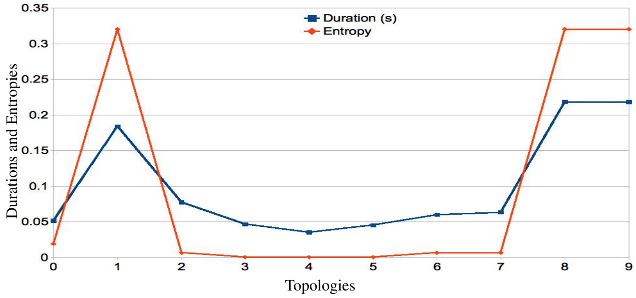


Figure 2: $H(\pi_k)$ and $\Delta(\pi_k)$ for $\pi_{k\in[0,9]} \in \{\mathcal{U}, \mathcal{D}, \mathcal{PL}\}$

sub-utility of factor $\Phi_j$, we distinguish two types of messages: from issue $i_k$ to $\Phi_j$ (1a), and from $\Phi_j$ to issue $i_k$ (1b).

$$\mu_{i_k \to \Phi_j}(i_k) = \sum_{\Phi_{j'} \in \mathcal{N}(i_k) \setminus \Phi_j} \mu_{\Phi_{j'} \to i_k}(i_k) \tag{1a}$$

$$\mu_{\Phi_j \to i_k}(i_k) = \max_{i_1} \ldots \max_{i_{k'} \neq k} \ldots \max_{i_n} \Bigg[ \phi_j(i_1, \ldots, i_k, \ldots, i_n) + \sum_{i_{k'} \in \mathcal{N}(\Phi_j) \setminus i_k} \mu_{i_{k'} \to \Phi_j}(i_k) \Bigg] \tag{1b}$$

We start from the leaves of the hypergraph, *i.e.*, the issues. At $t = 0$, the content of the initial messages is defined according to $\mu_{i_k \to \Phi_j}(i_k) = 0$ and $\mu_{\Phi_j \to i_k}(i_k) = \phi'_j(i_k)$, with $\phi'_j(i_k)$ being the sub-utility of $i_k$ in the factor $\Phi_j$.

**Complexity.** We identify the parameters that could potentially affect the complexity of the utility space and thus the probability of finding optimal contract(s). We distinguish the constraint-issue connectivity, $\pi$. It refers to the number $\varphi_j$ of issues involved in each $c_j$. Thus, a utility space (resp. hypergraph) profile is a tuple of the form $(n, m, \pi)$. This parametrization will be used in the study of the complexity. If we take the distribution $\pi$ to be a propagation topology, or hypergraph traversal, we can assess the corresponding complexity using information entropy (Hadfi and Ito 2013).

## 3 Experimental Results

When exploring $G$, it is possible to adopt several propagation topologies that differ in performance. Other than the initial topology defined by $\pi$, we can adopt other topologies as propagation strategies. The goal is to find the topology that minimizes the exploration cost when searching for optimal contracts. Let us consider the strategies $\{\pi_k\}_{i=0}^{9}$ that are either uniform ($\mathcal{U}$), deterministic ($\mathcal{D}$) or power-law ($\mathcal{PL}$). Their corresponding durations and complexities are illustrated in Figure 2. Both entropy ($H$) and duration ($\Delta$) fluctuate similarly when describing the complexity of the underlying strategy. Besides, $\mathcal{U}$ is the most complex topology since it possesses the highest entropy and duration, as opposed to $\mathcal{D}$ and $\mathcal{PL}$.

Firstly, let us consider the general propagation mechanism (AsynchMP) that naively implements (1), and show how it outperforms the Simulated Annealing (SA) approach in (Ito, Hattori, and Klein 2007) for optimal contracts finding. For the profile $(40, [20, \ldots, 100], \pi(\Phi_j) \leq 5 \forall j)$, Figure 3 shows the resulting difference in the performances.

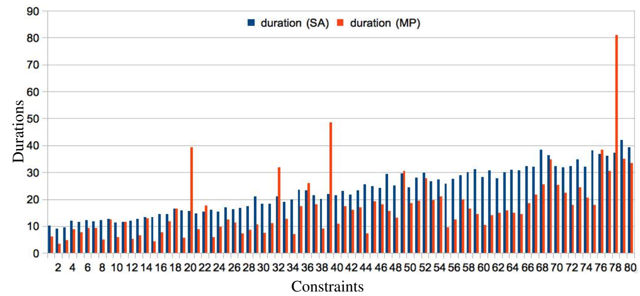


Figure 3: AsynchMP vs. SA

Now, we propose to improve AsynchMP by adopting a $\mathcal{PL}$ topology as a new propagation strategy, namely AsynchMPi, that prioritizes high degree constraints' nodes. To this end, we evaluate both AsynchMP and AsynchMPi for profiles $\{(100, 100, \pi_k)\}_{k\in[1,5]}$ with $\pi_k(\Phi_j) \leq p_k$, $p_k \in [5, 10]$ and $k \in [1, 100]$. Figure 4 shows that restricting the message passing to high degree nodes results in a drastic decrease in the duration of the search time. For small connectivity values ($\pi_1 = \pi_2$) the search process takes approximately the same amount of time despite the large number of issues and constraints. In fact, assessing the complexity of a utility space (resp. utility hypergrpah) must take into consideration the connectivity function $\pi$. In this sense, neither the dimension $n$ nor the number of constraints $m$ could objectively reflect this complexity unless we consider $\pi$.

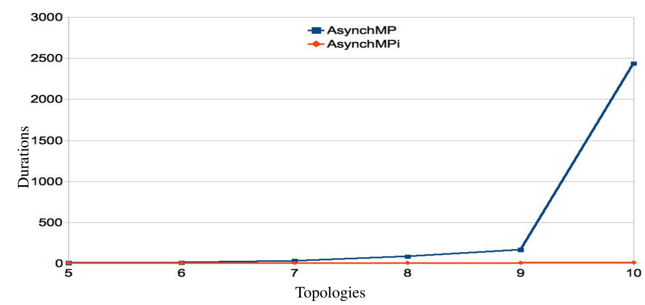


Figure 4: Duration when high degree nodes are used

## References

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