T_i)^2]$ - Distance from target dispatch

Subject to:

Equations (1) - (17)

Simplification and Kron Reduction

In Model 2, the number of variables is O($|N|T$). The model dimension can be reduced by constructing a smaller equivalent network and computing the corresponding admittance matrices. Since a line closing only affects a subset of connected complements in the network, we disregard islands that are not related to the current switching. An admittance preserving reduction known as Kron reduction (Ward 1949) is then performed. This technique removes all buses that are not under study by translating their properties into new lines. In our case, since we are only interested in rotor dynamics, Kron reduction is used to remove all non-generator buses. This results in two admittance matrices Y^o and Y^c representing the reduced network before and after line closing. The number of variables in Model 2 is then O($|G| {\cdot} T$), where G represents the number of generators on the island corresponding to the line closings. Figure 3 shows an example of Kron reduction on a 6-bus network with three generators (G1 to G3), three loads (L1 to L3), and five transmission lines. After performing the Kron reduction, only three buses (with generators) are left. Three virtual transmission lines are constructed and loads L1 to L3 are re-adjusted to maintain an equivalent transfer admittance and power flows.

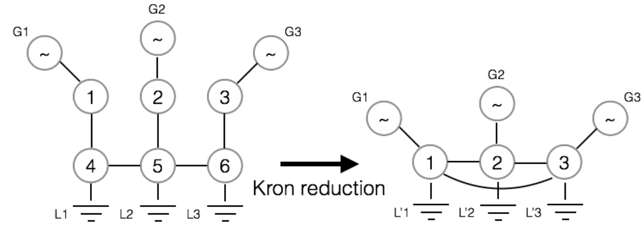

Figure 3: An example network illustrating Kron reduction

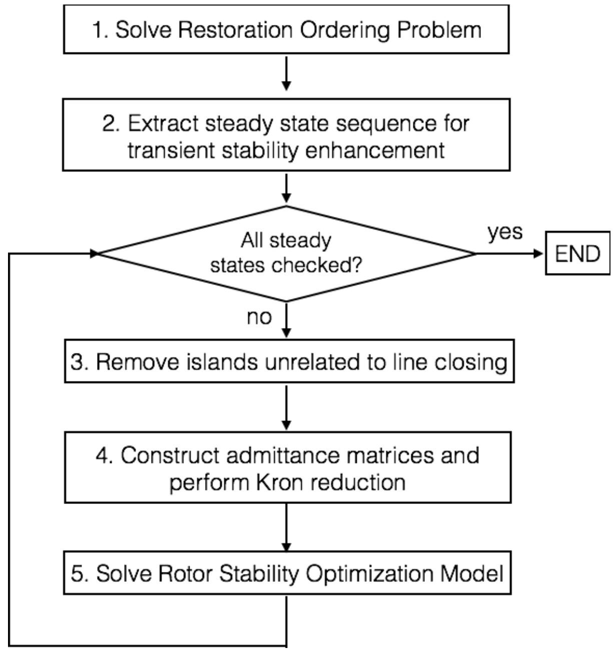

Figure 4: Flow chart for the stability enhancement routine

Kron reduction is inspired by Kirchhoff's Current Law (KCL) (Kundur, Balu, and Lauby 1994). Given an admittance matrix Y, a vector of voltages V, and a vector of current injections I, the KCL law (in matrix form) states that:

$$I = YV$$

Since the current injections will be zero for non-generator buses, we can use Gaussian elimination to delete corresponding rows from the system of equations. This will allow us to generate a reduced Y-bus matrix representing the network. Following the Gaussian elimination, we obtain the following set of equations allowing to remove a non-generator bus n from the network.

$$\forall i, j \in N - \{n\} : Y_{ij}^{'} = Y_{ij} - \frac{Y_{in} Y_{nj}}{Y_{nn}}$$

Stability Enhancement Routine

Figure 4 presents our overall approach to power restoration with transient stability. Step 1 solves the ROP to obtain a restoration plan defined as a sequence of steady states. Since this solution may be computed using approximation techniques, it may be necessary to convert the potentially non-AC feasible steady states into AC feasible solutions. A standard load flow procedure can be used in the conversion process. Step 2 extracts the sub-sequence of steady states corresponding to line switchings within an energized island. One can safely ignore lines connecting two islands as this is covered by standard approaches (e.g., monitoring periodic oscillations of the phase angle variables or using a synchronization check relay)(Kundur, Balu, and Lauby 1994). Steps 3 and 4 remove islands which are not related to the line-switching operations and perform a Kron reduction (as discussed in the last section) in order to reduce the size of Model 2. Finally, our optimization model is run in Step 5 to find generator dispatches that ensure transient stability during the switching operations.

Experimental Results

We now present experimental results to demonstrate the benefits of the proposed approach. The experimental results study whether the steady states returned by the ROP are transient-stable under various increasingly strict rotor angle limits and for various reactance values. They also study the magnitude of changes in generator dispatches to ensure transient stability, i.e., how close the transient-stable dispatches p_{i1}^{e} and q_{i1}^{e} and the target dispatches p_i^T and q_i^T are. Our model is implemented in AMPL (Fourer, Gay, and Kernighan 2002; AMPL 2014), and uses Ipopt 3.11 (Wächter and Biegler 2006) as a nonlinear solver. We have tested our model on 5 MATPOWER benchmarks (Zimmerman, Murillo-S andnchez, and Thomas 2011). Note that the ROP was solved using the utilization heuristic (Coffrin, Van Hentenryck, and Bent 2012).

Table 1 presents experimental results for various generator reactances and maximum rotor swing limits $\overline{\delta}$. The table presents aggregate results obtained by summing up all the objective values after solving Model 2 for all of the restoration steps (after filtering and simplification). Hence, each value provides a measure on the degree of dispatch (measured in per-unit (Kundur, Balu, and Lauby 1994)) required to obtain transient stability. In other words, larger values indicate large adjustments in dispatch. The table also indicates, in parentheses, the number of restoration steps in which Model 2 is not solved by IPOPT within one hour. Note that, in general, a rotor angle limit of 90 degrees is considered acceptable, the goal of the current experiment is to push the limits on this bound as far as possible.

The results indicate that the steady-state dispatches are in general transient-stable except for the 39-bus and 57-bus benchmarks. Fortunately, in general, it is possible to ensure transient stability with minor changes in generator dispatches. The changes in dispatch increases with stricter limits on rotor angles. In extreme cases, where the bound is set to 1 degree, IPOPT fails to find transient stable dispatches for the 14-bus, 39-bus, and 57-bus benchmarks. This indicates that adjusting generator dispatch may not be sufficient for an arbitrary degree constraint, but those limits are extremely strict.

Overall, the results show that the proposed approach produces transient-stable restoration plans with minimal adjustments in generator dispatches. These adjustments are however necessary to guarantee transient stability, highlighting the benefits of the proposed approach.

Conclusion

Automated power restoration is a key component in future smart grids. Finding the optimal restoration plan is only one step towards system recovery. Guaranteeing transient stability of all switching operations is equally critical. In this paper, we propose an algorithmic procedure for validating

Table 1: Generator total dispatch adjustments on 5 Matpower benchmarks ($T = 400, \Delta = 0.005$)

	6 Bus				14 Bus				30 Bus			
	Maximum Rotor Swing				Maximum Rotor Swing				Maximum Rotor Swing			
Gen. Reactance	90	40	10	1	90	40	10	1	90	40	10	1
0.02	0.00002	0.00002	0.00002	0.17967	0.00000	0.00000	0.00000	6.53902	0.00004	0.00003	0.00004	2.91547
0.06	0.00003	0.00003	0.00003	0.10593	0.00000	0.00000	0.00000	8.67193	0.00004	0.00003	0.00004	3.07084
0.10	0.00003	0.00002	0.00002	0.00843	0.00000	0.00000	0.00000	8.18311	0.00004	0.00004	0.00004	3.76922
0.14	0.00002	0.00003	0.00002	0.30335	0.00000	0.00000	0.00000	6.09923 (1)	0.00004	0.00005	0.00004	4.17696
0.20	0.00003	0.00003	0.00003	1.01248	0.00000	0.00000	0.00000	3.66457 (3)	0.00004	0.00004	0.00004	4.07680
	39 Bus				57 Bus							
	Maximum Rotor Swing				Maximum Rotor Swing							
Gen. Reactance	90	40	10	1	90	40	10	1				
0.02	0.00001	0.00001	20.52951	7.95879 (6)	0.00000	0.00000	0.66260	131.39311				
0.06	0.00001	0.00001	81.80927 (1)	0.00002 (7)	0.00000	0.00000	1.36766	1.12417 (21)				
0.10	0.42052	0.28436 (1)	78.24781	48.13747 (5)	0.23766	0.23766	39.33088	1.14312 (21)				
0.14	0.49691	0.49698	60.35684	41.99449 (5)	0.68299	0.67222 (1)	73.84672	1.26118 (21)				
0.20	0.00002	0.00002	34.26957	69.35875 (3)	0.83704	0.83744	120.01411 (1)	1.41655 (21)				

and enhancing transient stability of such restoration plans. Stability is measured using the nonlinear dynamic swing equations corresponding to rotor angles of power generators. Computational scalability is then improved using preprocessing and reduction routines. Numerical experiments are carried on state-of-the-art benchmarks, assessing the scalability of the approach and validating the stability results.

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